Order Number 8726134

Precise measurement of the asymmetry parameter δ in muon decay

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University of California, Berkeley, 1987

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Precise Measurement of the Asymmetry Parameter δ in Muon Decay

By

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B.S. (University of California) 1982

DISSERTATION

Submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

PHYSICS

in the

GRADUATE DIVISION

OF THE

UNIVERSITY OF CALIFORNIA, BERKELEY

Approved: Mark Strovink .... April 21, 1987
Chairman .... Date

Kenneth M. Crowe 4/24/87

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DOCTORAL DEGREE CONFERRED
MAY 15, 1987

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Precise Measurement of the Asymmetry Parameter δ in Muon Decay

Brian Konrad Elliott Balke

ABSTRACT

Highly polarized μ^+ from the surface muon beam at TRIUMF have been analyzed by means of the muon spin rotation technique to determine the parity-violating muon decay asymmetry as a function of the daughter positron's momentum. The primary result is a determination of the muon decay parameter δ to high precision ($\delta = 0.7486 \pm 0.0038$ combined error), consistent with the prediction $\delta = 0.75$ of the standard model of the weak interactions. The implications of this measurement for generalized four-fermion contact interaction models of muon decay are discussed. The data are also used to constrain the parameters in certain left-right-symmetric and supersymmetric extensions of the standard model, and to limit the existence of lepton-number-violating scalar decays of the muon ($\mu \to e\sigma$, where $m(\sigma) < 80$ MeV/c^2).

Mark Strovink

This is dedicated to the ones I love.

ACKNOWLEDGEMENTS

I would like to recognize the members of the TRIUMF E247 collaboration: Drs. Alex Jodidio, Chris Oram and David Stoker, and Profs. John Carr, Bruno Gobbi, Kirk Shinsky (tragically deceased), Herbert Steiner, Mark Strovink and Robert Tripp. Without their contributions, this experiment would not have been possible. Of particular significance were the efforts and advice of Alex Jodidio and David Stoker, upon whose work the analysis described here was based. I would also like to express my respect and admiration for the critical expertise and Physical acumen of my adviser, Prof. Strovink. His example manifests itself throughout this work.

Significant contributions to the work of the collaboration as a whole should also be recognized. We are indebted to the entire TRIUMF management and staff for their strong support of this experiment. In its early stages we benefited from discussions with J. Brewer, R. Cahn, K. Crowe, K. Halbach and W. Wenzel, and from the technical contributions of C. Covey, R. Fuzesy, F. Goozen, P. Harding, M. Morrison, and P. Robrish. This research was supported in part by the U.S. Department of Energy through contracts Nos. DE-AC03-76SF00098 and AC02-ER02289.

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I. INTRODUCTION

Since its formulation more than a decade ago, the standard $SU(2)_L \times U(1)$ model of the electro-weak interactions[1] has been unfailingly upheld by experiment. Among the tests of the theory, muon decay has proven to be a very sensitive probe into the structure of the purely leptonic weak interactions, despite the fact that much of the decay information is carried away by the two final-state neutrinos. In the most comprehensive generalization with massless neutrinos of the standard model description of muon decay, only ten combinations of the nineteen possible parameters can be measured[2]. Determining the accessible parameters accurately is therefore critical if the generalized theory is to be constrained. The recent advent of highly polarized, high-flux "surface" muon beams has opened new possibilities, allowing high-precision measurements of the parameters which characterize parity violation. This potential has recently been exploited in a search for right-handed currents in muon decay, through a measurement of the degree of parity violation at the decay endpoint[3]. In the work reported here, the surface muon technique is used in a precise measurement of the parity violating decay asymmetry as a function of the positron momentum.

Our experiment improves on the last reported measurement of this type[4] with a two-fold increase in both statistics and muon polarization. However, as we sought to cover the same positron momentum range, the small momentum acceptance of our spectrometer required a wide range of field settings and a demanding calibration procedure. Therefore, in comparison to Ref. 4, this experiment is more sensitive to systematic error, though still achieving more than a factor of two improvement in precision.

In terms of the general muon decay theory, our asymmetry measurements are directly interpreted as a measurement of the decay parameter δ. These measurements are also used to set limits on certain extensions of the standard model, including supersymmetric theories with light sneutrinos and left-right symmetric models with massive right-handed neutrinos. Lastly, the positron energy spectrum from unpolarized muon decay is searched for evidence of lepton-number violating scalar decays, $\mu \rightarrow e\sigma$.

We begin with a discussion of muon decay and the muon spin rotation (μSR) experimental technique (Sec. II). Sections III and IV describe the apparatus and the event reconstruction in some detail. In Section V we discuss the momentum calibration. Section VI covers the asymmetry fits and corrections. The last two sections discuss the implications of the results: Sec. VII in terms of δ, and Sec. VIII in terms of the alternative physics models.

II. EXPERIMENTAL OVERVIEW

II.A Muon Decay

The experiment was designed to measure the parity-violating component of the stopped μ^+ decay rate as a function of the decay positron's momentum. The four-fermion[5] tree-level muon decay rate (Fig. 1a) is

$$\frac{d^2\Gamma}{dx\, d\cos\theta} \propto (x^2 - x_0^2)^{1/2} \{ 6x(1-x) + \frac{4}{3}\rho(4x^2-3x-x_0^2) + 6\eta x_0(1-x) \qquad \text{(II.1)}$$
$$+ \xi\cos\theta\, (x^2 - x_0^2)^{1/2} [2(1-x) + \frac{4}{3}\delta(4x-3-m_e x_0/m_\mu)] \},$$

where θ is the angle between the μ^+ spin direction and the outgoing e^+ momentum, $x = E_e/E_e(\max)$, $x_0 = m_e/E_e(\max)$, and $E_e(\max) = (m_\mu^2-m_e^2)/2m_\mu = 52.83$ MeV is the maximum positron energy. The term containing ξ is parity violating. Since Eq. (II.1) integrates over all neutrino variables, sums over positron spins and factors out the overall decay rate, it exhibits only four of the ten measurable muon decay parameters[2]. Experimental[6] and standard-model ("V-A") values of these parameters are shown in Table 1.

Several modifitions to (II.1) were made in our analysis. As shown in Fig. 2, internal radiative corrections[7] (Fig. 1b) have a percent-level effect on the decay rate. These corrections have been included in the analysis, although for brevity's sake not in (II.1) nor in subsequent equations. Similarly, bremsstrahlung[8,9] and Bhabha[8] interactions (Fig. 1c) of the positrons with material in the apparatus alter the reconstructed spectrum in the same sense, and by roughly the same magnitude, as the internal radiative corrections. These external radiative corrections are discussed in Sec. VI.C and App. A. Next, in observing an ensemble, we must average over the polarization of the initial state muons. Then, in (II.1), ξ becomes ξP_μ, where P_μ is the mean muon polarization along the axis of choice, and θ becomes the angle between that axis and the outgoing positron direction. Lastly, in the equations below (though not in the analysis) we neglect m_e with respect to $E_e(max)$ and m_μ. The decay rate is then

$$\frac{d^2\Gamma}{dx\, d\cos\theta} \propto x^2 \{ 6(1-x) + \frac{4}{3}\rho(4x-3) + \xi P_\mu \cos\theta \, [2(1-x) + \frac{4}{3}\delta(4x-3)] \}. \quad \text{(II.2)}$$

II.B Experimental Method

To maximize the experimental sensitivity to parity violation, we used highly polarized "surface" muons from the M13 channel at the TRIUMF cyclotron[10] (Fig. 3). Muons from pion decay at rest were transported through an anti-symmetric channel and focused onto one of two aluminum stopping targets. Apart from scattering and possible small effects of right-handed currents, the incoming μ^+ spin direction was exactly opposite the μ^+ momentum direction, which was measured for every event. The muons passed through ≈60 mg/cm² of material before the target, and ≈111 mg/cm² within it before coming to rest. They were slightly depolarized by interactions with electrons while stopping, and afterwards via spin coupling to the magnetic moments of the aluminum nuclei.

As in a classic μSR experiment, the stopped muon spin was precessed and the decay rate measured as a function of time. Figure 4 shows the apparatus. Since the spin-precession field was applied perpendicular to the beam axis, the expected time dependence was an exponential with sinusoidal modulation ($\cos\theta \rightarrow \cos(\omega t+\theta_0)$ in Eq. II.2). Two different field strengths yielded either 9.6 MHz ("fast") or 5.9 MHz ("slow") precession frequencies. The muon spin direction at the instant of decay was known from the incoming muon direction, the precessing field strength, and the time interval between the arrival of the muon and the detection of the positron ("muon decay time").

Decay positrons were tracked and momentum-analyzed downstream of the target. The track was sampled by proportional and drift chambers as it traced a half turn in a solenoidal magnetic field. The solenoid focused the positron into a cylindrical dipole spectrometer, where it was bent by roughly 98°. Energy loss and straggling occurred in $\approx$240 mg/cm^2 of material between the decay point and the spectrometer, which was evacuated to reduce the effect of Coulomb scattering on the momentum resolution. Highly redundant track measurements before and after the spectrometer were used to determine the positron momentum in an accepted range of ±20% around the momentum setting Y_S. To cover the positron momentum range in $y = p_e/p_e(\max)$ from 0.36 to 1.00, data were taken at six different values of the central field $B_S = B_{S1}\Phi_S$, with B_{S1} = 3186.7 G and Φ_S = 0.42, 0.50, 0.60, 0.72, 0.86 or 1.00 (Note: "Φ" denotes the magnet field strength relative to a reference value; upper case "Y", referred to as the magnet setting, denotes the "characteristic" momentum for particles transmitted at a given Φ; and lower case "y" denotes a particle momentum). The solenoid field was scaled with the spectrometer field to maximize the total downstream angular acceptance.

Two properties of the spectrometer made necessary an extensive calibration procedure. First, the large fringe field of the dipole magnet resulted in substantial errors in the positron momentum reconstructed using the first-order magnet optics. Corrections for the fringe field effects were formulated using the endpoint of the muon decay spectrum. This reference point made possible a precise ordering of events in y at a given spectrometer setting Y_s, though there was no guarantee that the momenta found were correct. Secondly, non-scaling of the central and fringe fields meant that Y_s did not scale simply with the central field B_s, and that the final adjustments to the previous momentum determination varied non-trivially with the spectrometer setting. An absolute calibration was made using the π-μ decay point and the μ-e decay endpoint as benchmarks, and the M13 beamline as a momentum source. Procedural complications and inconsistencies in this part of the calibration produced the dominant systematic error.

In addition to the μSR and calibration data, data were collected at all six spectrometer settings with a large (0.3T) spin-holding field applied parallel to the beam axis in the target region. These "parallel" data had an exponential muon decay time spectrum unmodulated by muon spin rotation, and were used to set decay time cuts and to calibrate the muon decay time clocks. Lastly, data were taken in many special runs to position the wire chambers and to check stopping range and energy loss calculations. A total of 3.0×10^7 spin-precessed triggers, 3.6×10^6 spin-held triggers, and 4.8×10^6 momentum calibration events were collected in a single three-week run. These data were written onto four hundred 1600 BPI tapes. Analysis was carried out on the computer facilities at Lawrence Berkeley Laboratory, Northwestern University, and the University of Colorado at Boulder.

II.C μSR Analysis

Taking the initial (pre-precession) "axis of choice" to be opposite the incoming muon's direction, (II.2) implies the instantaneous μSR rate,

$$\frac{d^4R}{dx\ dt\ d\Omega_e\ d\Omega_\mu} \propto e^{-t/\tau_\mu} A_e(x,\Omega_e)\ A_\mu(\Omega_\mu)\ x^2\ \{\ 6(1-x) + \frac{4}{3}\rho(4x-3)$$
$$- \xi P_\mu(t)\ \hat{p}_e\ \bar{O}(\omega t)\ \hat{p}_\mu\ [2(1-x) + \frac{4}{3}\delta(4x-3)]\ \},$$

where t is the interval after the muon arrival and τ_μ is the muon decay lifetime; Ω_e and Ω_μ represent the positron and muon direction variables relative to the beam axis; $\hat{p}_e$, $\hat{p}_\mu$ are the corresponding positron and muon unit direction vectors; A_e and A_μ are the positron and muon acceptance functions, normalized to unity at all x when integrated over the angular variables; $\bar{O}$ is a matrix descibing the muon spin rotation; ω is the muon spin precession frequency; and P_μ is the muon polarization, including spin relaxation effects in the target. Integrating over angular variables:

$$\frac{d^2R}{dx\ dt} \propto e^{-t/\tau_\mu}\ x^2\ \{\ 6(1-x)+\frac{4}{3}\rho(4x-3)\ -\ \xi P_\mu[2(1-x)+\frac{4}{3}\delta(4x-3)]$$
$$\times \int d\Omega_e A_e(x,\Omega_e)\hat{p}_e\ \ \bar{O}(\omega t)\ \int d\Omega_\mu A_\mu(\Omega_\mu)\hat{p}_\mu\ \}\ . \qquad \text{(II.3)}$$

For an isotropic decay, $A_e(x,\Omega_e)$ and its moments (such as the first integral above) can be determined from the measured distribution of decay particles. In a μSR experiment, such a sample is isolated by integrating the signal over a decay time interval $[t_1,t_2]$ such that

$$\int_{t_1}^{t_2} dt\ e^{-t/\tau_\mu}\ \bar{O}(\omega t) \int d\Omega_\mu\ A_\mu(\Omega_\mu)\ \hat{p}_\mu = 0. \qquad \text{(II.4)}$$

This requires that the average muon direction lies in the plane of the spin precession. If so, the anisotropic term in (II.3) averages to zero, and in the large statistics limit:

$$\int d\Omega_e A_e(x,\Omega_e)\hat{p}_e = \langle\hat{p}_e(x)\rangle_{[t_1,t_2]} \quad , \qquad \text{(II.5)}$$

where the right-hand side is the average direction of the decay positrons detected in the interval $[t_1,t_2]$. Similarly, if the average positron direction is nearly in the plane of precession, for that same time interval

$$\int d\Omega_\mu A_\mu(\Omega_\mu)\hat{p}_\mu = \langle\hat{p}_\mu\rangle_{[t_1,t_2]} \quad . \qquad \text{(II.6)}$$

For our apparatus both the muon and positron directions lay very nearly in the precession plane, and (II.5) and (II.6) were excellent approximations.

As no absolute determination of the rate was made, the final fit was to the form (using (II.5) and (II.6))

$$\frac{d^2R}{dx\ dt} = N(x)e^{-t/\tau_\mu}\{1 - M(x)\frac{P_\mu(t)}{P_\mu(0)}\langle\hat{p}_e(x)\rangle_{[t_1,t_2]}\bar{O}(\omega t)\langle\hat{p}_\mu\rangle_{[t_1,t_2]}\}. \qquad \text{(II.7)}$$

Here N(x) is the spectrum normalization, and

$$M(x) = \xi P_\mu(0) \frac{2(1-x) + \frac{4}{3}\delta\ (4x-3) + \alpha\, h(x)}{6(1-x) + \frac{4}{3}\rho\ (4x-3) + \alpha\, g(x)} \qquad \text{(II.8)}$$

is the muon decay asymmetry, with h and g describing radiative corrections[7,8]. As ρ is already well known[6], measuring M(x) primarily determines δ. In principle, a precise measurement of the zero point x_Z of the asymmetry, for which $M(x_Z) = 0$, would determine δ. At this point

$$\left.\frac{d\delta}{dx}\right|_{x_Z} = \frac{3}{2[4x_Z-3]^2} \times \frac{1}{2}\ . \qquad \text{(II.9)}$$

The final factor of 1/2 gives the reduction in sensitivity found empirically when fitting the data simultaneously for ξP_μ, as was necessary in this experiment. As $x_Z \approx 0.5$, one estimates that a determination of δ to ± 0.002 requires a precision approaching 0.002 in the momentum calibration.

We can also estimate the necessary precision in calculating the internal and external radiative corrections. For positrons which travel straight down the beam axis, Fig. 5 shows the motion of the zero point as the radiative corrections are applied. For positrons moving through the solenoid with finite at an angle with the axis, the effect of the external radiative corrections is less than that shown: Bhabha and bremsstrahlung energy loss caused a discontinuity in the track curvature, and such events were susceptible to track quality cuts. In a detailed Monte Carlo simulation of the experiment, a 30% reduction in the effects of the external radiative corrections on M(x) was found. Conservatively, then, Fig. 5 implies that an uncertainty of 10% in the radiative corrections can be allowed for a measurement of δ to 0.002.

The next-order QED internal radiative corrections to muon decay have not been completely calculated, but partial results[11] indicate that these will be at most 3-5% of the order α contributions. The uncertainty in the external radiative corrections[9] is also estimated to be 3-5%. These are safely within the 10% limit.

III. APPARATUS

III.A M13 Beamline

As described above, the M13 beamline (Fig. 3) served both as a source of highly polarized muons and as a spectrometer for our momentum calibration. With regard to the first use, a detailed description can be found in Ref. 3, which we summarize here. With the TRIUMF cyclotron delivering ≈130μA of 500 MeV protons at the 2mm graphite production target at position 1AT1, 15,000 Hz of muons impinged on our stopping target at the final focus F3. The beamline was tuned to accept a 1% FWHM momentum bite at 29.5 MeV/c, thus selecting the muons from pion decay at rest closest to the surface of the production target. This sample of muons, 98% of the total, was highly polarized. The remaining 2%, "cloud" muons from pion decay in flight, were nearly unpolarized. The small acceptance of the beamline (0.3 msr from a 1 cm spot at 1AT1) allowed the second sample to be eliminated by cuts on the muon arrival time at F3 with respect to the arrival time of the proton pulse at 1AT1. A few ns into the 43 ns deadtime between pulses, nearly all pions were either at rest in the graphite or had passed out of the volume accepted by the beamline. After subtracting their beamline transit time, the cloud muons were therefore prompt with respect to the proton signal. After their elimination, the final sample of muons was more than 99.6% polarized[3].

Evaluation of the M13 beamline as a spectrometer requires a more detailed understanding of its design. The production target was viewed at 135° by the beamline (Fig. 3). Positively-charged particles emanating from the target (Fig. 6) were guided to a momentum-dispersed focus at F1 by the combined action of the quadrupole doublet Q1-Q2 and the bending magnet B1. The horizontal and vertical jaws just after Q2 (opened to 12.0 and 2.0 cm, respectively) defined the accepted solid angle. The bend angle in B1 was roughly 60°, and scaled with the vertical field integral and horizontal component of the particle momentum y_h as

$$\theta \propto \int B_z dl \;/\; y_h. \tag{III.1}$$

Therefore, for a point source at 1AT1, every point along the momentum-selecting slit S1 at F1 would receive a different momentum, with a measured dispersion of 1.22cm/(%$\Delta p/p$). In reality, the source had a finite size, which varied with the particle species, and each point in S1 received a finite range of momenta. Thus the B1 setting, S1 position and width, and source size completely determined the momentum distribution of particles arriving at the final focus F3, given that the rest of the beamline was properly tuned to match.

The aperture chosen for S1 was dependent upon the experimental situation. For main data taking it was set to 1.2 cm, giving $\Delta p/p$=1.0% FWHM with a flux well matched to our trigger efficiency. For calibration runs the aperture was set to 0.6 cm, yielding $\Delta p/p$ = 0.6% FWHM. Using a still smaller S1 aperture would have reduced the beam flux without appricianly narrowing the momentum spread, due to the finite size of the source. A sample positron spectrum is shown in Fig. 7 (the curve is described in Sec. IV.2).

As the particles continued past F1, they were focused at F2 by the quadrupole triplet Q3-Q4-Q5. Slit S2 at F2 served only to eliminate off-momentum particles that had scattered into the beamline at F1, although if closed more tightly it could have defined the momentum bite. Finally, the B2-Q6-Q7 combination focused the spot at F2 to the final focus F3, with B2 undoing the momentum dispersion at F2. We see by the symmetry of the beamline that the total magnification was unity, and since there were three image inversions (once at every focus) for those particles accepted by S1 the spatial distributions of the source at 1AT1 and the final focus at F3 were simply mirror images. Beam envelopes at F3 are shown in Fig. 8. The cartesian coordinates are u in the horizontal direction (in the plane of the bend), v vertical, and w along the beam axis, with the notation u' = du/dw, etc. The effect of the jaws at Q2 in defining the phase space is clear.

Several chromatic effects were expected in the final particle distribution at F3, two of which were large enough to be readily apparent in the data. Particles with finite vertical slope in the benders were biased towards larger momentum (Fig. 9a), as the bend angle depended only on the horizontal projection of their momentum (III.1). Particles that appeared on the outside of the bend at F3 (negative u in Fig. 9b) had higher momentum than particles on the inside, consistent with the smaller bend in reaching F3 from F2. This effect was also clearly seen. A surprise, though, was the step at low u. We attribute this to the presence of approximately 250 mg/cm^2 of extra, unexpected material upstream of P2 through which a small portion of the beam passed. The asymmetry in the muon decay vertex distribution (Fig. 10) supports this conclusion: muons at low u were stopped before P2 by the extra material, and so did not activate the event trigger.

There were several known sources of deviation from ideal behavior in the beamline. First, the computer-assisted beamline monitor allowed the quadrupole current settings to be changed only in discrete steps of 0.5–1% of the Y=1.00 setting. Second, the benders induced a hysteresis-dependent dipole component in their nearest neighbor quadrupoles (Q2 and Q6). Finally, particle flux was usually maximized for off-center positioning of the slits S2, with weak dependence on the beamline setting Y_b. Particle trajectories were therefore not guaranteed to be symmetric about Q4. This last effect helped to prompt a complete re-alignment of the beamline shortly after the experiment was completed.

The beamline was well monitored. A radiation protection monitor centered the proton beam on 1AT1 to within 1mm. Hall probes mounted on the pole faces were used to scale and monitor the quadrupoles. High sensitivity Hall probes[12] were positioned in the central, shoulder and fringe field regions of B1 and B2. Furthermore, the absolute field in each bending magnet was measured in the range from 600 to 1800 G using NMR probes with a sensitivity of 0.1 G. The combined precision was such that the scaling of the bender field integrals with the central field strength was monitored to within 0.1%.

III.B μSR Apparatus

The detector was triply segmented, with each segment devoted to a specific purpose in the experiment. The μ-arm tracked and timed the arrival of particles from M13, and differentiated positrons from the heavier beam species. The e-arm accurately measured positron tracks downstream of the target. The spectrometer measured the positron momentum. Considered in turn below, each segment consisted of a set of detectors built around a magnet. Reference should be made throughout to Fig. 4, the diagram of the apparatus. Further details may be found in Ref. 13.

III.B.1 μ-Arm

The μ-arm detectors, in the region upstream of the stopping target, served to time and track the arrival of beam particles. Low-mass detectors were required so that the low energy muons would come to rest not in the detectors themselves, but rather in the non-depolarizing aluminum stopping target.

Proportional chambers P1 and P2 provided the tracking function. Each had one horizontal and one vertical wire plane, with 2 mm separation between the anode wires. The chambers were separated by 8.9 cm, providing angular resolution of better than 22 mrad. The cathode planes, made of aluminized mylar sheets, provided signals that were used in forming the event trigger. For this purpose, at low flux P1 was 90% and P2 95% efficient for positrons, and nearly 100% efficient for muons.

To eliminate beam halo, veto counter V1 (not shown in Fig. 4) was placed just upstream of S1. The scintillator had a 1.5" diameter hole centered on the beam axis. As were all the scintillators, it was viewed by photomultipliers from both left and right. The combined output was used as a veto in the event trigger.

Timing information was provided by scintillator S1 and proportional chamber A (identical to P2). The signal from S1 gated the muon decay clock. Cathode signals from the A chamber were used (with 99.5% efficiency) to signal the presence of extra beam particles after the decay clock had been started.

The second component of the μ-arm segment was the aluminum stopping target. The calculated residual range for beam muons was 111.0±6.9 mg/cm² into the aluminum[14], where the standard deviation is dominated by the statistical variation in the muon range[15]. Two targets were used, of thicknesses 151.0 and 181.0 mg/cm², providing five and ten standard deviations of extra stopping power. They were kept thin to minimize the energy straggling of the decay positrons. The thicknesses were also such that the event trigger could cleanly differentiate beam muons and pions, which came to rest in the target, from beam positrons, which suffered less than $\Delta x = 0.01$ of energy loss. Most importantly, though, the targets were non-depolarizing. The lattice structure of aluminum is such that the muon spin couples only in an average fashion with the spins of the nearby conduction electrons. As a result, the spin-spin interactions which would most rapidly depolarize the muons are almost completely absent. Residual depolarization due to coupling with the nuclear spins is briefly discussed in Sec. VI.B and VII.E.

The final component of the μ-arm was the polarimeter magnet. Two sets of coils in the target region provided a longitudinal spin-holding field and vertical μSR field. The longitudinal field (for which a pair of cylindrical coils are indicated in Fig. 4) was concentrated by pole tips. The field strength was 0.3T, with transverse component less than

0.6%. The spin-precessing field, applied by a pair of horizontal coils mounted above and below the target, was measured to be uniform to better than 0.4%, with a longitudinal component predicted by computer simulation to be less than 1%. The perpendicular field was either 70 G ("slow" precession) or 110 G ("fast").

The magnet setting and field configurations were changed many times during the course of the experiment. To ensure reproducibility, the current in the coils was monitored to 0.5% by shunt voltage readings at the power supply. Also, a longitudinal remanent field of nearly 40G remained whenever the field was switched from the spin-holding to the μSR configuration. Using Hall probes of the concentrator type[11] as monitors, this field was zeroed to within 0.1 G by adjusting a small current passed through the longitudinal field coils.

III.B.2 e-Arm

The e-arm detectors, lying between the target and the end of the solenoid, served to track and time positrons from both muon decay and the beamline. Emphasis was placed on high spatial and angular accuracy in extrapolation back to the target.

Proportional chamber P3 (identical to P2) provided a rough position measurement near the target, and its cathode signals were used in the event trigger. Drift chambers D1 and D2 were placed as close to the target as possible (29 cm and 50 cm, respectively) given the constraints imposed by the magnet design. D1 was cone shaped, and, as in all the drift chambers, had alternating planes offset by half the sense wire spacing to aid in resolving left-right hit ambiguities. The two vertical wire planes of D1, nearest the target, provided 150 μm resolution. The two horizontal wire planes, for reasons not well understood, had only 800 μm resolution. D2 was cylindrical, with two horizontal and two vertical planes all achieving 160 μm resolution. As in the μ-arm, the e-arm chambers were very efficient: more than 95% of all tracks recorded at least nine of ten possible hits.

The two scintillators S2 and V2 corresponded in purpose to S1 and V1 in the μ-arm. S2 timed the positron signals, gating the drift chamber electronics as well as stopping the decay time clock. V2 vetoed off-axis positrons headed towards the longitudinal field magnet pole tips.

The solenoid magnet was 50 cm long with a bore of 11 cm radius. Its field was designed to maximize the downstream acceptance by focusing particles of momentum equal to the spectrometer setting Y_s to the center of the spectrometer. An acceptance of more than 250 msr was achieved. The peak solenoid field was nearly 10 kG, and the total lens strength was 0.5 T-m at the highest spectrometer setting. The field shape was only partly mapped, but computer simulations gave an approximate shape which was shown in track reconstruction studies to be sufficiently accurate for our event reconstruction (Sec. IV.B). As with the μ-arm magnets, shunt readout at the power supplies ensured field reproducibility - to within 0.02% near the chambers, and 0.4% downstream.

III.B.3 Spectrometer

The spectrometer drift chambers D3 and D4 provided high precision track measurements for use in the momentum reconstruction. The chambers were mounted on the spectrometer vacuum box, each facing the magnet center at a radius of roughly 120 cm. The angle between the two chamber axes was the bend angle for particles with momentum equal to the spectrometer setting. D3 was a stack of three 11 inch diameter cylindrical chambers similar to D2. Tracks were sampled six times in each of the transverse coordinates, with a hit resolution of 160 μm. D4 was a single rectangular chamber, with a sensitive area 30 inches wide by 23 inches high. Six planes of vertical sense wires and four planes of horizontal wires all achieved resolutions of 250 μm. Both chambers were better than 97% efficient in all planes.

Behind D4 was a wall of three 39 inch wide by 8 inch high scintillators (S3). These provided a fast signal to the trigger logic to indicate that a positron had successfully traversed the spectrometer. The vertical segmentation and the photomultiplier left-right timing difference allowed a check on the track information registered in D4.

The spectrometer magnet was a cylindrically symmetric dipole with 37 inch diameter pole faces and a 14.5 inch gap. At its highest setting (Y_s=1.00) the central field was 0.32 T. Field measurements were made in the magnet midplane using an NMR probe in the central field and high-sensitivity Hall probes[12] at the central, 70% and 30% field points (Fig. 11). The measurements were sufficient to monitor the field integral for typical tracks to within 0.1%.

The spectrometer was horizontally focusing. For positrons with momentum equal to the spectrometer setting ($y = Y_S$), the focal planes were 104 cm from the magnet center, their normals forming an angle of 98°. To first order, positron momenta were found from the sum of the horizontal coordinates at the focal planes using the measured momentum dispersion of 0.93 cm/(%$\Delta y/y$). Since this first-order determination did not depend on the track angles, the effect on the momentum resolution of Coulomb scattering in the 0.005 inch mylar vacuum box windows was minimized by placing them near the focal planes.

Owing to the large gap between the pole faces (0.4× their diameter), the fringe component of the field was large (Fig. 11). In the midplane, the field was purely vertical, and uniform out to 20 cm. The field weakened only slowly at larger radii, having a residual strength of 10% as far out as twice the pole radius. At large vertical displacement (large z, in the polar coordinate system with origin at the magnet center), a radial field component and vertical field intensification became apparent near the edge of the pole face. By symmetry, it was expected that the radial field strength would behave like a polynomial in odd powers of z, and the vertical field bump in even powers of z. We found empirically that it was sufficient to take $B_r \propto z$ and $\Delta B_z = B_z - B_z(z=0) \propto z^2$. The radial and vertical field bumps are shown in Fig. 11 at $z = 8$ cm, a typical displacement (particles were accepted over ±16 cm).

IV. EVENT RECONSTRUCTION

The event reconstruction proceeded in several stages. The trigger allowed us to differentiate between beam muons, beam positrons, and positrons from muon decay in our target. Simple electronic trigger logic reduced the rate of events written to tape to about 0.1% of the beam flux. The second reconstruction stage sorted the raw timing and chamber information, grouped the hits in segments, and selected the cleanest tracks. The final stage refined the track fits, calculated linkups between the various segments to check the event coherence, and began the momentum reconstruction. Roughly 5% of the events written to tape survived for the asymmetry analysis (Sec. VI).

IV.A The Trigger

Three triggers were required in the experiment, distinguishing μ stops (momentum calibration), μ decays (main data sample), and beam positron "straight throughs" (momentum calibration and chamber alignment). The trigger elements (Fig. 12) were divided into two classes: "upstream" of the target (P1, P2, S1 and V1), and "downstream" (P3, S2, V2 and S3). The logic was built around two signals.

$$\mathrm{BEAM} = \mathrm{P1}\cdot\mathrm{P2}\cdot\mathrm{S1}\cdot\overline{\mathrm{V1}}$$

indicated a fiducially allowed upstream track, while

$$\mathrm{SAGANE} = \mathrm{P3}\cdot\mathrm{S2}\cdot\overline{\mathrm{V2}}\cdot\mathrm{S3}$$

indicated a fiducially allowed downstream track. A beam straight-through was recognized by the simultaneous signal

$$\text{ST THRU} = \text{BEAM}\cdot\text{SAGANE}.$$

μ stops were identified as BEAM signals without any downstream activity:

$$\mu\ \text{STOP} = \text{BEAM}\cdot\overline{\text{P3}}\cdot\overline{\text{S2}}\cdot\overline{\text{V2}}.$$

Each μ STOP trigger opened a 10 μs gate, during which a SAGANE signal without A or V1 activity (DECAY e^+ in Fig. 12) triggered a μ DECAY.

The μ DECAY trigger had two major contaminants. The first, due to multiple muons in the stopping target, was mostly eliminated by an "extra before" flag, which tagged for later rejection events with BEAM signals up to 10 μs before the μ STOP. The second background was from straight throughs detected after a legitimate μ DECAY (the two sets of drift chamber hits could possibly be confused in the reconstruction). These events were identified as "extra afters" by activity detected in the A, P1 or P2 counters during the remainder of the 10 μs μ-decay gate. The effectiveness of these cuts is seen in Fig. 13, for worst-case data in which the spectrometer acceptance overlapped the beam momentum ($y = 0.55$).

A critical component of the trigger was the FAST CLEAR, which allowed a fast reset of the timing electronics when a μ DECAY was vetoed by a straight through, or by a decay positron with momentum outside the spectrometer acceptance. Without this measure, at low spectrometer settings the dead time from unaccepted μ decays would have swamped the trigger. Finally, the BUSY signal disabled the trigger logic while the online analysis processed an event.

IV.B Chamber Calibration and Track Reconstruction

To the extent possible, chamber calibration was accomplished empirically. The drift chamber space-time relationships were first estimated under the assumption that each cell was uniformly illuminated, and then fine tuned for every run during a preliminary stage of the analysis to minimize plane-by-plane the track fit residuals vs. drift time[13]. The wire chambers were aligned transversely using beam positrons collected with the solenoid field off. The tracks were fit to straight lines in the solenoid chambers (P1-D2), in D3 and in D4. The plane positions were adjusted until all residual distributions were centered to within 50 μm.

For muon decay events, four track segments were reconstructed: the muon track in P1 and P2, the positron track in P3-D2 within the solenoid, and tracks in D3 and D4 at the spectrometer entrance and exit. Initially, all were fit to straight lines, with left-right ambiguities in the drift chamber hits resolved locally. After the first stage of the momentum reconstruction (Sec. V.C), the solenoid track was fit to a curve using transport matrices calculated in a first-order optics approximation to the solenoid field[16]. Occasionally the left-right ambiguities in D1 and D2 were resolved improperly in the initial straight-line fit. These errors inflated the curved track chi-squared, but the corresponding effect on the reconstructed positron target angles and position were shown by the Monte Carlo simulation to be negligible.

Many track quality cuts were applied. Muon tracks were required to be unambiguous in all planes except one (of two) in P2, where the best hit could be chosen by comparison with the positron track. Unambiguous positron hits were required in P3, and all events with multi-track positron signatures (twice the expected number of hits) were rejected. Chi-squared cuts were made on all fit tracks. The extrapolated muon and positron tracks were required to join at the target; the transport matrix formalism was used to check the matching of the P3-D2 and D3 tracks in mid-solenoid; the D3 and D4 tracks were matched vertically in mid-spectrometer, and checked for large differences in vertical angle characteristic of pole-face scattering; and agreement was required between hit positions in S3 and D4. Cuts were also made on the reconstructed positron coordinates at all apertures in the apparatus. None of these cuts eliminated more than a few percent of the events, as they were tuned to reject only genuine backgrounds (multiple tracks, scattered positrons, etc.), as opposed to events with small reconstruction inaccuracies. Lastly, the fringes of the muon and positron target angle distributions, where the angular reconstruction was assumed to be poor, were conservatively cut to prevent contamination of the critical averages (II.4) and (II.6). Excluding events with $\cos\theta_\mu < 0.99$ and $\cos\theta_e < 0.975$ (θ the angle with respect to the beam axis) reduced the sample by 30%.

IV.C Relative Momentum Reconstruction

Prior to the spectrometer calibration of Sec. V.B.4, our momentum reconstruction was limited to the determination of the positron momentum y in the spectrometer relative to the spectrometer setting Y_s. We therefore introduce the "relative positron momentum"

$y_r = y/Y_s$. The determination of y_r had several increasingly accurate stages, starting in the first stage with an approximate model of the spectrometer optics. In this model, as described below, events at all field scaling factors Φ_s in the range $0.75<y_r<1.25$ were analyzed as though the spectrometer field was the $\Phi_s=1.00$ field. The accuracy and resolution at this early stage were limited both by the nature of the algorithm and by the precision with which physical parameters such as the field map and the absolute positions of D3 and D4 were known. Even so, the method succeeded in providing a fairly accurate ordering of events by momentum at a given spectrometer setting. It was left to the calibration procedures described in Sec. V to fine-tune the relative ordering of events and to formulate the corrections which would enable a precise conversion of y_r to y.

The essence of the algorithm was the approximate reconstruction of the positron motion projected into the midplane of the spectrometer. The field was divided into three annuli separating the effects of the fringe, the "bump", and central components (Figs. 11,14). The projected radius of curvature was approximated as being constant along each of the five track segments within the annuli, and infinite outside. The vertical slope was taken to be uniform except to account for the effects of the radial field near the fringe-bump interfaces, where the slope was changed to match the track in height along the interior path length.

The reconstruction proceeded in an iterative fashion. Starting with the momentum from first-order optics (Sec. II.B) and assuming typical curvatures, the track was extrapolated forward from D3 to the exit focal plane and backward from D4 to the same point (Fig. 14). The difference Δu in horizontal coordinate was used to re-estimate y_r using the spectrometer dispersion. From the calculated trajectory, a radius of curvature R appropriate to the new momentum was found in each region (i=1-5) using the ϕ_s=1.00 field map of Fig. 11:

$$R_i = \frac{y_r \, cp_e(\max) \, \langle \cos\theta_z \rangle^2{}_i}{0.3 \left(\int_i B_z \langle \cos\theta_z \rangle_i \, ds + \int_i B_r \frac{dz}{ds} \, ds \right) / s_i}$$

with $cp_e(\max)$ equal to 52.83 MeV, B in kG, θ_z the helix angle, and s_i the path length in segment i. For these purposes the field integral in the straight track region was added to that in the outer annulus. Then with the new momentum and radii of curvature, a second comparison Δu could be made. The iteration was continued until the last correction to y_r was less than 0.0005. For most events, convergence was achieved in three or four passes.

This model accounted for all but 1-4% of the 5-15% fringe field effects in the reconstruction. Although the final resolution σ_y was a factor of ten again smaller than that achieved at this stage, the importance of this first step should not be underestimated. The μ-e edge studies (Sec. V.A) used to formulate final corrections to the relative momentum were sensitive only to the high side of the momentum resolution function. While the effect of a symmetrical resolution in y on M(x) (Eq. II.8) is of order $\sigma_y{}^2$ at all x, an asymmetrical resolution acts like momentum straggling and can have a lower-order effect. Our confidence that the finally corrected resolution is highly symmetrical is enhanced by the fact that this initial momentum determination supplied most of the correction needed and was physically based.

V. MOMENTUM CALIBRATION

The momentum calibration had three goals. The first two were the determination to high resolution and accuracy of the relative momentum $y_r = y/Y_s$ introduced in Sec. IV.C. For a precise conversion of y_r to y, the final goal was an accurate calibration of the spectrometer setting Y_s as a function of the spectrometer central field strength. In attaining these goals, two sets of measurements were used. Examination of the endpoint of the momentum spectrom from muon decay enabled improvement of the resolution in y_r by a factor of 10 over that achieved with the model of Sec. IV.C. However, this procedure merely ordered the y_r; it did not ensure that the values obtained were actually correct. Using the π-μ and μ-e decay reference points as benchmarks and the beamline as a momentum source, straight-through positrons were analyzed to determine final corrections to the relative momentum scale and to calibrate the spectrometer.

We provide a brief dictionary of the notation used in the discussion that follows. Upper case 'Y' denotes a magnet setting, and lower case 'y' denotes a particle momentum. All 'y' are absolute ($y=1$ at the muon decay endpoint) except y_r, the relative momentum introduced in Sec. IV.C. 'Φ' denotes a central field scaling factor relative to $B_s = 3186.7$ G in the spectrometer magnet or $B1 = 1555.4$ G in the first beamline bender. For the reference field strengths chosen, Φ and Y were approximately equal. Energy loss in the material of the apparatus, equal to momentum loss for the relativistic positrons, is indicated by Δx. The subscript 'b' refers to the beamline while the subscript 's' refers to the spectrometer. The calibration points in π-μ or μ-e decay are indicated by subscripts '$\pi\mu$' or 'μe'.

V.A Relative Calibration

By varying the spectrometer central field, the $y = 1.00$ edge for spin-precessed data was swept through the spectrometer volume, simulating positrons of differing relative momentum y_r at a constant Φ_S. Nine samples of 1.2×10^5 events were collected, corresponding to relative momenta y_r between 0.84 and 1.17 (Tbl. 2). For each run, the values of y_r obtained from the initial determination discussed in Sec. IV.C were plotted versus various track parameters (field integral, impact parameter b, and mean-squared deviation from the midplane $\langle z^2 \rangle$). Since the phase space of positrons accepted by the spectrometer varied slowly with y_r, up to a scaling factor the momentum distributions near the endpoint (Fig. 15) should have been the same for all values of a parameter. Whenever the edge distribution did show correlations with a parameter, a further ad hoc correction Δy_r was made to y_r as a function of that parameter to remove the correlations. For example, when the parameter was $\langle z^2 \rangle$, this correction took the form $\Delta y_r = f(y_r) \langle z^2 \rangle$ where f is a polynomial of fourth order in y_r. Next, any correlations of y_r with the coordinates measured in chambers D3 and D4 were eliminated using the same procedure. The corrections were smallest (<1%) near y_r=1.00, where the focusing action of the spectrometer was most effective. At low y_r, for which the fringe and bump irregularities had strongest effect, the corrections were larger, but still less than 4%.

Using data in a time range with zero average muon polarization (Eq. II.4), the momentum resolution achieved was determined by fitting the reconstructed spectrum to the radiatively corrected, unpolarized standard model spectrum smeared by a gaussian resolution function. An offset of the endpoint from the expected position and corrections for the spectrometer acceptance were also included in the fit. The final resolutions, scaled by the spectrometer setting, are given in Tbl. 2. They ranged from about 0.1% to 0.2%. Representative fits are shown in Fig. 15, where a preliminary conversion from y_r to the initial positron momentum allows comparison of the reconstructed endpoints to y=1.00.

V.B Absolute Calibration

Although the idea of the absolute calibration was simple, its execution was rather involved. Therefore we outline only the main ideas and measurements here, leaving a precise description of the calibration fit to App. B.

The central assumption of the calibration was that the beamline and spectrometer momentum settings (Y_b and Y_s) were linear in the magnet field strengths:

$$Y_b = m_b\,(B1 - B1_0) \qquad (V.1)$$

$$Y_s = m_s\,(B_s - B_{s0}) \qquad (V.2)$$

Y_b is the mean of the momentum distribution (assumed to be gaussian) accepted by the beamline when the central field value in the first M13 bending magnet B1 is B1 = 1555.4 Φ_b G. Although we select B1 in the discussion as a calibration standard, we note that the symmetry of the beamline made B2 an equally good choice. Y_s is the factor which converts y_r to y when the central field in the spectrometer is $B_s = 3186.7\ \Phi_s$ G. Of the four parameters to be determined in the calibration, two were the conversion factors m_b and m_s. The beamline zero point offset $B1_0$ was a correction for the effects of field hysteresis in the bending magnets and for offsets in the central field measurements; the spectrometer offset B_{s0} was a correction for those effects in the spectrometer, and for systematic shifts in the reconstructed momenta as well.

Under these assumptions we needed two reference points to calibrate the beamline, which could then be used as a momentum source to calibrate the spectrometer. In Fig. 6, two such benchmarks stand out: the decrease in muon flux at the π-μ decay edge ($p_{\pi\mu} = 29.78$ MeV/c or $y_{\pi\mu} = 0.5639$) and the decrease in positron flux at the μ-e endpoint at $y_{\mu e} = 1.00$. Conveniently, these nearly span the range of momenta reconstructed in this experiment, $0.36<y<1.00$. Secs. V.B.1 and V.B.2 describe how these calibration points were determined.

We completed the calibration with a procedure complementary to that of Sec. V.A, passing beam positrons of varying momentum through the spectrometer at fixed Φ_s. This was done for each of the six values of Φ_s used for main data collection. From these data, we determined the final corrections to the relative momentum of Sec. V.A and the spectrometer calibration curve (Eq. V.2) needed to convert relative to absolute momenta. We discuss this procedure and its results in Secs. V.B.3 and V.B.4.

V.B.1 Beamline π-μ Calibration Point

The π-μ calibration determined the bending magnet settings ($B1_{\pi\mu}$, $B2_{\pi\mu}$) which centered the beam acceptance at the pion decay momentum. This was done by sweeping the beam setting Y_b (Eq. V.1) across the momentum distribution of muons from π^+ decay at rest in the 2mm carbon target at 1AT1 (Fig. 6). The delta-function muon momentum spectrum was smeared to a theta-function by energy loss inside the target, with an endpoint at $y_{\pi\mu}$=0.5639 (the theta-function approximation is valid only near the endpoint). As Y_b was changed, the muon flux at F3 varied in proportion to the overlap of the muon spectrum with the beamline momentum acceptance. Thus $B1_{\pi\mu}$ was the field strength at which the muon flux was exactly midway between the muon fluxes measured when the acceptance completely overlapped or completely excluded the theta function.

The detailed procedure was as follows. First, the beamline was carefully tuned near Y_b=0.5639 by maximizing the muon flux and centering the muon spot at F3. Then B1 was scanned from 864 G to 894 G in small steps while scaling the rest of the beamline magnet fields in proportion to B1. At each point we measured the ratio of rates

$$F = (\mu\ \mathrm{STOP})/(\mathrm{BEAM}) = (R(\mu^+)+R(\pi^+)) \ / \ (R(\mu^+)+R(\pi^+)+R(e^+)) ,$$

where R is the flux of each beam particle species (Figs. 6 and 16). The flux of protons is not included in the expression because they stopped in the beamline vacuum window before P1. The approximation depends on the chamber efficiency: a positron which did not register in the downstream chambers P3, S2 or V2 generated a μ STOP rather than a BEAM signal (Fig. 12).

The measurement was made twice, at the beginning and end of the experiment. The fit to the data shown in Fig. 16 is described in App. C. The results were $B1_{\pi\mu}$ = 875.6 G and $B2_{\pi\mu}$ = 954.3 G for the earlier and $B1_{\pi\mu}$ = 874.8 G and $B2_{\pi\mu}$ = 952.6 G for the later measurement. The statistical uncertainty in the fit values is estimated to be 0.2 G, much less than the difference between the two measurements. Possible reasons for the difference are considered in App. C, with the conclusion that one π-μ calibration cannot be preferred over the other. The results were therefore averaged, and their difference applied as a systematic error.

V.B.2 Spectrometer μ-e Calibration

In principle, the μ-e edge in the beamline could have been used as the second beamline calibration point, taking the ratio (BEAM)/(μ STOP) near Y_b=1.00. Had this been possible, both m_b and $B1_0$ in (V.1) could have been fixed. However, the rapidly rising heavy particle flux, whose exact shape was not well known, undermined this measurement. Instead, the necessary second absolute calibration point was obtained for the spectrometer using the μ-e edges in the data (Fig. 15b). The beamline calibration was then completed by analyzing beam positrons collected when Y_b was near 1.00, and comparing the reconstructed momentum with the edge position.

The most convenient way to make the μ-e calibration was to choose Y_s=1.00 at Φ_s=1.00 (B_s = 3186.7 G in Eq. V.3), and to shift the reconstructed momentum scale of Sec. V.A to put the μ-e endpoint at the proper value. Using the Φ_s=1.00 data taken intermittently throughout the course of the experiment, the decay edge was analyzed using the procedure described at the end of Sec. V.A and illustrated in Fig. 15. Subtracting $\Delta x_s = 0.0083$ of energy loss upstream of the spectrometer vacuum box, the $y=1.00$ decay endpoint was required to appear at $(y-\Delta x_s)=y_r=0.9917$ on the reconstructed momentum scale. This condition was satisfied individually for each of the seven thin target runs analyzed with a reproducibility in y of 0.0002.

V.B.3 Calibration Data

In an important preliminary to the calibration, we verified that the digital beamline monitor gave adequate control of the beamline parameters which could affect deviations in Y_b from (V.1). The reconstructed momentum distributions for straight-through positron runs collected with small variations in the beamline parameters were analyzed to yield the results in Tbl. 3. For comparison, the systematic error in the calibration data points described below was $\sigma_Y=0.0005$ (App. B). In the upper half of the table, Y_b is seen to be relatively insensitive to the aperture parameters which determined the width and center of the positron momentum bite. In the lower half of Tbl. 3 we list the effect of changing the currents in the quadrupole magnet Q1-Q7 by one least count on the monitor. As the beamline setting was varied during the calibration, these data anticipated the fluctuations in Y_b due to the inability to precisely adjust the magnet currents. Again, the effects seen are quite small. Finally, the shift in Y_b correlated with a change in the central field in B2 is given. For variations in B2 typical of those needed for steering the positron spot onto F3 during the calibration data taking (see below), this effect alone was sufficient to explain the systematic error in the calibration points. Although the source of this final instability was not determined, the overall conclusion was that the beamline was stable enough to provide a calibration to 0.001 in y (Sec. II.C. See also App. C).

For the calibration itself two runs were performed, one early and one late in the experiment. Data were collected as follows: the spectrometer and solenoid were powered in one of the six data taking configurations (Φ_s = 0.42, 0.50, 0.60, 0.72, 0.86 and 1.00). In the earlier calibration, the beamline was then tuned at each of five settings: Φ_b/Φ_s = 1.18, 1.09, 1.00, 0.92 and 0.84. In the later calibration, the format was changed to allow direct comparison with the edge scan data (Sec. V.A), taken immediately before. At Φ_s = 1.00, 0.72, 0.60 and 0.42, only three points at Φ_b/Φ_s = 1.09, 1.00 and 0.92 were taken. At Φ_s = 0.86 and 0.50, nine points were taken at Φ_b/Φ_s = 1.17, 1.13, 1.09, 1.05, 1.00, 0.95, 0.92, 0.88 and 0.85.

To stabilize hysteresis effects, all magnets were saturated at maximum field strength before being reset to their operating values, although B1 and B2 thereby induced a dipole component in Q2 and Q6. After setting B1, the field strengths in the other magnets were found by scaling the values at the π-μ calibration point, except for B2, which required adjustments of typically 0.05% of the Φ_s=1.00 setting to center the positron spot on F3. 30 000 straight through triggers were taken at each setting. The analysis proceeded as for muon decays, with the standard aperture cuts and momentum reconstruction (Sec. IV). The phase space (u,v,u',v') of positrons arriving at the target was not entirely stable, and because of the momentum correlations with these coordinates (Fig. 9) further cuts were made on the fringes of the phase space to minimize fluctuations in the beam momentum distribution. The cuts on the vertical slope v' also increased the similarity between the positron momentum distribution and the muon momentum distribution at the π-μ edge: the high momentum fringe at large v'^2 would not have been filled by the π-μ theta function.

Correlating the reconstructed momentum distribution with the beamline setting Y_b required much care. First, measures of the reconstructed peak position (Fig. 7) which were insensitive to the straggling tails were calculated. One measure (y_r(ave)) took consecutive averages over a decreasing range, and was sensitive to the distribution of events within the peak. Another measure (y_r(fit)) fit a parabola to the distribution of events within $\Delta y_r/y_r = 0.4\%$ of the peak center, and was primarily sensitive to the edges of the peak. Secondly, corrections were needed for the absolute shift Δx_b of the peak from the beamline setting Y_b due to energy loss and straggling before measurement in the spectrometer. Assuming a gaussian beamline momentum acceptance, these were $\Delta x_b = 0.0104$ when using y(fit) (the parabolic peak measure applied to the true momentum distribution) and 0.0099 when using y(ave) (App. A.3). The assumption was checked by fitting straggled gaussians to the data distributions, and calculating y(ave) and y(fit) for both the simulated and the actual data. The differences were usually less than 0.0001 in y(ave) and 0.0003 in y(fit), which are negligible. Fig. 7 shows a sample fit in the extreme: the discrepancy in y(fit) is 0.0003.

V.B.4 Calibration Results

Combining all the measurements described thus far in Sec. V, the calibration fit determined the four parameters in Eqs. V.2 and V.3 and a set of continuous curves which gave final corrections to the relative positron momenta y_r. A detailed description of the fit, which was rather complex, is left to App. B. We concentrate here on estimating and understanding the systematic uncertainties of the calibration with a graphical comparison of the fits to the early and late data, shown side-by-side in Fig. 17.

After using the π-μ calibration point to eliminate m_b in (V.2), we relied on the data of Sec. V.B.3 to determine $B1_0$ in the fit of App. B. Figs. 17a and 17b show these beamline calibration results. The abscissa is the central field strength B1 in the beamline bender B1. The ordinate is the difference between the beamline setting Y_b and the value expected by simple scaling (i.e. $B1_0$=0 in Eq. V.1) from the π-μ calibration point. The line shows the effect on (V.1) of including the zero-point offset $B1_0$ from the calibration fit of App. B. The triangle near 880 G is the π-μ point. The circles are the calibration data points in Figs 17c and 17d nearest the vertical line. Since the corrections to y_r were constrained to be zero there, these data points determined the magnet zero-point offsets (see below). As anticipated in Sec. V.B.2, the points nearest the μ-e momentum (larger values of B1) constrained the fit for $B1_0$. The substantial difference between Figs. 17a and 17b was due to the π-μ calibration discrepancy of Sec. V.B.1, and fed directly into the determination of the spectrometer zero-point offset.

With m_s in (V.2) fixed by the μ-e calibration point, the fit of App. B determined B_{s_0} and the final corrections to y_r. Figures 17c and 17d show these spectrometer calibration results. Note that Δx is the energy loss between the point at which a positron entered the apparatus and the spectrometer vacuum box, and y here is the initial positron momentum. Thus the abscissa is the true positron momentum in the spectrometer, divided by Y_s to allow comparison of data taken at different values of the spectrometer central field. On the ordinate we plot the final corrections to the momentum determination (if both B_{s_0} and the corrections to y_r remaining after Sec. V.A were zero, $y = y_r \Phi_s$). The curves show the results of the fit to the data described in App. B. To clarify the presentation only the Φ_s=1.00 data (open circles) are displayed at the correct vertical position. The Φ_s=0.86, 0.72, 0.60, 0.50 and 0.42 data are displaced downwards, respectively, by 0.0025, 0.005, 0.0075, 0.01 and 0.0125. Also plotted are the relative calibration points of Sec. V.A (crosses). These points have been corrected for the fit spectrometer zero-point offset (for them the ordinate is $y_r Y_s-(y-\Delta x)$). Therefore the crosses are normalized to Φ_s=1.00, and comparison with the curve for Φ_s=1.00 indicates the fit residuals for these data. Because of the chage in the ordinate, the crosses show differences between Figs. 17c and 17d due to the difference in the value of B_{s_0} fit to the two data sets.

To decouple the determination of B_{s_0} and the corrections to y_r, we fixed the latter to zero at 0.9917 on the horizontal scale (the vertical line in Figs. 17c and 17d). Thus corrections to the six points nearest this line could be made only by adjusting the zero-point offsets $B1_0$ and B_{s_0}. Since the μ-e calibration (Sec. V.B.2) fixed

Y_s=1.00 at Φ_s=1.00, the influence of B_{s_0} is limited to the curves at other values of Φ_s. From the constraint on the corrections to y_r, only changes in B_{s_0} can move those curves up or down at the vertical line. Thus, comparing the early and late curves for Φ_s=0.50 (open triangles) at the vertical line, we see that the uncertainty in (V.2) was roughly 0.0010 near Y_s=0.50. Equation (II.9) implies a corresponding uncertainty in δ of 0.0008.

Away from the vertical line, the differences between the earlier and later sets of curves in Figs. 17c and 17d show that the corrections to y_r were also not reproduced. Overlaying curves for the same Φ_s at their intersections with the vertical line (to remove the effects of differing B_{s_0}), the disagreement is seen to be as large as 0.0014 in the Φ_s=1.00 data near the low end of the horizontal scale. Although it is our suspicion that the discrepancy is due to a systematic downwards shift in the later Φ_s=0.86 data, and should therefore be reflected already in the determination of B_{s_0}, we conservatively accepted the difference in the curves as a systematic error in the calibration.

In establishing the final source of uncertainty in the calibration, we turn from Fig. 17 and recall that the symmetry of the beamline meant that the field strength in either B1 or B2 could have been used in (V.1). Though we chose B1 in the discussion, from a certain standpoint B2 might actually have been preferred. For B2, the object at F2 and the image at F3 were both fixed, the latter by adjusting the current in B2 to position the positron spot on the target. In B1, only the image at F1 was fixed - the object source at 1AT1 was dependent on the particle species. This implied that the π-μ calibration for B1 using muons was not necessarily accurate for positrons. However, considering the potential for asymmetries in the beamline configuration (Sec. III.A), that the positron spot at F3 was subjected to cuts not applied to muon events, and that the relative centering of the muon and positron spots at F3 was biased by material upstream of P2 which stopped muons but not positrons (Fig. 10), the same doubt was also cast on B2. We used the difference between calibrations using B1 and B2 to estimate the importance of these effects.

In summary, there were sixteen possible ways of calibrating the spectrometer, choosing one of the combinations:

$\{y_r(\text{ave}), y_r(\text{fit})\} \times \{\text{Early,Late}\}$ data $\times$ $\{\text{Early,Late}\}$ $\pi\mu$ $\times$ $\{B1, B2\}$.

The $\{y_r(\text{ave}), y_r(\text{fit})\}$ option checked the gaussian beamline momentum bite assumption; the {Early,Late} differences accounted for uncertainty in determining the corrections to y_r and for calibration difficulties at the π-μ point; and the {B1,B2} option accounted for possible differences in muon and positron behavior in the apparatus. Table 4 summarizes the combinations used in Sec. VII to determine systematic errors by comparing calibrations between which only one of the options varied.

VI. ASYMMETRY FITS

Because the data were divided into distinct yet complementary samples, the asymmetry analysis required several steps. The main division in the data was between the spin-held and μSR samples, with the latter further subdivided into four samples of (thin, thick) target × (fast, slow) precession. Replacement of the μ-decay clock at the midpoint of the experiment required a further subdivision because of differences in the clock calibration parameters. Finally, data in each class were collected at six different values of the spectrometer setting Y_s.

The analysis was organized to optimize the sensitivity with which the time-related parameters in (II.7) could be determined, thereby minimizing errors in the asymmetries. In the initial fits, data from all spectrometer settings were combined. The spin-held data calibrated the counting rate of the clock being used, while the combined μSR data determined its offset relative to the muon arrival time. The data samples from both clocks were then combined in the remaining fits. The combined thick and thin target data determine the polarization function $P_\mu(t)$ and the precession frequency ω for the fast and slow samples. With these quantities determined, the four μSR samples were fit separately by spectrometer setting for the decay asymmetries, as required by the dependence of the external radiative corrections on the target thickness and Y_s.

VI.A Spin-held Data and Fits

The spin-held data are displayed in Fig. 18. Whenever the spectrometer setting was changed (see below), typically one or two runs of spin-held data were collected along with eight runs of μSR data. After the first data-taking period (just before the later calibration), the μ decay clock began to fail for large decay times. Although the time range lost (from 9.0 - 10.0 μsec) represented only 1.1% of the data, possible further loss in functionality necessitated replacement. Naturally, the rate at which the two clocks counted was not the same. Cross calibration was performed by fitting a separate muon decay lifetime τ_μ for each clock (Tbl. 5) using the exponential decay rate formula

$$R(t) = N_0 \exp(-t/\tau_\mu). \qquad \text{(VI.1)}$$

Then the second clock readout was scaled to make the lifetimes equal.

The spin-held data were also used to identify early time ranges during which "ringing" in the μ-arm proportional chamber cables caused μ-stop events to self-veto the μ-decay trigger. The fit to (VI.1) was repeated while varying the lower limit of the time range fitted. τ_μ decreased monotonically as times up to 1.4 μsec were excluded, after which it stabilized for the data in Fig. 18 at 2.200(8) μsec. The accepted value of the muon decay lifetime is τ_μ=2.197 μsec. Deviation of the measured decay rate from an exponential can easily be seen at early times in Fig. 18. In the μSR analysis described below, only the time range from 1.4 to 8.8 μsec was fit.

VI.B μSR Data and Fits

The μSR data were collected with cyclical variation of the spectrometer central field scaling Φ_S in the pattern Φ_S=1.00, 0.72, 0.50, 0.42, 0.60, 0.86; or the reverse. Two runs were taken at each setting for each of the combinations [{thin, thick} targets × {fast, slow} precession] except at Φ_S=1.00, where only one run of each class was taken. Two different target thicknesses were used in order to check the effect of external radiative corrections. Two different precession frequencies were used to check the fit procedure, which required making precise time averages.

After all cuts described in Sec. IV.B and V.A were applied, the data were binned in x and t, and fit to (II.7) - appropriately averaged over the range of x and t in each bin - by minimizing the Poisson maximum-likelihood χ^2 [17]:

$$\chi^2 = 2\sum_{bins}[e_i - o_i + o_i \ln(o_i/e_i)]. \qquad \text{(VI.2)}$$

o_i is the fit and e_i the measured number of events in bin i.

Still to be clarified in (II.7) is the precise form of the polarization term $P_\mu(t)$. Selecting an appropriate form for the relaxation due to spin-spin coupling with the magnetic moments of the aluminum nuclei was problematic, as the data did not have the statistical power to differentiate between the two common forms: gaussian and the Kubo-Tomita "motional narrowing" [18] (the latter parameterizes a smooth variation between exponential and gaussian forms of the relaxation). The situation was less favorable in this experiment than in the endpoint analysis already reported, where a different trigger made it possible to use data at earlier decay times. As discussed below, we therefore chose to use the same depolarization fits obtained in Ref. 13. Still, it was impossible to choose between the gaussian and Kubo-Tomita fits made there, which disagreed by nearly 1% in the extrapolation to t=0. We (arbitrarily) selected the simpler gaussian form of the spin relaxation in the fits described below:

$$P_\mu(t) = P_\mu(0)\, e^{-G t^2}.$$

The implications of this choice are discussed in Sec. VII.D.

Lastly, the clock offset relative to the muon stopping time ($t_{0,1}$ and $t_{0,2}$ for clocks 1 and 2 respectively) had to be known to calculate both $P_\mu(t)$ and the precession phase ωt. With τ_μ, the parameters G, ω, $t_{0,1}$ and $t_{0,2}$ completely described the time-dependence of the decay rate (Eq. II.7). A two-stage fitting procedure determined their values. In both stages, the fast and slow precession frequency data were fit separately, but the data for both targets and from all spectrometer settings were combined. For each of 16 bins in x covering the range $0.36<x<1.00$, events in the time range from 1.4 to 8.8 µsec were binned in 40 nsec intervals. In the first fitting stage, the data were split by the clock used. The fit parameters were G, ω, and t_0, eoa N(x) and M(x) (Eq. II.8) for each of the 16 energy bins. The results for G, ω and t_0 are given in Tbl. 5. In the second stage, with t_0 determined for each clock the combined data were fit by precession frequency for the final determination of ω and G.

As seen in Tbl. 5, the fit value of G disagreed by 1.6 standard deviations for data with slow and fast precession frequencies. To assess this possible discrepancy we examined data from our previous µSR analysis[13] at the decay spectrum endpoint, for which the same two aluminum targets were used. As noted above, the polarization curve was better determined by those earlier data, which gave $G = 0.00378(20)\ \mu sec^{-2}$. Since the difference between this single value of G and those in Tbl. 5 did not significantly affect the fit values of ω and t_0, we used it in the fits described below.

Although we combined the data from both clocks for the final asymmetry fits, the correlation of cuts on the external radiative corrections (Secs. II.C and VI.C) with the spectrometer field strength Φ_S required that each spectrometer setting be fit separately. The energy ranges fit (Tbl. 6a) were determined by the range over which the relative momentum calibration was performed. To keep the size of the data sample in each energy bin from varying with the spectrometer setting, the bin size was fixed at 0.02 in x. The free parameters in the fit were only the normalization N(x) and asymmetry M(x) for each x bin. Of the parameters that were fixed, the uncertainties in ω, $t_{0,1}$ and $t_{0,2}$ did not significantly propagate into the fitted asymmetries, while the error in G had a smaller effect than changing the gaussian for the motional narrowing form of the relaxation (see above, and Sec. VII.D). The fit M(x) are exhibited in Tbl. 6b, where the errors are statistical and do not include the error in G.

The sensitivity of the data is demonstrated in Fig. 20, where data from different clocks and targets are combined to show the statistical power of the full sample. In the high x data (Figs. 20a and 20d), the effect of spin-spin relaxation in suppressing the μSR signal at large decay times is clearly visible.

VI.C Monte Carlo Simulation and Asymmetry Corrections

As mentioned in Sec. II.C, a detailed Monte Carlo simulation of the experiment was required to calculate the asymmetry shifts due to external radiative corrections. When these corrections are included, the average asymmetry M_f expected at an energy x_f and spectrometer setting Y_s is given by:

$$M_f(x_f,Y_s) = \frac{1}{N(x_f,Y_s)} \int d\Omega_e \int_{x_f}^{1} dx_i \, \frac{d\Gamma(x_i)}{dx_i} \, M_i(x_i) \int d\varepsilon \, \frac{dI(x_i,\varepsilon)}{d\varepsilon} \tag{VI.3}$$

$$\times \, \delta(x_f+\varepsilon-x_i) \, A(\hat{p}_e,x_i,\varepsilon,Y_s) \, \frac{\hat{p}_e\cdot\langle\hat{p}_\mu\rangle}{\langle\hat{p}_e(x_f,Y_s)\rangle\cdot\langle\hat{p}_\mu\rangle} \, .$$

with

$$N(x_f,Y_s) =$$

$$\int d\Omega_e \int_{x_f}^{1} dx_i \, \frac{d\Gamma(x_i)}{dx_i} \int d\varepsilon \, \frac{dI(x_i,\varepsilon)}{d\varepsilon} \, \delta(x_f+\varepsilon-x_i) \, A(\hat{p}_e,x_i,\varepsilon,Y_s) \, .$$

$d\Gamma/dx$ is the isotropic component of (II.1) and M_i is the asymmetry (II.8), both calculated for standard-model values of the decay parameters and including internal radiative corrections; $dI(x_i,\varepsilon)/d\varepsilon$ is the probability per interval $d\varepsilon$ for the positron energy to change from x_i to $x_f=x_i-\varepsilon$ due to bremsstrahlung and Bhabha scattering (App. A); A is the spectrometer acceptance, which depends on Y_s and the externally radiated energy loss ε; and the last factor accounts for the difference in angular distribution between straggled and unstraggled positrons. The averages of the unit momentum vectors are given by (II.5) and (II.6).

Equation VI.3 was evaluated stochastically using a Monte Carlo simulation of the thin-target data. For each of the six spectrometer settings, decay positrons were generated in the range from $0.8Y_S<y<1$, and in the cone $\cos\theta_e>0.97$ around the solenoid axis. For the muons, randomly selected muon tracks from μ-stop triggers provided a realistic phase space. The resulting events were propagated in small steps through magnetic fields in the solenoid and spectrometer that were based on computer simulations of their magnetic properties. Cuts were applied at all apertures, and hits were accumulated at all chamber planes. The hits in drift chambers D1 and D2 were smeared by resolution functions designed to mimic the true curved fit resolutions, including long tails (extending up to 16 standard deviations) to simulate the effect of occassional errors in resolving left-right ambiguities (Sec IV.B). Simulated events were subjected to Coulomb scattering, mean energy loss, Bhabha scattering and bremsstrahlung. When the energy straggling (App. A) was calculated, the apparatus was decomposed by material along the particle path exactly as in the experiment. Each material was further decomposed into its constituent elements when evaluating $dI/d\varepsilon$ in (VI.3), and, for straggling in the target, corrections were included for the suppression of Bhabha events due to the multiple hit cuts in P3 (Sec. IV.B). Although momentum analysis of Monte Carlo positrons was not necessary, they were propagated completely through the spectrometer field to allow the vertical apertures at D4 to have effect. The agreement between the final energy scales in the data and simulation was carefully checked near $x=1.00$ using the endpoint fit of Sec. V.A, and at lower energies with special Monte Carlo runs in which positrons were generated at fixed x (App. A.2).

The final Monte Carlo event sample (400K events at each of the six spectrometer settings) was subjected to the standard analysis, except for track selection and momentum reconstruction. All cuts were applied. As expected, the link between the tracks in P3-D1-D2 and D3 proved sensitive to extreme straggling. However, cuts on the curved track χ^2 could not be used to reduce the straggled-event contamination because the above-mentioned left-right ambiguities completely dominated the χ^2. Asymmetry corrections were calculated for each x bin in the fit from the averages:

$$\langle M_i(x,Y_s)\rangle = \frac{1}{N_i} \sum_{\substack{\text{events at } Y_s \\ x-0.01<x_i<x+0.01}} M_i(x_i)$$

$$\langle M_f(x,Y_s)\rangle = \frac{1}{N_f} \sum_{\substack{\text{events at } Y_s \\ x-0.01<x_f<x+0.01}} M_i(x_f) \qquad \text{(VI.4)}$$

Here N_i is the total number of events which were generated with energy x_i in the bin centered at x, and N_f is the number of events (after correcting for energy loss) which had final energy x_f in that same bin. The second average in (VI.4) corresponds directly to (VI.3). Although $\langle M_f\rangle$ and $\langle M_i\rangle$ were both sensitive to systematic uncertainties due to the limited size of the simulated sample, the events used to calculate the two averages were highly correlated, and the difference $\Delta M = \langle M_i\rangle - \langle M_f\rangle$ was stable. Therefore, as described in Sec. VII, the difference $\Delta M = \langle M_i\rangle - \langle M_f\rangle$ was added to the fit asymmetry M for each x bin to correct for the effects of the external radiative corrections. The corrected data points were then compared to M(x) calculated from (II.8) with only internal radiative correction.

Table 6b includes the Monte Carlo simulation results for ΔM. The correction ranges from −0.0013 to −0.0115 with the estimated error varying over that range from 0.0002 to 0.0006. Since energy loss in the target affected only the positron momentum without introducing any track discontinuities downstream, the thin-target simulation results could also be applied to the thick target data, with corrections for the extra target material calculated analytically using (VI.3) with $A=1$ and $dI/d\epsilon$ for 30 mg/cm^2 of aluminum. These corrections are included in the tabulated results.

VII. DETERMINATION OF DECAY PARAMETERS

After applying the corrections for Bhabba scattering and bremsstrahlung, the measured asymmetries (Tbl. 6b) were used to determine δ and ξP_μ in a chi-squared fit to equation II.8. In evaluating the theoretical expression for M(x) the decay parameter ρ was set provisionally to 3/4 and h(x) and g(x) were evaluated from the expressions for the internal radiative corrections given in Ref. 7. The fit results are exhibited in Tbl. 7 and illustrated in Fig. 21. The average over the six calibration fits (Tbl. 4. Fit 2 was redundant and therefore excluded) gave

$$\delta = 0.7479(26), \quad \xi P_\mu = 0.9830(35), \qquad \text{(VII.1)}$$

where the error is purely statistical. The individual results by target and precession frequency were entirely consistent (Tbl. 7). Since the cloud muon cuts were not made (Sec. III.A), the muon polarization was expected to be ≈ 0.98. Applying the cuts reduced the data sample by 25%, which then fit to

$$\delta^* = 0.7451(30), \quad \xi P_\mu{}^* = 0.9969(40). \qquad \text{(VII.2)}$$

Taken together, δ and $\xi P_\mu{}^*$ agree well with the standard weak interaction predictions for muon decay (Tbl. 1). The difference between δ and δ^*, a 1.6 standard deviation effect for these two highly correlated samples, was assumed to be purely statistical in origin.

VII.A Checks on the Fit Procedure

To check for systematic effects in the fit, (II.3) was randomized for standard model values of the decay parameters and fit using the μSR analysis of Sec. VI.B. As in the Monte Carlo simulation, muon variables were taken from randomly selected μ-stop triggers, and positrons were generated in the range $\cos\theta_e > 0.97$ with respect to the solenoid axis. The generator was biased to yield a nearly flat energy spectrum for the decay positrons, as seen in the data. Two "combined" data samples, designed to resemble the slow or fast precession frequency data summed over spectrometer settings, verified that the parameters ω, G, and t_0 were accurately determined by the fit. Next, twelve "separate" data samples were generated, with 100 000 events at each of the six spectrometer settings. Each of the twelve samples had the statistical power of one half of the data. The asymmetry and decay parameter fits were performed, and the final results averaged to obtain

$$\delta(\text{check}) = 0.7494(44), \quad \xi P_\mu(\text{check}) = 1.0005(62),$$

in excellent agreement with the input values.

VII.B Systematic Errors

We now discuss the systematic errors in the determinations of δ and ξP_μ. These are summarized in Tbl. 8.

VII.B.1 Spectrometer Calibration

The uncertainties in the spectrometer calibration manifested themselves independently in the spectrometer zero-point offset B_{S_0} and in the curves which gave corrections to the relative momentum y_r. The five sources of error had their roots in: 1) the beamline π-μ calibration reproducibility (Sec. V.B.1); 2) the uncertainty in the spectrometer μ-e calibration (Sec. V.B.2); 3) peak measurement uncertainties (y_r(ave) vs. y_r(fit), (Sec. V.B.3); 4) the reproducibility of the magnet settings in the earlier and later calibrations (Sec. V.B.4); and 5) muon-positron differences in the beamline (Sec. V.B.4). Sources 2 and 3 were negligible. In evaluating the others, we took differences of the fit results in Tbl. 7 (as outlined in Tbl. 4) to find the calibration systematics given in Tbl. 8. The errors from sources 1 and 5, which fed directly into the spectrometer zero-point offset, were added linearly. The error from source 4, which primarily affected the y_r correction curves, was then added in quadrature. The final calibration errors were $\sigma_{cal,\delta}$ = 0.0020, and $\sigma_{cal,\xi}$ = 0.0031.

VII.B.2 Momentum Resolution

As mentioned in Sec. IV.C, the μ-decay edges used to fine tune the momentum reconstruction were not sensitive to asymmetries in the resolution. Since all significant reconstruction asymmetries that we found were associated with events on the fringes of the distributions in track positions within the spectrometer, such asymmetries could be detected as differences in the reconstructed μ-decay edge position in different parts of the positron phase space. Analyzing the edges in the $\phi_s = 1.00$ and 0.86 data taken with the thin target and in the edge scan runs described in Sec. V.A, we limited the existence of resolution asymmetries to levels corresponding to errors of $\sigma_\delta = 0.0002$ and $\sigma_\xi = 0.0013$.

Since the calibration analysis was based on data taken with ϕ_s near 1.00, the non-scaling of the spectrometer fringe field implied that the reconstruction was not necessarily accurate at lower ϕ_s. In particular, if the ad hoc corrections of Sec. V.A broke down, deterioration of the total resolution and a significant resolution asymmetry might have developed. We ruled out both possibilities by changing the shape of the spectrometer fringe field (Fig. 11) in the reconstruction algorithm of Sec. IV.C by an amount consistent with the fitted spectrometer zero point, re-analyzing the data with the standard ad hoc correction of Sec. IV.C and observing that the edge distributions did not change.

VII.B.3 Radiative corrections

There were three sources of uncertainty in the calculation of the radiative corrections. The single most important source accounted for possible effects of higher-order QED diagrams on the internal and external radiative corrections to muon decay indicated in Fig. 1. As discussed in Sec. II.C, the available estimates of the higher-order effects limit them to 3-5% of the known corrections. Conservatively, we took the larger number and assumed that the sign of the effect would be the same for both sets of corrections. Scaling ΔM in Tbl. 6b and the internal radiative corrections h(x) and g(x) in (II.8), we refit for the decay parameters to find the errors $\sigma_\delta = 0.0016$, $\sigma_\xi = 0.0022$.

Secondly, we checked that the description of the apparatus and the sample size in the Monte Carlo simulation were sufficient for an accurate evaluation of (VI.3). Since the statistical uncertainties in the asymmetry corrections were less than 1/20 of the errors in the data points themselves (Sec. VI.C, Tbl. 6b), the primary concern was that all important apertures in the apparatus were included in the simulation. Comparison of the phase-space distributions of simulated and real data showed that any omissions were minor. Modifying the phase space of the simulated events used in evaluating (VI.4) without destroying that similarity, we found no significant variation in the asymmetry corrections.

Lastly, we calculated the error in ΔM due to the assumption of standard-model values of the decay parameters in (VI.3) by recalculating the asymmetry corrections (VI.4) with $\delta = 0.7479$ in (II.8). The differences again were negligible.

VII.B.4 Energy Loss and Stopping Power Calculations

It was necessary to know the amount of positron energy loss in the target and other material in order to sensibly calibrate the spectrometer. Since the energy loss did not vary with the positron momentum, an incorrect value could mimic a zero-point offset in the magnets. Secondly, it was needed in the absolute calibration to correlate the initial momenta of decay and beam positrons, which suffered slightly different energy loss before reaching the spectrometer. Analyzing the momentum distribution of straight-through positrons collected with and without extra material in the target region, we found 10% agreement between the observed changes in the peak position and the most probable energy loss Δ_{mp} calculated with the algorithms outlined in App. A. Theoretical and experimental comparisons in Ref. 19 also show this level of agreement. Feeding this uncertainty into the momentum calibration had only a small effect on the fitted values of the decay parameters (σ_δ=0.0003, σ_ξ=0.0003).

The muon stopping range was needed to calculate the residual range of the decay positrons in the target, and thus affected the energy loss and straggling calculations. A direct experimental determination of the range could not be made because the cold-rolled aluminum from which the targets were made was not available in fine gradations of thickness. We therefore relied on the calculations of Ref. 14, which implied a total range of 171 mg/cm² for 29.5 MeV/c muons in our apparatus. Although there are few direct experimental checks of muon range calculations, experimental results for non-relativistic protons show 1% agreement with theory. Since the energy-loss mechanisms for low-energy muons and protons are identical, this implies a 2 mg/cm² uncertainty in the muon range. The corresponding effect on the positron energy loss and straggling calculations was negligible.

VII.C Corrections

The final significant systematic error in the determination of δ and ξP_μ arose from the uncertainty in the world average value of the decay parameter ρ, currently known to be 0.752±0.003[6]. Using this value in (II.8) and refitting for the decay parameters gave the shifts $\Delta\delta$ = +0.0007±0.0011, $\Delta\xi P_\mu$ = +0.0012±0.0017.

Track reconstruction errors affected the calculation of the acceptance-weighted averages of $\langle\hat{p}_\mu\rangle$ and $\langle\hat{p}_e\rangle$ in (II.5) and (II.6). Chamber alignment errors, finite hit resolution, the curved track reconstruction approximations and Coulomb scattering all contributed. The results (VII.1) and (VII.2) include corrections determined in the Monte Carlo simulation for these effects of $\Delta\delta$ = +0.0000, $\Delta\xi P_\mu$ = +0.0004. The errors in these corrections were estimated to be less than 0.0002.

We also included the correction for the muon depolarization effects carefully enumerated in Ref. 13, a factor of 1.0015±0.0005 multiplying the experimentally determined ξP_μ.

VII.D Spin Relaxation and Final Results

As discussed in Sec. VI.B, the form of the muon spin-relaxation curve $P_\mu(t)$ was a delicate point in the analysis. In fact, there is no complete theoretical analysis of spin-spin relaxation in room temperature metals[20]. Furthermore, the relaxation observed in our targets is greater than that which has been seen in other measurements of muon spin relaxation in aluminum[21]. Possibly this is due to imperfections of the crystal lattice introduced during the cold-rolling manufacturing process. Our experimental situation was thus even further removed from the regular-lattice models considered theoretically. To determine whether the theoretical uncertainty was critical, the two available forms of the relaxation (motional narrowing and gaussian) were used separately in the μSR fits of Sec. VI.B. Subsequent fits for the decay parameters showed a 1% difference in ξP_μ in the two cases.

Fortunately, only ξP_μ was directly affected by the uncertainty in the depolarization: after entirely eliminating the relaxation from the asymmetry fits, the value of δ obtained was unchanged, though ξP_μ was reduced to 0.95. A sensitive determination of ξP_μ using the μSR technique does remain a possibility, for the theoretical uncertainty is not critical when the relaxation is small. For example, in the gold target data in the earlier endpoint μSR measurement[13] the depolarization was less than 10% at t=10μsec and the fitted asymmetries for Kubo-Tomita and gaussian relaxation agreed to better than 0.1%. In comparison, the depolarization in cold-rolled aluminum was 30% at 10 μsec with a disagreement of 1% in the fitted asymmetries.

Because of this serious uncertainty, we choose only to quote a result for δ. Combining the systematic errors enumerated in Tbl. 8, the analysis with ρ=0.75 gives

$$\delta = 0.7479 \pm 0.0026 \pm 0.0026,$$

or, combining the errors in quadrature,

$$\delta = 0.7479 \pm 0.0037.$$

Using instead the world average value of ρ,

$$\delta = 0.7486 \pm 0.0026 \pm 0.0028,$$

and, again with the errors in quadrature,

$$\delta = 0.7486 \pm 0.0038.$$

The final result is in excellent agreement with the standard model of the weak interactions (Tbl. 1).

VIII. IMPLICATIONS

VIII.A Muon Decay Analysis

The primary implications drawn from muon decay measurements are limits on the coupling constants of the generalized effective four-fermi contact interaction. Several equivalent parameterizations of the theory exist[2,22]. Currently the most popular is the helicity-projection form of Ref. 22, in which the Hamiltonian takes the form

$$H = \frac{G_0}{\sqrt{4\pi}} \sum_{i=S,V,T} \sum_{\alpha,\beta=R,L} g^i_{\alpha\beta}\, \bar{e}\, O^i_\alpha \nu_e\, \bar{\nu}_\mu\, O^i_\beta \mu \;+\; \text{H.C.}$$

Here

$$O^S_{R,L} = 1\pm\gamma_5, \quad O^V_{R,L} = \gamma^\mu\,(1\pm\gamma_5), \quad O^T_{R,L} = \sigma^{\mu\nu}(1\pm\gamma_5)$$

are the possible Lorentz-covariant couplings, and the $g^i_{\alpha\beta}$ are complex coupling constants. Trivially, $g^T_{LL} = g^T_{RR} = 0$, leaving only ten complex parameters, representing twenty real parameters less one arbitrary phase, to be constrained by experiment. The standard model theory has the particularly simple parameterization

$$g^i_{\alpha\beta} = 0, \text{ except } g^V_{LL} = 1.$$

The recent experimental efforts of the SIN collaboration in measuring the electron polarization[23] and our earlier measurement of $\xi P_\mu \delta/\rho$ (Ref. 3) have yielded significant improvement in the precision of the constraints on the $g^i_{\alpha\beta}$ [24]. Substituting the final result reported here for the preliminary result on δ reported earlier[25], an analysis of the type carried out in Ref. 24 yields essentially the same limits as those reported by Stoker. The final experimental status is summarized in Tbl. 9. The branching ratio limits there were calculated from the normalization condition on the total lifetime

$$\sum_{\alpha,\beta=L,R} \left(\frac{1}{4} |g^S_{\alpha\beta}|^2 + |g^V_{\alpha\beta}|^2 \right) + 3 \left(|g^T_{RL}|^2 + |g^T_{LR}|^2 \right) = 1 ,$$

with the assumption of standard model dominance. A complete description of the analysis method is found in Ref. 24.

VIII.B Alternative Physics

Gauge-theoretical extensions of the standard model which have recently attracted attention include left-right (L-R) symmetric[26] and supersymmetric[27] models. In many variations of these models, no effect on muon decay is expected because the unobserved particles in the final state (ν_R or $\tilde{\nu}$) are expected to have masses much larger than the muon mass. Thus the masses of the virtual intermediates in the decay (W_R and $\tilde{W}$) can be constrained by muon decay only in cases in which the final state particles are less massive than the muon[28].

In L-R symmetric models[26], the electro-weak gauge group is expanded to $SU(2)_R \times SU(2)_L \times U(1)_Y$. Parity is unbroken at high energies, but at low energies, in the absence of W_L-W_R mixing, the difference in the W_L and W_R masses (M_R and M_L respectively) reduces the contributions of the right-handed sector by a factor $\epsilon_R = (M_L/M_R)^4$. When $M_R >> M_L$, the theory is then consistent with observed low-energy phenomenology. In muon decay, the effect of the right-handed sector is to decrease the parity violation illustrated in Fig. 2. In the simplest L-R models, δ remains exactly 3/4. To escape this result, one or both of the right-handed neutrinos in the final state must have a non-negligible mass. If $m(\nu_{eR})+m(\nu_{\mu R}) < m_\mu$, the shape of the asymmetry spectrum is altered because the endpoints of the left and right-handed decays do not coincide. Alternatively, if the right-handed decay is forbidden because one or both of the right-handed neutrinos are extremely heavy, mixing between the left and right families would still allow right-handed contributions to the decay, with $\delta=3/4$ [29].

In Fig. 22, we present limits for the first scenario in the particular case $m(\nu_{eR}) = 0$. We obtain 90% CL limits on the W_R mass (assuming no mixing with M_L) vs. $m(\nu_{\mu R})$. The strong limits near $m(\nu_{\mu R}) = 0$ were obtained from our previous endpoint rate analysis[3]. The data reported here extend the limit to $M_R > 160$ GeV/c^2 at $m(\nu_{\mu R}) = 50$ MeV/c^2. Details of the analysis are given in App. D.

In supersymmetric theories, the muon can decay via wino exchange into an electron and two sneutrinos. The decay spectrum has been analyzed by Buchmuller and Scheck[30]. With $m(\tilde{\nu}_e) = 0$, they use the experimental values of ξ, δ and ρ to constrain $M(\tilde{W})$ vs. $m(\tilde{\nu}_\mu)$. In the case $m(\tilde{\nu}_\mu) = 0$, they find

$$\rho = \frac{3}{4}\left[1 + \frac{\epsilon_S}{2 + 3\epsilon_S}\right], \qquad \delta = \frac{3}{4}\left[1 - \frac{3\epsilon_S}{2 + 3\epsilon_S}\right],$$

$$\xi = 1 + \frac{4\epsilon_S}{2 + 3\epsilon_S}, \qquad \epsilon_S = \frac{M(W)^4}{M(\tilde{W})^4}$$

The experimental values of ρ and δ combine to give the most sensitive limits. The result for δ reported here extends the 90%-confidence limit to $M(\tilde{W}) > 280$ GeV/c^2 in this case. For $m(\tilde{\nu}_\mu)$ finite, we obtain limits by comparing the theoretical asymmetry spectrum (standard model and supersymmetry combined) to our measured asymmetries. Fig. 23 shows the 90% C.L. limits on $M(\tilde{W})$ from this analysis, without the input of ρ and using even more conservative estimates for the systematic errors in the momentum calibration than were used for the determination of δ, the limit shown at $m(\tilde{\nu}_\mu) = 0$ is weaker than the one stated above. However, in the range of $m(\tilde{\nu}_\mu)$ shown in Fig. 23, our results are still much stronger than those from the best collider searches. For example, the limit from the ASP collaboration[31] is indicated by the straight line in Fig. 23. They obtain $M(\tilde{W}) > 61$ GeV/c^2 for $m(\tilde{\nu}) = 0$, and a comparable result out to $m(\tilde{\nu}) = 10$ GeV/c^2.

Details of the analysis used to obtain the limits on the mass of the $\tilde{W}$ is described in App. D.

VIII.C Rare Decays

Family extensions of the axion solution to the strong CP problem have been suggested which predict the decay $\mu \to e\sigma$, where σ is a pseudo-scalar[32]. Such a decay would appear as a peak in our positron energy spectrum, with a width dominated by the spectrometer resolution. We have searched for such peaks and set limits on the branching ratio $\Gamma(\mu \to e\sigma)/\Gamma(\mu \to e\nu\nu)$. Our data were sensitive for masses $m_\sigma < 80$ MeV and lifetimes $\tau_\sigma > 10^{-8}$ sec (For $\tau_\sigma < 10^{-8}$ sec, charged daughters from σ decay would register in the μ-arm chambers, vetoing the event).

Since to first order we were concerned only with establishing the existence or absence of a peak, cuts that protected against suppression of the μSR signal could be relaxed. The decay signal was tripled by eliminating the cosine cuts of Sec. IV.B and the "extra before" cuts of

Sec. IV.A, and by integrating the data over the full time range of the signal (Sec. VI.A). Using the time distribution of the spin-held data (Fig. 18) to estimate the efficiency vs. decay time of the trigger and the measured spin precession frequencies, the residual polarization of the final muon decay sample was calculated to be 3%, opposite to the beam direction. Our limits therefore require only a very small model-dependent adjustment depending on the chirality of the leptonic current.

The final sample for this analysis was binned in steps of 0.001 in x over the range $0.36<x<1.00$. Since the shape of the spectrum (Fig. 24a) was acceptance dominated (the discontinuities are due to acceptance cutoffs at different ϕ_s settings), a quadratic fit to the continuum was made using bins to either side of an eleven-bin range centered on the position of a possible scalar peak. Near the cutoffs in the spectrometer acceptance, more poorly calibrated data (not shown) just outside the cutoff was also included in the background fit. After subtracting the continuum contribution, we used the five bins centered on the eleven-bin range to limit the height of a possible peak. Statistical errors in the background fit were included. This procedure was repeated for ranges centered on each x bin.

In obtaining these limits, the line shape was fixed as a gaussian with $\sigma_x = 0.002$. This represents a crude but fair estimate of the momentum resolution, taking into account the uncertainties in the corrections to y_r in the absolute calibration as well as the spectrometer resolution given in Tbl. 2. We verified that the results were not sensitive to the precise value of σ_x.

After normalizing the fit results by the total muon decay rate, we obtained the 90%-confidence branching ratio limits shown in Fig. 24b. For clarity, limits only for every fifth bin have been plotted. For comparison, the limits obtained by Bryman et al[33] on the basis of earlier data, including data taken with this apparatus[3], are indicated by smooth lines. The limits reported here are more than a factor of two stronger across most of the spectrum.

IX. CONCLUSIONS

We have made a precise measurement of the muon decay asymmetry as a function of the positron energy. The main result is a new determination of the muon decay parameter $\delta = 0.7479 \pm 0.0026(stat) \pm 0.0026(sys)$. We have also set limits on the parameters in certain L-R symmetric and supersymmetric extensions of the standard model. Lastly, we have used the measured muon decay spectrum to set limits on the lepton-number violating decay $\mu \rightarrow e\sigma$.

We consider briefly the potential for improvement of these measurements. Without greater running time or muon flux, the statistical sensitivity could still be increased significantly. A large fraction of our events were cut out in order to avoid multiple-track ambiguities. Adding 2cm×2cm Si strip detectors just upstream and downstream of the stopping target and shifting P2 upstream to obtain 3-point determination of the incoming muon tracks would reduce these losses. The high resolution of the solid state detectors would allow us to reconstruct almost all configutations of upstream tracks, for example allowing unambiguous association of one decay positron with one of a number of stopped muons. Also, additional drift chamber planes with multiple-hit time digitizers downstream of the target would have improved the ability to reconstruct multiple positron tracks and to differentiate straight-through from decay positrons. With these improvements, it would seem possible to increase the event sample by a factor of nearly four without increasing the running time or compromising the muon polarization.

A two-fold reduction in the systematic errors might also be possible. Complete calculations of the internal radiative corrections to order α^2 are needed. More critically, the errors in the absolute momentum calibration need to be reduced. In principle, the calibration method described here should be accurate to $\sigma_y = 0.0005$ (Tbl. 3 and App. B), rather than the 0.0014 actually obtained. Attaining that accuracy might require little more than greater consistency in the procedures and conditions under which the calibration runs were carried out (Sec. V.B.4 and App. C). On the other hand, extreme measures could be necessary, such as collimating the proton beam at 1AT1 and monitoring of the beam envelope and magnet fields with much more detail. Momentum calibration will be the critical issue in any new effort that seeks to improve substantially upon this result. A completely new calibration technique might be required.

Alternatively, it should be kept in mind that the mass scales (<300 GeV/c^2) for alternative physics probed by an improved version of this experiment may be directly accessible in the near future at the Fermilab collider[34]. At lower energies, measurements of semi-leptonic weak processes[35] can be more sensitive than the δ parameter to universal (rather than purely leptonic) breakdowns of the standard model. However, it follows from just these observations that a measurement of δ significantly different from the value of 3/4 predicted by the standard model would be all the more exciting. Whatever the source, any physics beyond the standard model would be worth the greatest effort in its discovery.

APP A. ENERGY LOSS AND STRAGGLING

The most demanding aspect of the experimental analysis was to accurately model the interactions of positrons with the 240-270 mg/cm^2 of material downstream of the muon decay point. The results were needed in the Monte Carlo simulation and in fitting measured positron spectra (such as the decay edges of Sec. V.A and the straight-through distributions of Sec. V.B.3) with theoretical distributions. The important phenomena were bhabha scattering and bremsstrahlung. Both diverge as the energy loss $\varepsilon = x_i - x_f$ goes to zero:

$$d\sigma_e/d\varepsilon \propto 1/\varepsilon^2, \quad d\sigma_\gamma/d\varepsilon \propto 1/\varepsilon.$$

These divergences are cut off by atomic binding effects, but simplify modelling of the energy loss by a dividing the energy loss spectrum into two parts[8,36]. The most probable energy loss Δ_{mp} accounts for the effects of the "divergent" part of the spectrum. These "low-energy" interactions occur in large enough number that they can be treated by ensemble variables. The straggling curve $f(\chi) = f(x'-\Delta_{mp})$, which peaks at $x' = \Delta_{mp}$ (here x' is the final energy after all straggling events have occurred), has two components: a nearly gaussian peak which describes fluctuations around Δ_{mp}, and a tail for the low-probability "catastrophic" scatters.

The cutoff ε_{min} between "low-energy" and "catastrophic" scatters depends on the experimental situation, and in particular on the accuracy of the event reconstruction. ε_{min} should be small compared to the total resolution σ_t, which combines the gaussian widths of the straggling curve and the spectrometer resolution. For this experiment, the minimum of σ_t was 0.0009 at x=1.00. The cutoff was made at ϵ_{min} = 0.0001. Unfortunately, for the algorithm we used to calculate the straggling curve, events with these values of ϵ still made a significant contribution to the most probable energy loss, and the calculations of Δ_{mp} and $f(\chi)$ could not in fact be completely decoupled. We describe the necessary corrections to Δ_{mp} in the last section of this appendix.

A.1 Straggling Curve

In this section, we discuss the modelling of straggling events with energy loss ϵ>0.0001 to determine the straggling curve $f(\chi)$.

To start, bremsstrahlung and Bhabba scattering cross-sections appropriate to our experimental conditions were culled from the literature. For Bhabba scattering, since the materials of the apparatus were light, the K-shell cutoff[36] K was small compared to ϵ_{min} (K = 163 eV = 3×10^{-6} in x for aluminum[19], the heaviest element in our apparatus). Atomic binding effects were therefore negligible, and the plane-wave approximation of the Bhabba scattering theory was accurate (cf. Ref. 8, Eq. B.1. The energy distribution for Bhabba scattering has been substituted for the corresponding Moller scattering expression):

$$\frac{dI_e}{d\epsilon} = \frac{a}{\epsilon^2} \left(1 - 2z + 3z^2 - 2z^3 + z^4 \right). \qquad (A.1)$$

$dI_e/d\epsilon$ is the scattering probability per gm/cm² in a material, and z is x_f/x_i. The coefficient a scales as Z/A, the atomic number over the atomic mass. For straggling in the aluminum stopping target, the formula was further modified to account for the suppression of Bhabha events by the cuts on multiple hits in P3 (Sec. IV.B. The other multiple track cuts had little additional effect in suppressing straggling events).

For the range of momentum transfers to which we were sensitive, the screening of the nucleus by the electron cloud had to be included when modelling bremsstrahlung interactions (Ref. 9, Formula 3CS):

$$\frac{d\sigma_\gamma}{d\varepsilon} \propto \frac{1}{\varepsilon} \{ z + \frac{3}{4} (1-z)^2 \} \{ \frac{\phi(\nu)}{4} - \frac{1}{3}\ln Z - f(Z) \} ,$$

$$f(Z) = 1.2021 \ (Z/137)^2, \qquad \phi(\nu) = 20.4 - 4\nu/3, \qquad \nu = 0.97 \ \varepsilon/x_i x_f \ .$$

$\phi(\nu)$ is the average of the ϕ_1 and ϕ_2 plotted in Fig. 1 on p. 927 of Ref. 9. The first term in brackets is the uncorrected bremsstrahlung phase space; the second term in brackets gives the screening and coulomb wavefunction corrections. This was used to correct Tsai's formula (Ref. 8, Eq. B.2):

$$\frac{dI_\gamma}{d\varepsilon} = \frac{1}{X_0} \frac{b}{\varepsilon} \{\ldots\} \{\ldots\}, \tag{A.2}$$

$$b = \frac{4}{3} \{1 + \frac{1}{9} \frac{(Z+1)/(Z+\eta)}{\ln(183 \ Z^{-1/3})}\}, \quad \eta = \frac{\ln(1440 \ Z^{-2/3})}{\ln(183 \ Z^{-1/3})} .$$

X_0 is the radiation length in gm/cm², so this is the bremsstrahlung probability per gm/cm².

The straggling curve was calculated in several steps. First, the interaction probability $I(x_i, Z, A)$ for unit thickness of a given element was calculated by integrating (A.1) and (A.2) for $\varepsilon > 0.0005$. Since the total interaction probability was 90%, the apparatus was divided into segments $d(\rho l)$ for which $I d(\rho l) < 10\%$ to allow simulation of more than one catastrophic event. The positron spectrum was then straggled by the distribution (A.1) or (A.2) according to the probabilities $I_e d(\rho l)$ and $I_\gamma d(\rho l)$. In the Monte Carlo simulation a random number generator was used to determine whether and how much energy was lost. In fitting data spectra, the initial positron energy distribution was simply convoluted with the straggling distribution.

To save computational effort, straggling with ε between 0.0001 and 0.0005 was simulated separately. The energy loss was subdivided in small probability steps and the straggling spectra were approximated by $dI_e/d\varepsilon \propto 1/\varepsilon^2$, $dI_\gamma/d\varepsilon \propto 1/\varepsilon$. After accumulating the effects of all the material in the apparatus, the resulting spectrum was normalized to unity. Then, as appropriate to the application, the final distribution was either sampled randomly or used in a convolution to obtain the final positron distribution.

Lastly, the gaussian spreading of the peak was included when fitting the data by smearing, and in the Monte Carlo simulation by including events down to $\varepsilon = 0.00001$ in the calculation described in the paragraph above.

A.2 Most Probable Energy Loss

Δ_{mp} has been calculated for positrons by Rohrlich and Carlson[37]:

$$\Delta_{mp} = \zeta T \left[\ln \frac{\zeta T^2 2(\gamma+1)}{K^2} - \beta^2 + 0.37 - 2.8C\zeta \right] ,$$

$$\zeta = \frac{2\pi e^4}{mc^2\beta^2} N_A \frac{d(\rho l)}{T} \frac{Z}{A} , \qquad C = \beta^2[2-(\gamma+1)^{-2}] .$$

T is the electron kinetic energy, K is again the K-shell cutoff in the Landau theory, and β and γ are the standard relativity parameters. Unfortunately this expression does not include higher-order corrections to the energy loss, such as the "density effect" (polarization of the ionization medium[19]). To account for these effects, which can be substantial, we compared the theoretical expressions for Δ_{mp} and the mean energy loss $D_m = dE/d(\rho l)$ given in Ref. 37. For relativistic positrons:

$$\frac{\Delta_{mp} - D_m}{D_m} = \frac{1.37 + \ln\zeta}{\ln\frac{2T^2(\gamma+1)}{I^2} - 2} .$$

The material-dependent parameters, buried in the logarithms, were relatively constant in our case. For 250 mg/cm^2 of material, $\Delta_{mp} = 0.8D_m$. Taking D_m from the energy loss tables of Ref. 19, which include the higher-order corrections to the energy loss, we obtained a similarly corrected value of Δ_{mp}, accurate to better than 4%.

A different approach was used to generate the correct energy loss in the Monte Carlo simulation. The tabulated D_m was applied at each segment $d(\rho l)$ in the straggling simulation, and then decreased by the average energy loss in the applied straggling curve. The two methods were compared by generating simulated events with x_i = 1.00, 0.86, 0.72, 0.60, 0.50 and 0.42. The final peak positions agreed with the calculated values to within 0.0003 in x.

A.3 Corrections to Δ_{mp}

There were two places in which the calculation of Δ_{mp} had to be refined. First, not all of the most probable energy loss was taken out of the straggling algorithm, and the peak of the curve $f(\chi)$ shifted as a result of compounding the $1/\varepsilon$ and $1/\varepsilon^2$ spectra. This shift had to be eliminated from the edge fits of Sec. V.A. Secondly, because of their finite width, the peaks of the gaussian positron momentum distributions in the calibration data did not shift by exactly $\Delta x = \Delta_{mp}/E_e(max)$. A careful analysis was required to correlate the final peak position with the center of the initial, unstraggled distribution (Sec. V.B.3).

To calculate the first effect, a spike with initial energy 0.9998 was straggled using the algorithm described above. The final peak position was found to be 0.9994. The shift was the same for a spike with initial energy near 0.42. Therefore, independently of the energy, 0.0004 had to be added to the theoretical energies or momenta when making comparisons with the data. This correction was applied in the edge fits, and below.

Next, a gaussian momentum distribution with center at 0.9800 and FWHM of 0.6% was subjected to simulated straggling equivalent to that experienced by beam positrons before reaching the spectrometer. The peak measures y(ave) and y(fit) (Sec. V.B.3) of the final distribution were found to be 0.9785 and 0.9790. Since the most probable energy loss for beam positrons was 0.0093, these converted to data momenta of 0.9696 (= 0.9785-0.0093+0.0004) and 0.9701. Therefore, in correcting y(ave) and y(fit) to the initial gaussian center, the shifts Δx_b = 0.0104 (= 0.98-0.9696) and 0.0099 were applied.

APP B. CALIBRATION FIT

We describe here the calibration fit to the data of Sec. V.B. These data determined the beamline and spectrometer calibration curves (V.1) and (V.2) and the final corrections to the relative momentum y_r obtained after applying the results of Sec. V.A. Using the π-μ and μ-e benchmarks of Secs. V.B.2 and V.B.3, m_b and m_s were eliminated from (V.1) and (V.2) ($Y_{\pi\mu}$ = 0.5639, $Y_{\mu e}$ = 1.0000):

$$Y_b(B1,B1_o) = Y_{\pi\mu}\,\frac{B1 - B1_o}{B1_{\pi\mu}-B1_o}\,, \quad Y_s(B_s,B_{so}) = Y_{\mu e}\,\frac{B_s - B_{so}}{B_{s,\mu e}-B_{so}}\,.$$

As discussed in Sec. V.B.1, the two inconsistent measurements of $B1_{\pi\mu}$ required distinct calibration fits. The spectrometer μ-e measurement $B_{s,\mu e}$ = 3186.7 G was highly reproducible. The corrections to the relative momentum y_r were taken to be second-order in y_r and constrained to be zero at y_r=0.9917, as discussed in Sec. V.B.4. The transformation which converted y_r to the initial positron momentum y was therefore of the form:

$$y-\Delta x = Y_s\,\{y_r + a(y_r-0.9917) + b(y_r-0.9917)^2\} \qquad (B.1)$$

Δx is the energy loss experienced by the positron before reaching the spectrometer. a and b, the linear and quadratic coefficients of the corrections to y_r, were allowed to vary linearly with Y_s:

$$a = a_0 + a_1(1-Y_s), \qquad\qquad b = b_0 + b_1(1-Y_s)$$

The free parameters to be determined in the calibration fit were therefore $B1_o$, B_{so}, a_0, a_1, b_0 and b_1.

Each of the thirty calibration points (twenty-nine in the later data set) contributed three pieces of information to the fit (i indexes the data point): $B1^i$ (or $B2^i$), $y_r(\mathrm{ave})^i$ (or $y_r(\mathrm{fit})^i$), and $B_s{}^i$. With the energy loss experienced by the beam positrons in traversing the apparatus to reach the spectrometer calculated to be Δx_b=0.0104 (or 0.0099 for $y_r(\mathrm{fit})$), we made the identifications in (B.1):

$$y = Y_b(B1^i, B1_0), \quad Y_s = Y_s(B_s{}^i, B_{s0}), \quad y_r = y_r(\mathrm{fit})^i.$$

The relative calibration data of Tbl. 2 were also included in the fit, which contributing nine points (indexed by j):

$$y-\Delta x_s = 0.9917, \quad Y_s = Y_s(B_s{}^j, B_{s0}), \quad y_r = y_{r,\mu e}{}^j.$$

These helped to stabilize the corrections to y_r in the later calibration data, which were very sensitive to fluctuations in the points at Φ_s=1.00 and 0.42.

The results of a chi-squared fit to these data are exhibited in Tbl. 10 for both the early and late sets of calibration data. The change in B_{s0} when calibrating the beamline with B2 rather than B1 is also given (the corrections to y_r were unaffected). With a systematic error σ_Y = 0.0005 assigned to the data points, we obtained a χ^2/DOF ≈ 1. However, this measure of the calibration accuracy is misleading. As discussed in Sec. V.B.4, differences between the two calibrations were often nearly as large as 0.0015. Correspondingly, the differences between the parameters fit using the early and late data were all much larger than their fit one standard deviation errors. It was therefore our inability to reproduce the calibration conditions, rather than the fundamental limitations of the beamline design, which limited the precision.

APP C. π-μ CALIBRATION SHIFT

As discussed in Sec. VII.B.1, the shift in the beamline π-μ calibration point between the early and later calibration runs was an important contributor to one of the larger errors in the determination of δ. Here we complete the description of the analysis of the π-μ calibration point begun in Sec. V.B.1, and then consider explanations for the shift. Three effects were expected to be accounted for by a successful explanation: the shift in the π-μ point; the difference in the relative shifts in B1 and B2 (0.09% vs. 0.17%); and the shift in the spectrometer zero point. A beamline hysteresis hypothesis satisfied all three criterion, but did not enable an unambiguous selection of one measurement over the other. Thus, despite the fact that the second measurement was clearly flawed, the two measurements were accepted with equal weight in the analysis.

The data for the two calibrations (Fig. 16) were fit to the form

$$F = \frac{(\mu STOP)}{(BEAM)} = \frac{N\ I + F_o}{1 + N\ I} . \qquad (C.2)$$

F_o is the pedestal signal above the $\pi\mu$ edge where the muon flux is dominated by pion decay in flight at 1AT1, $I = I(B1-B1_{\pi\mu},\sigma_b)$ is the overlap of the gaussian beamline acceptance and the theta-function momentum distribution for surface muons arising from pion decay at rest at 1AT1, σ_b is the gaussian width in Y_b of the beamline acceptance, and N is a flux factor for the surface muons. F_o, N, $B1_{\pi\mu}$ and σ_b were determined in a chi-squared fit to the data of Fig. 16. As discussed in Sec. V.B.1, the two data sets gave significantly different results:

for the earlier calibration $B1_{\pi\mu}$ = 875.6 G and $B2_{\pi\mu}$ = 954.3 G; while for the later $B1_{\pi\mu}$ = 874.8 G and $B2_{\pi\mu}$ = 952.6 G. The statistical uncertainty in the fit values was 0.2 G.

We now carefully consider Fig. 16. Since muons accounted for less than 2% of the beam flux above the π-μ edge (Fig. 6), the pedestal must have been due primarily to inefficiencies for positrons in the downstream chambers resulting in spurious μ-stop signals (Sec. V.B.1). Clearly, the dramatic increase in the threshold seen in the later calibration required a dramatic change in the experimental conditions. The increase in the proton current on 1AT1 from 30 μA (earlier calibration) to 130 μA (main data taking and later calibration), and a corresponding four-fold increase in M13 beam flux, stands out. We hypothesize that the downstream chambers began to saturate (i.e. become unresponsive to positron hits) at the higher flux. Since the loss in efficiency indicated was large (40%), it might seem reasonable to reject the second measurement altogether. However, that would leave us without a check on the π-μ calibration. Furthermore, inefficiency in the downstream chambers should only change the pedestal F_0 in (C.1). Saturation could produce a change in $B1_{\pi\mu}$ only in complicated scenarios in which the upstream chambers were inefficient for positrons, and that inefficiency was dependent on the muon flux. The rejection of straight-through positrons seen in Fig. 12, indicating high efficiency in the upstream chambers, ruled out such possibilities.

Secondly, we considered the potential effects of hysteresis in the beamline bending magnets. The calibration procedure (Sec. V.B.3) called for saturation of B1 and B2 at maximum current each time the central fields were changed, although dipole fields were thereby induced in Q2 and Q6. The complicating factor was that the strength of the induced dipole component in Q2, for example, depended on whether its current was set before or after B1 was saturated. Unfortunately, no particular procedure was adopted to ensure consistency in the data as concerns this point. In studies done after the experiment, the two cases were observed to give a 0.14% difference in $B1_{\pi\mu}$. This agrees well with the differences seen between the two π-μ measurements: 0.09% in $B1_{\pi\mu}$ and 0.17% in $B2_{\pi\mu}$.

Long-term magnet and power supply instabilities were also investigated, in this case using the measured positron endpoints in the Φ_S=1.00 data and the beam positron peaks in the Φ_S=0.50 and 0.60 data. Both the beamline and spectrometer were found to be very stable. As already noted (Sec. V.B.2), the endpoint in the spin-precessed data varied by only ±0.0002 over the course of the experiment. Similarly, analysis of the straight-through positron peak (Fig. 12, Sec. V.B.3) showed y(ave)=0.5508(3) for the Φ_S=0.50 runs, and y(ave)=0.5504(2) for the Φ_S=0.60 runs. Only one significant difference was seen, when the data were divided into samples taken before and after the later calibration. The twenty earlier runs had y(ave) on average 0.0003 higher than the later eight (the earlier calibration fit results (App. B) were used in analyzing both samples). Since the cyclotron was retuned following maintenance just before the later calibration, such a shift was not surprising: slight changes in the proton spot at 1AT1 and thus in the beamline momentum bite were to be expected (see below). However, the difference is not significant compared to the 0.0011 difference in the two beamline calibrations (Eq. V.1) at these momenta.

The observed stability of the spectrometer eliminated changes in the spectrometer field as an explanation of the early-late difference in B_{s_0}. Other effects could only cause the shift by introducing an error into the calibration curve (V.1) for the beamline. The beamline hysteresis effect described above is a good example. In that case, one of the calibrations is simply wrong. When it is used to calibrate the momentum of beam positrons, the systematically incorrect results yield an apparent shift in B_{s_0}. Conversely, motion of the proton spot on 1AT1 cannot explain the change in B_{s_0}. Changes of less than 1 mm were allowed by the monitoring equipment (Sec. III.B), and might have been expected when the proton current was increased from 30 to 130 μA or following cyclotron retuning. Any systematic horizontal motion would have been reflected in the source distributions seen by M13, and would have appeared as a shift of up to 0.0005 in Y_b at the π-μ calibration point (Sec. III.B). This effect could not explain the change in B_{s_0}, though, since the motion of the proton spot would have had the same effect on the muon and positron sources. Although the calibration curve (V.1) might have changed, it would still yield correct results for the transmitted positron momentum.

In conclusion, though many effects were clearly involved in creating superficial differences between the two π-μ calibration measurements, it seemed likely that hysteresis effects in Q2 and Q6 were mainly responsible for the corresponding changes in the spectrometer calibration results, and in particular for the change in B_{s_0}. Unfortunately, it is not possible under this hypothesis to choose one of the calibration points over the other. Furthermore, we checked that the individual calibration points and data sets were not directly correlated. For example, fitting both calibration data sets independently using the first π-μ measurement gave consistent results for the spectrometer calibration curve (V.2). After accounting for the 0.0003 shift in the beamline setting following cyclotron retuning, the difference in (V.2) was only 0.0004 in y at ϕ_s=0.50 (in comparison, the systematic error on the data points was 0.0005 in y and the π-μ calibration discrepancy corresponded to a difference at ϕ_s=0.50 of 0.0013 in y). Fitting both data sets independently with the second calibration point also gave consistent results. Therefore, no correlation of the individual π-μ calibrations with the individual calibration data sets could be made. All possible permutations were considered with equal weight in evaluating systematic errors on the decay parameters in Sec. VII.

APP D. CONSTRAINTS ON ALTERNATIVE PHYSICS

We describe here the asymmetry analysis (Sec. VIII.B) used to set limits on the parameters in L-R symmetric and supersymmetric theories for cases in which they contribute to muon decay.

In general, there are three unknown masses in these theories: the masses of the two undetected particles in the final state (n_e and n_μ, where n is either ν_R or $\bar{\nu}$), and the mass of the gauge particle (Ω, which is either W_R or $\tilde{W}$). To simplify the analysis, we set $m(n_e) = 0$. This leaves two parameters in the theory: $\varepsilon = (m(W_L)/m(\Omega))^4$, and $r = (m(n_\mu)/m_\mu)^2$.

D.1 L-R Symmetric Model

Shrock has calculated the muon decay spectrum for massive neutrinos[38]. Adapting his results for a V+A decay with $m(\nu_{eR})=0$, we have

$$\frac{d^2\Gamma_R}{dx\, d\cos\theta} \propto \Theta(1-r_R-x)\ \varepsilon_R\ E^2\ [\ 3-2x + r_R \frac{3-x}{1-x}$$

$$- \cos\theta\ (1-2x - r_R \frac{1+x}{1-x})\ ]$$

$$E = 1 - \frac{r_R}{1-x}\ , \qquad \varepsilon_R = \frac{m(W_L)^4}{m(W_R)^4}\ , \qquad r_R = \frac{m(\nu_{\mu R})^2}{m_\mu^2}\ .$$

We have normalized the decay spectrum with (II.1), so the isotropic and anisotropic components of the combined spectrum can be seen to be

$$R_R(\mathrm{iso}) = 3-2x + \alpha h(x) + \theta(1-r_R-x)\ \varepsilon_R\ E^2\ (\ 3-2x + r_R \frac{3-x}{1-x}\ )$$

$$R_R(\mathrm{ani}) = 1-2x + \alpha g(x) - \theta(1-r_R-x)\ \varepsilon_R E^2\ (\ 1-2x - r_R \frac{1+x}{1-x}\ )\ . \tag{D.1}$$

h(x) and g(x) represent only the internal radiative corrections. The asymmetry is then found to be (cf. (II.8))

$$M_{new}(x) = P_\mu \frac{R(\mathrm{ani})}{R(\mathrm{iso})}\ . \tag{D.2}$$

We compared this to the measured asymmetries (Tbl. 6) after making the corrections for external radiative effects.

D.2 Supersymmetric Models

Buchmuller and Scheck[30] have calculated the Wino-mediated muon decay spectrum:

$$\frac{d^2\Gamma_S}{dx\ d\cos\theta} \propto \theta(1-r_S-x)\ \varepsilon_S\ E^3\ [\ 3-x + \cos\theta\ (1+x)\ ]$$

$$E = 1 - \frac{r_S}{1-x}\ , \qquad \varepsilon_S = \frac{m(W_L)^4}{m(\tilde{W})^4}\ , \qquad r_S = \frac{m(\tilde{\nu}_\mu)^2}{m_\mu^2}\ .$$

Corresponding to (D.1) we have

$$R_S(\mathrm{iso}) = 3-2x + \alpha h(x) + \theta(1-r_S-x)\ \varepsilon_S\ E^3\ (\ 3-x\ )$$

$$R_S(\mathrm{ani}) = 1-2x + \alpha g(x) - \theta(1-r_S-x)\ \varepsilon_S E^3\ (\ 1+x\ )\ . \tag{D.3}$$

We used these expressions in (D.2) and compared to the data, as described next, to set the mass limits.

D.3 Description of the Fit

After averaging over the targets and precession frequencies, we had 58 data points for comparison with (D.2) (for reasons that will be clear below, we could not combine the spectrometer settings). For a given value of r, we mapped out the χ^2 distribution as a function of ε for $M(\Omega)$ between $M(W_L)/2$ and infinity. We converted the χ^2 to probability using the approximation, which holds that for large N_D (number of degrees of freedom), that $\sqrt{(2\chi^2)} - \sqrt{(2N_D-1)}$ is normally distributed with unit standard deviation[39]. From the final probability distribution we found the 90% confidence limits on ε, and thus the gauge mass.

To calculate the χ^2, we first had to know $E_{ij} = \sigma_i \sigma_j$, the 58×58 error matrix for the asymmetries. The σ_i were dominated by statistics and the uncertainty in the calibration and external radiative corrections. The contribution from the radiative corrections was taken to be 5% of the corrections themselves: $\sigma_{i,rad} = 0.05\ \Delta M(x,Y_s)$ (see Tbl. 6b). The contribution from the calibration was found from the discrepancy $\Delta x(x,Y_s)$ between the first and seventh fits of Tbl. 4 (the pair which showed the greatest disagreement):

$$\sigma_{i,cal} = \left.\frac{\partial M}{\partial x}\right|_x \Delta x(x,Y_s) \ .$$

This was a conservative estimate, as not all components of the error were correlated (Sec. VII.C.1). The total error matrix was then

$$E_{ij} = \delta_{ij}\sigma^2_{i,stat} + \sigma_{i,cal}\sigma_{j,cal} + \sigma_{i,rad}\sigma_{j,rad} \ ,$$

and the χ^2 for given ε and r was calculated (using (D.2))

$$\chi^2 = \frac{\sum_{i,j} M_{new}(x_i)\,(E^{-1})_{ij}\,M_{data}(x_j)}{\sum_{i,j} M_{new}(x_i)\,(E^{-1})_{ij}\,M_{new}(x_j)}.$$