MUON DECAY: COMPLETE DETERMINATION OF THE INTERACTION AND COMPARISON WITH THE STANDARD MODEL

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Present experiments determine the most general (local, derivative-free, lepton-number conserving) leptonic four-fermion interaction hamiltonian of the normal and inverse muon decay. Numerical results are given for all ten complex coupling constants and nine T-violating amplitudes with respect to the "helicity projection form".

The interaction of electrons, muons and their neutrini at low energies is successfully described by the most general, local, derivative-free lepton-number conserving four-fermion interaction hamiltonian [1]. It contains ten complex coupling constants or, since a common phase is arbitrary, nineteen independent real parameters. Without any further assumptions these are completely determined, as this paper will show, in the following way: one parameter by the muon-lifetime measurement, sixteen by further muon-decay measurements, and the remaining two by inverse muon decay. The Fierz transformations [2] allow various forms for the hamiltonian. We will choose the "helicity projection form" [3–5] to represent the results. This form (abbreviated HPF) uses fields of definite handedness. Since the charged weak interaction is seen to be dominated by a coupling to left-handed fermions, the HPF is closest to a physical interpretation, in which $V-A$, the charged current interaction adopted by the standard model, is taken as a basis, from which deviations are looked for.

We denote the matrix element by

$$M \sim \sum_{\substack{\gamma=S,V,T \\ \epsilon,\mu=L,R \\ (n,m)}} g^{\gamma}_{\epsilon\mu} \langle \bar{e}_{\epsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle (\bar{\nu}_{\mu})_m | \Gamma_{\gamma} | \mu_{\mu} \rangle . \quad (1)$$

γ labels the type of interaction: Γ^S, Γ^V, Γ^T (scalar–pseudoscalar, vector–axialvector, tensor); the indices ϵ and μ indicate the chiral projections (left-handed, right-handed) of the spinors of the experimentally observed particles, $\epsilon \,\hat{=}\,$ electron, $\mu \,\hat{=}\,$ muon; n indicates the helicity of the electron antineutrino, m that of the muon neutrino. n and m are uniquely determined for given γ, ϵ, μ. See table 1.

We normalize the coupling constants $g^{\gamma}_{\epsilon\mu}$, by taking out a common factor which is determined by the total decay rate, to

$$A \equiv 4(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2) + 16(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2) + 48(|g^T_{RL}|^2 + |g^T_{LR}|^2) = 16 . \quad (2)$$

From the relations to the observables, which have been worked out previously [3–5], we deduce

$$Q_{RR} \equiv \tfrac{1}{4}|g^S_{RR}|^2 + |g^V_{RR}|^2 = 2(b+b')/A , \quad (3)$$

$$Q_{LR} \equiv \tfrac{1}{4}|g^S_{LR}|^2 + |g^V_{LR}|^2 + 3|g^T_{LR}|^2 = [(a-a') + 6(c-c')]/2A , \quad (4)$$

$$Q_{RL} \equiv \tfrac{1}{4}|g^S_{RL}|^2 + |g^V_{RL}|^2 + 3|g^T_{RL}|^2 = [(a+a') + 6(c+c')]/2A , \quad (5)$$

$$Q_{LL} \equiv \tfrac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2 = 2(b-b')/A , \quad (6)$$

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Table 1
The complete set of coupling constants $g^{\gamma}_{\epsilon\mu}$ of the muon-decay interaction (1) determined from experiments without model assumptions. The "standard model" sets $g^{V}_{LL} = 1$, all others to zero. An additional $V + A$ interaction is represented by g^{V}_{RR}. The $Q_{\epsilon\mu}$ are the relative rates. The $\tau^{\gamma}_{\epsilon\mu}$ are normalized T-violating amplitudes. 68% (90%) CL is given.

Handednesses of electron (ϵ) and muon (μ) $\epsilon\mu$	Relative abundance $10^3 \times Q_{\epsilon\mu}$	Type of interaction γ	Handednesses of ν_μ (m) and $\bar{\nu}_e$ (n) mn	Coupling constant $10^3 \lvert g^{\gamma}_{\epsilon\mu} \rvert$	Equations used to derive limit for $\lvert g^{\gamma}_{\epsilon\mu} \rvert$	Main input	Corresponding coupling constant of refs. [3,4]	Violation of T-invariance ($\lvert\tau^{\gamma}_{\epsilon\mu}\rvert \leqslant 1$) $10^3 \times \lvert\tau^{\gamma}_{\epsilon\mu}\rvert$
RR	<1.4(2.0)	S	LR	<74(91)	(3)	$\xi\delta/\rho$	h_{21}	<74(91)
		V	RL	<37(45)	(3)		g_{11}	<74(91)
LR	<2.7(3.9)	S	LL	<116(137)	(4)		h_{11}	<116(137)
		V	RR	<52(62)	(7)	$\xi\delta/\rho$, ρ, δ	g_{21}	<104(125)
		T	LL	<34(40)	(4)		f_{11}	<116(137)
RL	<36(45)	S	RR	<378(448)	(5)		h_{22}	<371(436)
		V	LL	<97(114)	(8)	$P_L(\xi')$	g_{12}	<192(227)
		T	RR	<95(112)	(5), (8)		f_{22}	<325(383)
LL	>960(949)	S	RL	<743(961)	(14)	S, h_{ν_μ}	h_{12}	<690(843)
		V	LR	>928(877)	(13)	Q_{RR}, Q_{LR}, Q_{RL}	g_{22}	–

$$B_{LR} \equiv \tfrac{1}{16}|g^S_{LR} + 6g^T_{LR}|^2 + |g^V_{LR}|^2 = (a - a')/2A\,, \quad (7)$$

$$B_{RL} \equiv \tfrac{1}{16}|g^S_{RL} + 6g^T_{RL}|^2 + |g^V_{RL}|^2 = (a + a')/2A\,, \quad (8)$$

$$I_\alpha \equiv \tfrac{1}{4}g^V_{LR}(g^S_{RL} + 6g^T_{RL})^* + \tfrac{1}{4}g^{V*}_{RL}(g^S_{LR} + 6g^T_{LR}) = (\alpha + i\alpha')/2A\,, \quad (9)$$

$$I_\beta \equiv g^V_{LL}g^{S*}_{RR}/2 + g^{V*}_{RR}g^S_{LL}/2 = -2(\beta + i\beta')/A\,. \quad (10)$$

The parameters $\{a/A, ..., \beta'/A\}$ on the right-hand side of eqs. (3)–(10) have been introduced [6] to express all possible results of the measurements on the electron (positron) in the decay of polarized (and unpolarized) muons. Their values are known [7] for positive muon decay. The main property of eqs. (3)–(10) is, that they contain the maximum possible number of positive semidefinite quadratic forms of the coupling constants, which are determined (as linear combinations) by the observables. We note:

$$0 \leqslant Q_{\epsilon\mu} \leqslant 1 \quad (\epsilon, \mu = \mathrm{R, L})\,,$$

$$\sum_{\epsilon\mu} Q_{\epsilon\mu} = 1\,. \quad (11)$$

Thus a $Q_{\epsilon\mu}$ is the relative probability for the decay of a muon of handedness μ into an electron of handedness ϵ, irrespective of the neutrino helicities. Since the experiments evaluate the three quadratic forms Q_{RR}, Q_{LR}, Q_{RL} to be zero, within error, upper limits follow immediately for the absolute values of all of the corresponding eight coupling constants $g^\gamma_{RR}, g^\gamma_{LR}$, g^γ_{RL}, γ = S, V, T. However $Q_{LL} \neq 0$. Inspection shows, as already pointed out in ref. [4], that muon-decay measurements without neutrino observations *do not*, by themselves, determine the interaction to be $V - A$, that is $g^S_{LL} = 0, g^V_{LL} = 1$. (In the extreme, they are compatible with a pure scalar type interaction $g^V_{LL} = 0, g^S_{LL} = 2$, but the two unobserved neutrini would both have to have the "wrong" helicity.)

We can resolve this remaining ambiguity, however, by including the data from the inverse muon decay $\nu_\mu + e^- \to \mu^- + \nu_e$. The total rate S, normalized to the rate predicted by V–A, is found to be $S = 0.98 \pm 0.12$ [8,9]. This rate also has been expressed in terms of the most general four-fermion interaction in ref. [10]. The upper limits deduced from normal muon decay show that the influence of eight of the coupling constants on S is negligible. We get in the HPF form

$$S = |g^V_{LL}|^2(1 - \epsilon) + \tfrac{3}{64}|g^S_{LL}|^2\epsilon\,, \quad (12)$$

where $\epsilon = h - (-1)$ is the deviation of the helicity h of the muon neutrino in pion decay from -1. The helicity has been measured for ν_μ [11] and for $\bar{\nu}_\mu$ [12]. ϵ is known to be $< 4.1 \times 10^{-3}$ (90% CL). (This conclusion has been reached without any assumption on the pion-decay mechanism, except for angular-momentum conservation [13].)

Since certainly $|g^S_{LL}|^2 \leqslant 4$, the second term in eq. (12) can be neglected, and we obtain

$$|g^V_{LL}|^2 = S\,, \quad (13)$$

which leads to a lower limit for $|g^V_{LL}|$. Since furthermore $\frac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2 \leqslant 1$, we get an upper limit

$$|g^S_{LL}|^2 \leqslant 4(1 - S)\,. \quad (14)$$

We have thus found experimental, model-independent bounds for all coupling constants $g^\gamma_{\epsilon\mu}$: a lower bound for $|g^V_{LL}|$ from inverse muon decay and upper bounds for all the other $|g^\gamma_{\epsilon\mu}|$ from normal muon decay.

The numerical results of table 1 are based on the values of ref. [7], except that $\delta = 0.7502 \pm 0.0043$ [14] has been used, on S from ref. [8], and on h_{ν_μ} from ref. [13]. [In accordance with eq. (1) the table is expressed in terms of μ^- decay although the experiments have measured μ^+ decay.] The limits found for the absolute values of the complex coupling constants imply also limits for their T-violating imaginary parts. In order to give a quantitative representation for the limits of T invariance, we introduce

$$\tau^\gamma_{\epsilon\mu} = \operatorname*{extr}_{\gamma'\epsilon'\mu'} \; [2\,\mathrm{Im}\{(g^{\gamma'}_{\epsilon'\mu'}/M^{\gamma'})(g^\gamma_{\epsilon\mu}/M^\gamma)^*\}]$$

with the range $-1 \leqslant \tau^\gamma_{\epsilon\mu} \leqslant +1$, M^γ being the theoretical maximum of $|g^\gamma_{\epsilon\mu}|$, ($M^S = 2, M^V = 1, M^T = 1/\sqrt{3}$). $\mathrm{extr}_{\gamma'\epsilon'\mu'}[x]$ designates the value of x which maximally deviates from zero with respect to the choice of the primed indices. Upper limits for all nine $|\tau^\gamma_{\epsilon\mu}|$ are calculated according to

$$|\tau^\gamma_{\epsilon\mu}| \leqslant 2|g^\gamma_{\epsilon\mu}/M^\gamma|\,[1 - |g^\gamma_{\epsilon\mu}/M^\gamma|^2]^{1/2}\,,$$

and are displayed in table 1. These results are compatible with α'/A and β'/A, which have been deduced from a direct measurement of the T-violating component of the transverse polarization of the positron from μ^+ decay [7].

To discuss eqs. (9), (10), we note that $|I_\alpha|^2 \leqslant B_{RL}B_{LR}$ and that $|I_\beta|^2 \leqslant Q_{RR}Q_{LL}$. The first of these inequalities sets a bound to $|\alpha + i\alpha'|$ and has been used to improve the results for β/A and β'/A in the numerical analysis of the transverse-polarization measurement [7]. The second inequality is fulfilled by the data far away from the equality sign, confirming the independence of the information contained in β/A and β'/A from the other measurements.

To estimate the influence of various experiments on the coupling constants, the following expressions have been derived:

$$Q_{RR} = \tfrac{1}{2}[Y4\rho/3 - O(Y)] \cong \tfrac{1}{2}[Y - O(Y)] \,, \qquad (15)$$

$$Q_{LR} \cong \tfrac{1}{2}[Y + R + D/3 - O(Y)] \,, \qquad (16)$$

$$Q_{RL} = \tfrac{1}{2}[X - O(Y)] \,, \qquad (17)$$

$$Q_{LL} \equiv 1 - (Q_{RR} + Q_{RL} + Q_{LR}) \,, \qquad (11')$$

$$B_{LR} \cong \tfrac{1}{8}[Y + 4R + 4D/3 - O(Y)] \,, \qquad (18)$$

$$B_{RL} \cong \tfrac{1}{8}[X + 3R - D - O(Y)] \,, \qquad (19)$$

where $X = 1 - \xi'$, $Y = 1 - \xi\delta/\rho$, $R = 1 - 4\rho/3$, $D = 1 - 4\delta/3$ are the small, measured deviations from the $V-A$ values. $O(Y)$ allows one to judge the accuracies of the estimations: $0 \leqslant O(Y) \leqslant |Y|$. Since these expressions do not take into account the constraints discussed in ref. [7], they tend to somewhat underestimate the limits.

We may now directly read off the connection of an experimental result to a particular coupling constant. For instance, a possible $V+A$ admixture, given by g^V_{RR}, contained in Q_{RR}, is limited by the accuracy of the $P_\mu\xi\delta/\rho$ measurement, or, an improvement on the knowledge of possible admixtures of scalar, vector or tensor type couplings to a right-handed electron (Q_{RL}) needs an improvement of the basically most difficult of the present normal decay experiments, the measurement of the longitudinal electron polarization, ξ'.

We wish to emphasize that all the results of this paper on the coupling constants as summarized in table 1, have been obtained entirely from experiments on normal and inverse muon decay *without* any model assumption besides the most general, local, derivative-free, lepton-number conserving four-fermion interaction.

Various specific theoretical models make predictions for the couplings. The standard model sets $g^V_{LL} = 1$, all other $g^\gamma_{\epsilon\mu} = 0$. Left–right symmetric models with an intermediate boson coupling to $V+A$ currents [15] also allow $g^V_{RR} \neq 0$ [3]. A model motivated by supersymmetry with an additional muon decay into an electron and two (unobserved) photini mediated by heavy scalar leptons, proposed in ref. [16], is described by the four $g^S_{\epsilon\mu}$, $\epsilon, \mu = $ L, R.

The decay of a muon into an electron and two (unobserved) supersymmetric light scalar neutrini, mediated by a wino, as studied by Buchmüller and Scheck [17], constitutes an example of an interaction, which leads to electron observables that *cannot* be represented accurately by the parametrization of the four-fermion interaction. (There exists no value of the δ parameter which would correctly describe the energy dependence of the decay asymmetry of the fully left-handed electron.)

A variety of further theoretical models has been discussed and compared to experiments in ref. [4].

In summary, we have shown that the weak interaction in muon decay is completely determined by existing experiments: Measurements on normal muon decay (without detection of the neutrini), on inverse muon decay, and helicity measurements of the muon neutrino yield a lower limit for $V-A$ and upper limits for *all* other interactions (including e.g. $V+A$). Present experimental errors still allow substantial contributions from interactions other than $V-A$. Our results also include upper limits for all nine independent, T-violating amplitudes in the muon-decay interaction.

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