BETA DECAY AND MUON DECAY BEYOND THE STANDARD MODEL

Peter Herczeg
Theoretical Division, Los Alamos National Laboratory
Los Alamos, NM 87545, USA

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I. INTRODUCTION

Nuclear beta decay and the usual decay of the muon are among the oldest tools of particle physics. Their study helped to develop the electroweak component of the Standard Model (SM) [1,2]. Today their main role is to probe for possible deviations from the predictions of this model. Although the SM is consistent with all observations, for many theoretical reasons, and especially because of the large number of undetermined parameters in the model, the existence of new physics is expected.

In beta decay and muon decay new physics can manifest itself through the effects of neutrino mass and mixing, and through new contributions to the decay interaction. Comparison of the superallowed $0^+ \to 0^+$ beta decay rates and the muon decay rate provides the ud-element of the Kobayashi-Maskawa (KM) matrix. Limits on charged-current universality, expressed through the unitarity relation for the matrix elements of the KM matrix, set constraints on some classes of new decay interactions, and also on some additional types of new physics.

In this article we shall review and discuss new beta decay and muon decay interactions, and the role of the beta decay and muon decay experiments in obtaining information on them. Experimental aspects of searches for new interactions in beta decay and muon decay are discussed in Ref. [3] and Ref. [4], respectively. The subject of charged-current universality is reviewed in Ref. [5]. For the status of searches for neutrino mass and heavy neutrinos in beta decay and for analyses of limits on neutrino mixing from muon decay we refer the reader to the articles in Ref. [6].

Section II and Section III of this review deals with beta decay and muon decay, respectively. In Section IV we summarize our conclusions.

II. BETA DECAY

II.1. Introduction

In the SM the $d \to u e^- \overline{\nu}_e$ (and $u \to d e^+ \nu_e$) transition underlying beta decay arises from W-exchange, and has the V-A form [7]

$$H = (G U_{ud}/\sqrt{2})\, \overline{e}\gamma_\lambda(1-\gamma_5)\nu_e\, \overline{u}\gamma^\lambda(1-\gamma_5)d \;+\; \text{H.c.}\,, \qquad (1)$$

where $G/\sqrt{2} = g^2/8M_W^2$, and U_{ud} is the ud-element of the Kobayashi-Maskawa matrix. The field $\frac{1}{2}(1-\gamma_5)\,\nu_e$ in the interaction (1) represents a massless two-component neutrino, which is the $T_z = +1/2$ state of the $SU(2)_L$ doublet involving the electron.

Although the SM interaction dominates beta decay, the data still allow sizable contributions from possible new interactions [3]. We shall consider here only such contributions from new physics which appear already at the tree-level [8]. Tree-level contributions can be described at the quark level by nonderivative local four-fermion couplings [9]. The most general form of such interactions for the $d \to u e^- \nu_e$ transition can be written as

$$H_\beta = H_{V,A} + H_{S,P} + H_T\,, \qquad (2)$$

where

$$H_{V,A} = \bar{e}\gamma^{\lambda}(1-\gamma_5)\nu_e^{(L)}\,[a_{LL}\,\bar{u}\gamma_\lambda(1-\gamma_5)d + a_{LR}\,\bar{u}\gamma_\lambda(1+\gamma_5)d] + \bar{e}\gamma^{\lambda}(1+\gamma_5)\nu_e^{(R)}\,[a_{RR}\,\bar{u}\gamma_\lambda(1+\gamma_5)d + a_{RL}\,\bar{u}\gamma_\lambda(1-\gamma_5)d] + \text{H.c.}\,, \quad (3)$$

$$H_{S,P} = \bar{e}(1-\gamma_5)\nu_e^{(L)}\,[A_{LL}\,\bar{u}(1-\gamma_5)d + A_{LR}\,\bar{u}\,(1+\gamma_5)d] + \bar{e}(1+\gamma_5)\nu_e^{(R)}\,[A_{RR}\,\bar{u}(1+\gamma_5)d + A_{RL}\,\bar{u}(1-\gamma_5)d] + \text{H.c.}\,, \quad (4)$$

$$H_T = \alpha_{LL}\,\bar{e}\,\frac{\sigma_{\lambda\mu}}{\sqrt{2}}\,(1-\gamma_5)\,\nu_e^{(L)}\,\bar{u}\,\frac{\sigma_{\lambda\mu}}{\sqrt{2}}\,(1-\gamma_5)d + \alpha_{RR}\,\bar{e}\,\frac{\sigma_{\lambda\mu}}{\sqrt{2}}\,(1+\gamma_5)\,\nu_e^{(R)}\,\bar{u}\,\frac{\sigma_{\lambda\mu}}{\sqrt{2}}\,(1+\gamma_5)d + \text{H.c.} \quad (5)$$

Our notation in Eqs. (3) - (5) is such that the first and the second subscript on the coupling constants gives, respectively, the chirality of the neutrino and of the d-quark. The fields e, u and d are mass-eigenstates. Note that there are no tensor couplings of the α_{LR}- and α_{RL}-type, due to the relation $\sigma_{\lambda\mu}\gamma_5 = \frac{1}{2}\,i\epsilon_{\lambda\mu\alpha\beta}\,\sigma^{\alpha\beta}$. The interactions (3) - (5) are time reversal invariant only if all the coupling constants can be made real.

In Eqs. (3) - (5) we have assumed that in all interaction terms the electron couples only to the neutrino states $\nu_e^{(L)}$ and $\nu_e^{(R)}$, where $\nu_e^{(L)}$ is the neutrino state in the $W^+ \to e^+\nu_e^{(L)}$ amplitude and $\nu_e^{(R)}$ is a right-handed singlet state. Couplings involving other neutrino states are possible, but for these in most cases additional constraints apply [10].

In general $\nu_e^{(L)}$ and $\nu_e^{(R)}$ are, respectively, linear combinations of the left-handed and the right-handed components of the neutrino mass-eigenstates ν_i

$$\nu_e^{(L)} = \sum_i U_{ei}\,\nu_{iL}\,, \quad (6)$$

$$\nu_e^{(R)} = \sum_i V_{ei}\,\nu_{iR}\,, \quad (7)$$

where $\nu_{iL} = \frac{1}{2}(1-\gamma_5)\,\nu_i$, $\nu_{iR} = \frac{1}{2}(1+\gamma_5)\,\nu_i$; U_{ei} and V_{ei} are (in a basis where the charged leptons are diagonal) elements of the neutrino mixing matrix [11].

Let us consider the decay of the nucleon due to the interaction (2) in the case when only a single neutrino mass-eigenstate is involved, i.e. when $\nu_e^{(L)}$ and $\nu_e^{(R)}$ are, respectively, the left-handed and right-handed components of a (Dirac) mass-eigenstate ν_e. Neglecting the induced form factors (see Ref. [12]), the effective interaction describing $n \to pe^-\overline{\nu}_e$ is given by

$$H_\beta^{(N)} \simeq H_{V,A}^{(N)} + H_S^{(N)} + H_T^{(N)}\,, \quad (8)$$

where

$$H_{V,A}^{(N)} = \bar{e}\gamma_\lambda(C_V + C_V'\gamma_5)\,\nu_e\,\bar{p}\gamma^\lambda n + \bar{e}\gamma_\lambda\gamma_5(C_A + C_A'\gamma_5)\,\nu_e\,\bar{p}\gamma^\lambda\gamma_5 n + \text{H.c.}\,, \quad (9)$$

$$H_S^{(N)} = \bar{e}(C_S + C_S'\gamma_5)\,\nu_e\,\bar{p}n + \text{H.c.} \quad (10)$$

$$H_T^{(N)} = \bar{e}\,\frac{\sigma_{\lambda\mu}}{\sqrt{2}}\,(C_T + C_T'\gamma_5)\,\nu_e\,\bar{p}\,\frac{\sigma_{\lambda\mu}}{\sqrt{2}}\,n + \text{H.c.} \quad (11)$$

In Eqs. (9) - (11)

$$C_V = g_V\,(a_{LL} + a_{LR} + a_{RR} + a_{RL})\,, \quad (12)$$

$$C_V' = g_V(-a_{LL} - a_{LR} + a_{RR} + a_{RL})\,, \quad (13)$$

$$C_A = g_A(a_{LL} - a_{LR} + a_{RR} - a_{RL})\,, \quad (14)$$

$$C_A' = g_A(-a_{LL} + a_{LR} + a_{RR} - a_{RL})\,, \quad (15)$$

$$C_S = g_S(A_{LL} + A_{LR} + A_{RR} + A_{RL})\,. \quad (16)$$

$$C_S' = g_S(-A_{LL} - A_{LR} + A_{RR} + A_{RL})\,, \quad (17)$$

$$C_T = 2g_T(\alpha_{LL} + \alpha_{RR})\,, \quad (18)$$

$$C_T' = 2g_T(-\alpha_{LL} + \alpha_{RR})\,, \quad (19)$$

where the constants $g_V \equiv g_V(0)$, $g_S \equiv g_S(0)$ and $g_T \equiv g_T(0)$ are defined by

$$\langle p|\bar{u}\,\gamma_\lambda d|n\rangle = g_V(q^2)\overline{u}_p\,\gamma_\lambda\,u_n \quad (20)$$

$$\langle p|\bar{u}\,\gamma_\lambda\gamma_5 d|n\rangle = g_A(q^2)\overline{u}_p\,\gamma_\lambda\,\gamma_5\,u_n \quad (21)$$

$$\langle p|\bar{u}d|u\rangle = g_S(q^2)\overline{u}_p\,u_n \quad (22)$$

and

$$\langle p|\bar{u}\sigma_{\lambda\mu}d|n\rangle = g_T(q^2)\overline{u}_p\,\sigma_{\lambda\mu}\,u_n\,. \quad (23)$$

CVC predicts $g_V = 1$, and in the absence of new interactions the experimental value of g_A is $g_A = -1.2573 \pm 0.0028$ [13]. In the Hamiltonian (8) we did not include the combination from the part of $H_{S,P}$ (Eq. (4)) involving the pseudoscalar quark current, since these terms give no contribution to the beta decay observables in the nonrelativistic approximation for the nucleons. The interaction (8) is identical with the general beta decay interaction considered in Ref. [14].

In the general case when $\nu_e^{(L)}$ and $\nu_e^{(R)}$ are linear combinations of the mass-eigenstates, the observed beta decay probability is the sum of the probabilities of decays into the energetically allowed neutrino mass-eigenstates. In the following we shall assume that the neutrinos that can be produced in beta-decay are light enough that the effect of their masses on the decay probability can be neglected. In particular, we shall neglect the terms arising from the interference between amplitudes involving neutrinos of different chirality. As it is easily seen, under the above assumption the effect of neutrino mixing can be taken into account by multiplying in observables the coupling constants $\frac{1}{2}(C_K - C'_K)$ $(K = V, A, S, T)$ (which describe the interactions of $\nu_e^{(L)}$) by $\sqrt{u_e}$, and the coupling constants $\frac{1}{2}(C_K + C'_K)$ $(K = V, A, S, T)$ (describing the interactions of $\nu_e^{(R)}$) by $\sqrt{v_e}$, where

$$u_e = \sum_i{}' |U_{ei}|^2 \qquad (24)$$

$$v_e = \sum_i{}' |V_{ei}|^2 \qquad (25)$$

The prime on the summation in Eqs. (24) and (25) indicates that the sum extends only over the neutrinos that are light enough to be produced in beta decay. Without changing the notation, we shall understand in the following that in the expressions for the observables given in Ref. [14] the substitutions $\frac{1}{2}(C_K - C'_K) \to \frac{1}{2}(C_K - C'_K)\sqrt{u_e}$, $\frac{1}{2}(C_K + C'_K) \to \frac{1}{2}(C_K + C'_K)\sqrt{v_e}$ have always been made. Note that if all the neutrinos are light, we have $u_e = v_e = 1$, as a consequence of the unitarity of the neutrino mixing matrix.

The terms in the Hamiltonians (9) - (11) involving the right-handed neutrino state $\nu_e^{(R)}$ can manifest themselves in beta decay only if either the right-handed neutrinos are sufficiently light, or (for Majorana neutrinos) if there is mixing between the heavy right-handed neutrinos and the light ones. In the latter scenario the effects of the $\nu_e^{(R)}$-terms are expected to be suppressed by the light-heavy neutrino mixing angles, which should be small. For the interactions of light right-handed neutrinos there are stringent constraints from nucleosynthesis in the standard big bang model, and from the energetics of supernova 1987A.

In the standard big bang cosmology an experimental upper limit on the primordial ^{4}He abundance gives a constraint on the number of light ($\lesssim 1$ MeV) neutrino species [15]. The most recent analysis [16] of nucleosynthesis set the limit $N_\nu \leq 3.3$ on the effective number of light neutrinos. An implication of this result is that if light right-handed neutrinos exist, their interactions must be weaker than the strength of the weak interactions [17]. For one light right-handed neutrino (in addition to three known left-handed neutrinos) the limit $N_\nu \leq 3.3$ yields a decoupling temperature of $T_d \gtrsim 200$ MeV [17]. Since one has roughly $T_d \simeq (G'/G)^{2/3}$ [17], where G' is the strength of the interactions involving the right-handed neutrino, this implies $G' \lesssim 4 \times 10^{-3} G$ for the $\nu_e^{(R)}$-terms in the Hamiltonian (8). A more quantitative estimate of the upper limit on the strength of the beta decay interactions involving $\nu_e^{(R)}$ would require an analysis of the interaction of $\nu_e^{(R)}$ with the pions which, to our knowledge, has not been yet done.

A stringent constraint on the interactions of light ($\lesssim 10$ MeV) right-handed neutrinos comes from the observed neutrino pulse from the supernova 1987A [18,19]. The observed $\overline{\nu}_e$-luminosity is consistent with the standard supernova model, and this implies severe constraints on possible new cooling mechanisms of the supernova core. The requirement that the process $e^- p \to \nu_e^{(R)} n$ does not carry away most of the energy that can be radiated by the supernova leads for V,A interactions to the conclusion [19] that the coupling constants have to satisfy either the upper bound

$$\left(\tfrac{1}{6} g_V^2 |\eta_{RR}^{(e)} + \eta_{RL}^{(e)}|^2 + \tfrac{1}{2} g_A^2 |\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2\right)^{1/2} \lesssim 1.2 \times 10^{-5} , \qquad (26)$$

or the lower bound

$$\left(\tfrac{1}{6} g_V^2 |\eta_{RR}^{(e)} + \eta_{RL}^{(e)}|^2 + \tfrac{1}{2} g_A^2 |\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2\right)^{1/2} \gtrsim 2 \times 10^{-2} , \qquad (27)$$

where $\eta_{Rk} = a_{Rk}\sqrt{v_e}/a_{LL}\sqrt{u_e}$ $(k = L, R)$. For other types of beta decay interactions involving $\nu_e^{(R)}$ the constraints are probably similar. The present experimental limits are not far from ruling out the range (27) (see Section II.2.1). The bounds from the supernova on the beta decay interactions could be evaded if the right-handed neutrino has some additional interaction with electrons or nucleons which can trap them. A special interaction of this kind, which moreover does not cause conflict with the limit from nucleosynthesis on the effective number of light neutrinos, has been suggested in Ref. [20].

In the following while we shall bear in mind the constraints on right-handed neutrinos from nucleosynthesis and the supernova, we shall not invoke them in our discussions, since they do not diminish the importance of terrestrial experiments.

To conclude this section, we shall list the expressions from Ref. [14] for the few observables which we shall need to refer to in the subsequent discussions.

For allowed decays the $e^\mp$ longitudinal polarization P_L in the direction parallel to $\vec{p}_e$ in allowed transitions is

$$P_L = G \frac{p_e}{E_e} / (1 + b \frac{m_e}{E_e}) , \qquad (28)$$

where

$$\begin{aligned} G\xi = {} & 2|M_F|^2[\mp Re(C_S C_S'^* - C_V C_V'^*) - \frac{\alpha Z m_e}{p_e} Im(C_S C_V'^* + C_S' C_V^*)] \\ & + 2|M_{GT}|^2[\mp Re(C_T C_T'^* - C_A C_A'^*) \\ & + \frac{\alpha Z m_e}{p_e} Im(C_T C_A'^* + C_T' C_A^*) , \end{aligned} \qquad (29)$$

$$\begin{aligned} \xi = {} & |M_F|^2(|C_S|^2 + |C_S'|^2 + |C_V|^2 + |C_V'|^2) \\ & + |M_{GT}|^2(|C_A|^2 + |C_A'|^2 + |C_T|^2 + |C_T'|^2) \end{aligned} \qquad (30)$$

and b is the Fierz interference term, given by

$$b\xi = \pm 2(1-\alpha^2 Z^2)^{1/2}\, Re[|M_F|^2(C_S C_V^* + C'_S C'^*_V) - |M_{GT}|^2(C_T C_A^* + C'_T C'^*_A)]\,. \tag{31}$$

The coefficients D and R of the correlations $\langle\vec{J}\rangle \cdot \vec{p}_e \times \vec{p}_\nu / J E_e E_\nu$ and $\vec{\sigma} \cdot \langle\vec{J}\rangle \times \vec{p}_e / J E_e$ ($\vec{\sigma}$ = electron spin, $\vec{J}$ = nuclear spin) can be written as $D = D_t + D_f$, $R = R_t + R_f$, where D_t, R_t represent the T-violating contributions and D_f, R_f are the T-invariant contributions due to electromagnetic final state interactions. D_t and R_t are given by

$$D_t\xi = 2\delta_{J'J} M_F M_{GT} \left(\frac{J}{J+1}\right)^{1/2} Im(C_S C_T^* + C'_S C'^*_T + C_V C_A^* + C'_V C'^*_A)\,, \tag{32}$$

$$R_t\xi = \pm 2\lambda_{J'J}|M_{GT}|^2\, Im(C_T C'^*_A + C'_T C^*_A) + 2\delta_{JJ'} M_F M_{GT} \left(\frac{J}{J+1}\right)^{1/2} Im(C_S C'^*_A + C'_S C^*_A + C_V C'^*_T + C'_V C^*_T)\,, \tag{33}$$

where $\lambda_{J'J}$ is an angular momentum factor, defined in Ref. [14].

In the next section we shall consider new beta decay interactions of V,A structure, first the model independent aspects (Section II.2.1) and then V,A interactions in left-right symmetric models (Section II.2.2), in models with exotic quarks and leptons (Section II.2.3), and the V,A interactions arising from the exchange of leptoquarks (Section II.2.4). In Sections II.3 and II.4 we discuss, respectively, scalar and tensor interactions added to the SM interaction. The special case of scalar and tensor interactions from leptoquark exchange is considered in Section II.5.

II.2 New V,A Interactions

II.2.1 Model Independent Considerations

In this section we shall discuss the parameters which describe the general beta decay interaction involving vector and axial-vector currents, and consider the constraints on them which come from beta decay itself, and from processes or observables to which the new V,A beta decay interactions contribute in first order.

The most general form of the Hamiltonian for $d \to u\, e^- \nu_e^{(L,R)}$ constructed from vector and axial-vector currents is given in Eq. (3). For given neutrino states $\nu_e^{(L)}$ and $\nu_e^{(R)}$ the Hamiltonian (3) contains 8 real parameters (four complex coupling constants). One of these is an overall phase, which does not enter the observables. We can choose therefore a_{LL} to be real and positive. Defining $\eta_{ik} = a_{ik}/a_{LL}$ ($ik = LR, RR, RL$), a set of the remaining six parameters is, for example, $|\eta_{LR}|, |\eta_{RR}|, |\eta_{RL}|$, the phases $e^{i\varphi_L} = \eta_{LR}/|\eta_{LR}|$, $e^{i\varphi_R} = \eta_{RL}\eta^*_{RR}/|\eta_{RR}||\eta_{RL}|$ and $e^{i\varphi_{RL}} = \eta^*_{RL}/\eta_{RL}$.

We shall not consider further the phase φ_{RL} since in observables terms prop[ortional] to sin φ_{RL} are proportional to neutrino mass. The reason is that such ter[ms] arise from the interference of amplitudes involving neutrinos of different chiraliti[es].

Let us consider the beta decay of the nucleon in the framework of the Han[...] tonian (9). We can write (9) as (cf. Eqs. (12) - (15))

$$H_\beta^{(N)} = a_{LL} g_V (1+\eta_{LR})[\bar{e}\gamma_\mu(1-\gamma_5)\,\nu_e^{(L)}\,\bar{p}\gamma^\mu(1-\lambda\gamma_5)n + \bar{e}\gamma_\mu(1+\gamma_5)\,\nu_e^{(R)}\,\bar{p}\gamma^\mu(x+\gamma_5\lambda y)n] + \text{H.c.}\,, \tag{3}$$

where [21]

$$\lambda = \left(\frac{g_A}{g_V}\right)\frac{1-\eta_{LR}}{1+\eta_{LR}}\,, \tag{3}$$

$$x = \frac{\eta_{RR}+\eta_{RL}}{1+\eta_{LR}}\,, \tag{36}$$

and

$$\lambda y = \left(\frac{g_A}{g_V}\right)\frac{\eta_{RR}-\eta_{RL}}{1+\eta_{LR}}\,. \tag{37}$$

As follows from Eq. (34), normalized observables (such as asymmetries o[r] polarizations) can involve 5 parameters: $|\lambda|, |x|, |y|$, the phase of λ, and the relativ[e] phase of x and λy. The rate depends also on a_{LL}. As seen from Eq. (34), as long a[s] the induced form factors are neglected (as we do here), the number of parameter[s] at the nucleon level remains the same as at the quark level, since the only chang[e] is that $1+\eta_{LR}$ is replaced at the nucleon level by $g_V(1+\eta_{LR})$, and $(1-\eta_{LR})$ and $(\eta_{RR}-\eta_{RL})$ get multiplied by g_A.

In the following we shall keep in λ, x, y and λy only the lowest order terms in the η_{ik}'s. In this approximation we have

$$Re\,\lambda \simeq (g_A/g_V)(1-2\,Re\,\eta_{LR})\,, \tag{38}$$

$$Im\,\lambda \simeq -2(g_A/g_V)\,Im\,\eta_{LR} \simeq -2(Re\,\lambda)Im\,\eta_{LR}\,, \tag{39}$$

$$x \simeq \eta_{RR} + \eta_{RL}\,, \tag{40}$$

$$y \simeq \eta_{RR} - \eta_{RL}\,, \tag{41}$$

$$\lambda y \simeq (Re\,\lambda)(\eta_{RR}-\eta_{RL})\,, \tag{42}$$

$$Re\,x^*\lambda y \simeq (Re\,\lambda)(|\eta_{RR}|^2 - |\eta_{RL}|^2)\,, \tag{43}$$

$$Im\,x^*\lambda y \simeq -(Re\,\lambda)Im\,\eta^*_{RR}\eta_{RL}\,. \tag{44}$$

For a_{LL} and the η_{ik}'s the substitutions in the observables required to take into account the presence of more than one neutrino mass-eigenstate and neutrino mixing are

$$a_{LL} \rightarrow a_{LL}^{(e)} \equiv a_{LL}\sqrt{u_e}$$
$$\eta_{RR} \rightarrow \eta_{RR}^{(e)} \equiv \eta_{RR}\sqrt{\tilde{v}_e} \qquad (45)$$
$$\eta_{RL} \rightarrow \eta_{RL}^{(e)} \equiv \eta_{RL}\sqrt{\tilde{v}_e}$$

In Eq. (45) $\tilde{v}_e = v_e/u_e$, where u_e and v_e have already been defined in Eqs. (24) and (25).

Observables in pure Fermi transitions can depend only on $Re\, C_V C_V'^*$, $Im\, C_V C_V'^*$ and $|C_V|^2 \pm |C_V'|^2$, and observables in pure Gamow-Teller transitions on $Re\, C_A C_A'^*$, $Im\, C_A C_A'^*$, and $|C_A|^2 \pm |C_A'|^2$. From these the only ones which are not associated with interference between amplitudes involving neutrinos of different chirality are $Re\, C_K C_K'^*$ and $|C_K|^2 + |C_K'|^2$ $(K = V, A)$. Thus normalized observables in pure beta decays can involve only

$$\frac{Re(C_V C_V'^*)}{|C_V|^2 + |C_V'|^2} = -\frac{1}{2}\frac{1-|x^{(e)}|^2}{1+|x^{(e)}|^2} \simeq -\frac{1}{2}(1 - 2|\eta_{RR}^{(e)} + \eta_{RL}^{(e)}|^2) , \qquad (46)$$

or

$$\frac{Re(C_A C_A'^*)}{|C_A|^2 + |C_A'|^2} = -\frac{1}{2}\frac{1-|y^{(e)}|^2}{1+|y^{(e)}|^2} \simeq -\frac{1}{2}(1 - 2|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2) , \qquad (47)$$

where $x^{(e)}$ and $y^{(e)}$ are the x and y in which the substitutions (45) have been made. The rates involve

$$|C_V|^2 + |C_V'|^2 \simeq 2|a_{LL}^{(e)}|^2 g_V^2 |1 + \eta_{LR}|^2 \left(1 + |\eta_{RR}^{(e)} + \eta_{RL}^{(e)}|^2\right) , \qquad (48)$$

or

$$|C_A|^2 + |C_A'|^2 \simeq 2|a_{LL}^{(e)}|^2 g_A^2 |1 - \eta_{LR}|^2 \left(1 + |\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2\right) . \qquad (49)$$

In mixed transitions normalized observables can also depend on

$$\frac{|C_A|^2 + |C_A'|^2}{|C_V|^2 + |C_V'|^2} = |\lambda|^2 \frac{1+|y^{(e)}|^2}{1+|x^{(e)}|^2} \simeq |\lambda|^2(1 - 4Re\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}) , \qquad (50)$$

$$\begin{aligned} Re(C_V C_A^* + C_V' C_A'^*)/(|C_V|^2 + |C_V'|^2) &= (Re\,\lambda + Re\,x^{(e)*}\lambda y^{(e)})/(1 + |x^{(e)}|^2) \\ &\simeq (Re\,\lambda)(1 - 2|\eta_{RL}^{(e)}|^2 - 2Re\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}) , \end{aligned} \qquad (51)$$

$$\begin{aligned} Re(C_V C_A'^* + C_V' C_A^*)/(|C_V|^2 + |C_V'|^2) &= (-Re\,\lambda + Re\,x^{(e)*}\lambda y^{(e)})/(1 + |x^{(e)}|^2) \\ &\simeq -(Re\,\lambda)(1 - 2|\eta_{RR}^{(e)}|^2 - 2Re\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}) , \end{aligned} \qquad (52)$$

$$\begin{aligned} Im(C_V C_A^* + C_V' C_A'^*)/(|C_V|^2 + |C_V'|^2) &= -(Im\,\lambda + Im\,x^{(e)*}\lambda y^{(e)})/(1 + |x^{(e)}|^2) \\ &\simeq 2(Re\,\lambda)(Im\,\eta_{LR} + Im\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}) , \end{aligned} \qquad (53)$$

and

$$\begin{aligned} Im(C_V C_A'^* + C_A' C_A^*)/(|C_V|^2 + |C_V'|^2) &= (Im\,\lambda - Im\,x^{(e)*}\lambda y^{(e)})/(1 + |x^{(e)}|^2) \\ &\simeq -2(Re\,\lambda)(Im\,\eta_{LR} - Im\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}) . \end{aligned} \qquad (54)$$

In nuclear beta decay observables g_V and g_A appear multiplied by the Fermi matrix element M_F and the Gamow-Teller matrix element M_{GT}, respectively.

$Im\,\eta_{LR}$, $Im\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}$. A general V, A interaction contributes to the time reversal violating component D_t of the T-odd D-correlation (cf. Eq. (32)). D_t is given by

$$D_t \simeq a_D\, Im(\eta_{LR} + \eta_{RR}^{(e)*}\eta_{RL}^{(e)}) . \qquad (55)$$

The quantity a_D is proportional to $(r\, Re\,\lambda)/(1 + r^2|\lambda|^2)$, where $r = M_{GT}/M_F$. For ^{19}Ne and for n-decay $a_D \simeq -1.03$ and $a_D \simeq 0.87$, respectively.

The best limit on D_t/a_D comes at present from ^{19}Ne-decay. The experimental value $D = (0.1 \pm 0.6) \times 10^{-3}$ [22] yields

$$|Im(\eta_{LR} + \eta_{RR}^{(e)*}\eta_{RL}^{(e)})| < 1.05 \times 10^{-3} \qquad (90\%\ c.l.) . \qquad (56)$$

The contribution D_f of the electromagnetic final-state interactions to D has been estimated for this case to be of the order of $2 \times 10^{-4} p_e/(p_e)_{max}$ [23]. The present experimental result for the D-coefficient in neutron decay is $D = -0.0005 \pm 0.0014$ [13]. The final-state interaction contribution D_f is smaller than for ^{19}Ne decay by an order of magnitude [23].

The couplings that contribute to D will also give contributions to the electric dipole moment of the neutron (D_n) and of the electron (D_e) (through two-loop diagrams involving the a_{LR} and $(a_{LL})_{SM}$, and/or the a_{RR} and a_{RL} couplings). The upper limits on $|Im\,\eta_{LR}|$ and $|Im\,\eta_{RR}^{(e)*}\eta_{RL}^{(e)}|$ from the experimental limits [13] on these observables are not likely to be stronger than $\sim 10^{-3}$. The $Im\,\eta_{LR}$-term combined with the weak interaction contributes also to ϵ'/ϵ. Again, the corresponding limit on $|Im\,\eta_{LR}|$ from the experimental value of ϵ'/ϵ is not likely to be more stringent than $\sim 10^{-3}$.

$|\eta_{RR}^{(e)}|$, $|\eta_{RL}^{(e)}|$. A constraint on $|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|$ comes from measurements of the longitudinal polarization (P_L) of the charged lepton in Gamow-Teller beta decays. The experimental result (see Ref. [3]) $P_L^{GT}(E_e/p_e) = -0.998 \pm 0.014$ yields

$$|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}| < 0.110 \qquad (90\%\ c.l.) \qquad (57)$$

A novel type of experiment which can be very sensitive to the presence of $\eta_{RR}^{(e)}$ and/or $\eta_{RL}^{(e)}$ is the measurement of the ratio of the longitudinal polarizations of positrons emitted parallel and antiparallel to the direction of the spin of a polarized nucleus [24]. The first experiment of this kind, which measured the polarization of the positron from polarized ^{107}In decay, yielded $|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2 = 0.0080 \pm 0.0052$ [25] , implying

$$|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}| < 0.122 \qquad (90\%\ c.l.)\ . \tag{58}$$

Comparisons of e^+-longitudinal polarizations in Fermi and Gamow-Teller decays [26] yielded $P_L^F/P_L^{GT} = 1.0010 \pm 0.0027$, implying (note that $P_L^F/P_L^{GT} - 1 \simeq -8Re\,\eta_{RR}^{(e)*}\,\eta_{RL}^{(e)}$)

$$-6.8 \times 10^{-4} \ < \ Re\,\eta_{RR}^{(e)*}\,\eta_{RL}^{(e)} \ < \ 4.3 \times 10^{-4} \qquad (90\%\ c.l.)\ . \tag{59}$$

$Re\,\eta_{LR}$. As $Re\,\eta_{LR}$ appears in lowest order only in the parameter λ, the uncertainties in the theoretical value of g_A (and in nuclear beta decay also the theoretical uncertainties in the ratio $|M_{GT}|^2/|M_F|^2$) prevent the possibility of setting stringent limits on $Re\,\eta_{LR}$ from beta decay.

In models where $a_{LL} \simeq (a_{LL})_{SM}$ a bound on $Re\,\eta_{LR}$ can be derived from the experimental value of the ratio

$$R_\pi \ = \ \frac{\Gamma(\pi \to e\nu_e) + \Gamma(\pi \to e\nu_e\gamma)}{\Gamma(\pi \to \mu\nu_\mu) \ + \ \Gamma(\pi \to \mu\nu_\mu\gamma)}\ . \tag{60}$$

R_π for the interaction (3) is then given by [27]

$$R_\pi \ = \ (R_\pi)_{SM}\ (u_e/u_\mu)\ [(1 - Re\,\eta_{LR})^2 \ + \ (Im\,\eta_{LR})^2 \ + \ |\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2]\ , \tag{61}$$

where $(R_\pi)_{SM} = 1.234 \pm 0.001$ [28] is the value of R_π in the SM. A recent experiment [29] measured R_π with an improved precision, obtaining

$$(R_\pi)_{expt} = (1.2265 \ \pm \ 0.0034(\mathrm{stat}) \pm 0.004(\mathrm{sys})) \times 10^{-4}\ . \tag{62}$$

Neglecting the last two terms in Eq. (61), the result (62) implies

$$0.9932\ (u_\mu/u_e)^{1/2} \ < |1 - Re\,\eta_{LR}| < 1.0007\ (u_\mu/u_e)^{1/2} \qquad (90\%\ c.l.)\ , \tag{63}$$

where u_μ is defined as u_e (Eq. (24)) except for the replacement $U_{ei} \to U_{\mu i}$, and we have assumed for simplicity that the u_e's for $\pi \to e\,\nu_e$ and beta decay are equal.

For $u_e = u_\mu$ (63) allows $Re\,\eta_{LR}$ to be [30] either in the range

$$-0.0007 < Re\,\eta_{LR} < 0.0068\ , \tag{64}$$

or in the range

$$1.9932 < Re\,\eta_{LR} < 2.0007\ . \tag{65}$$

We note that the range (65) is ruled out by the experimental value of $|C_A|^2 + |C_A'|^2/(|C_V|^2 + C_V'|^2)$ ($\simeq (1.27)^2$ (see Ref. [3])). The latter, deduced from the neutron lifetime, would require $g_A/g_V \simeq 4$ (see Eqs. (50) and (59)) which is unreasonably large with respect to the theoretical prediction of g_A based on the Adler-Weisberger relation.

If we allow $|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|$ and $|Im\,\eta_{LR}|$ to be as large as the upper limits in Eqs. (75) and (73) below, or in Eqs. (75) and (57), the only appreciable effect is the increase of the upper bound in (64) to 1.3×10^{-2}.

Comparison of the predicted value of M_W (using the values of G_F, $\sin^2\theta_W$, and Δr given in Ref. [13]) with the experimental one shows that the factor $(u_e\,u_\mu)^{1/2}$ cannot be smaller than unity by more than $\sim 1.5\%$. To account for possible new V,A contributions to muon decay we shall allow $(u_e\,u_\mu)^{1/2}$ to deviate from unity by 2%, which implies $0.98 \leq (u_e/u_\mu)^{1/2} \leq 1.02$. Allowing $(u_e/u_\mu)^{1/2}$ to take any value in this range has no appreciable effect on the range (65), but (64) becomes $(-2.1 \times 10^{-2}) \ < \ Re\,\eta_{LR} < 2.7 \times 10^{-2}$. If $|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}|^2$ is also present (the effects of $Im\,\eta_{LR}$ are negligible), we obtain finally for $Re\,\eta_{LR}$ the bound

$$-2.1 \times 10^{-4} \ < \ Re\,\eta_{LR} \ < \ 3.3 \times 10^{-2}\ . \tag{66}$$

The limit on $Re\,\eta_{LR}$ from R_π may be weaker if a new V,A interaction contributing to $\pi \to \mu\nu_\mu$ is also present. Note that one would obtain $R_\pi = (R_\pi)_{SM}$ if the coupling constants in the muonic interaction are equal to the coupling constants in the interaction involving the electron.

The Results of a Comprehensive Analysis of Beta Decay Data. A comprehensive analysis of beta decay data [3], which included experimental results on neutron-decay and ^{19}Ne-decay, yielded the following 90% c.l. upper limits:

$$|\eta_{RR}^{(e)}| \ < \ 0.104\ , \tag{67}$$

$$|\eta_{RL}^{(e)}| \ < \ 0.044\ , \tag{68}$$

$$|Re\,\eta_{RR}^{(e)}| \ < \ 0.104\ , \tag{69}$$

$$|Re\,\eta_{RL}^{(e)}| \ < \ 0.010\ , \tag{70}$$

$$|Im\,\eta_{RR}^{(e)}| \ < \ 0.104\ , \tag{71}$$

$$|Im\,\eta_{RL}^{(e)}| \ < \ 0.043\ , \tag{72}$$

$$|Im\,\eta_{LR}| \ < \ 0.0038\ , \tag{73}$$

and also [31]

$$|\eta_{RR}^{(e)} + \eta_{RL}^{(e)}| \ < \ 0.106\ , \tag{74}$$

$$|\eta_{RR}^{(e)} - \eta_{RL}^{(e)}| < 0.106 . \quad (75)$$

This analysis was done in the framework of the Hamiltonian (8), with $H_{S,P}^{(N)}$ and $H_T^{(N)}$ absent.

It should be noted that the central value for $\eta_{RR}^{(e)}$ ($(\eta_{RR}^{(e)})_{central} = 0.077$) turns out to be 2.3 σ away from zero [3]. It should also be noted that if the experimental result of Ref. [32] on the asymmetry parameter A_n in neutron decay is removed from the data set, $|\eta_{RR}^{(e)}|$ becomes consistent with zero at the 1σ level [33].

The constraints we have considered in this section are independent of the source of the interaction (3) (except for the constraint from R_π, as explained in the text). For specific mechanisms that give rise to an interaction of the form (3) and for particular models additional constraints will apply in general.

Right-handed V,A beta decay interactions can arise at the tree-level if there are new charged gauge bosons which have right-handed couplings to the ordinary leptons and/or quarks, or if there are new quarks and leptons which have right-handed couplings to the W and which mix with the ordinary quarks and leptons, or in models involving leptoquarks. We shall discuss these mechanisms in the subsequent sections.

II.2.2. Left-Right Symmetric Models

Left-right symmetric models [34] are attractive extensions of the standard electroweak model, which provide a framework for the understanding of the origin of parity violation in the weak interaction. The simplest models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [34,35]. $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models involve an additional neutral gauge boson, and also a new charged gauge boson. The coupling of the charged gauge bosons W_L and W_R to the quarks and the leptons is given by

$$\begin{aligned}\mathcal{L} &= (g_L/\sqrt{2})(\overline{Q}_L^{(u)}\gamma_\lambda U_L Q_L^{(d)} + \overline{n}_L\gamma_\lambda U^\dagger E_L)W_L \\ &\quad + (g_R/\sqrt{2})(\overline{Q}_R^{(u)}\gamma_\lambda U_R Q_R^{(d)} + \overline{n}_R\gamma_\lambda V^\dagger E_R)W_R + \text{H.c.} ,\end{aligned} \quad (76)$$

where g_L and g_R are the $SU(2)_L$ and the $SU(2)_R$ gauge coupling constants, respectively, $\overline{Q}^{(u)} \equiv (\overline{u},\overline{c},\overline{t})$, $\overline{Q}^{(d)} \equiv (\overline{d},\overline{s},\overline{b})$, $\overline{E} = (\overline{e},\overline{\mu},\overline{\tau})$; $\overline{n}_L \equiv (\overline{\nu}_{1L},\overline{\nu}_{2L},\ldots)$ and $\overline{n}_R \equiv (\overline{\nu}_{1R},\overline{\nu}_{2R},\ldots)$ contain all the neutrino mass-eigenstates (three in the case of Dirac neutrinos, and six if the neutrinos are Majorana fermions); $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$ ($\psi = Q^{(u)}, Q^{(d)}, \ldots$). The matrices U_L, U_R and U, V are the quark and leptonic mixing matrices, respectively. The fields W_L and W_R are linear combinations of the mass-eigenstates W_1 and W_2:

$$\begin{aligned}W_L &= \cos\zeta\, W_1 + \sin\zeta\, W_2 \\ W_R &= e^{i\omega}(-\sin\zeta\, W_1 + \cos\zeta\, W_2) ,\end{aligned} \quad (77)$$

where ζ is a mixing angle and ω is a CP-violating phase.

In $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models CP-violation is present already for two quark generations [36], due to CP-violating phases in U_R. For n generations U and U_R contain together $n(n-1)$ mixing angles and $n^2 - n + 1$ CP-violating phases [37].

The Hamiltonian responsible for nuclear beta decay [38] resulting from (76) is of the form (3) with

$$a_{LL}^{(e)} \simeq (g_L^2 \cos\theta_1^L / 8m_1^2)\sqrt{u_e} , \quad (78)$$

$$\eta_{RR}^{(e)} \simeq e^{i\alpha}(\cos\theta_1^R/\cos\theta_1^L)(g_R^2 m_1^2/g_L^2 m_2^2)\sqrt{\tilde{v}_e} , \quad (79)$$

$$\eta_{LR} \simeq -e^{i(\alpha+\omega)}(\cos\theta_1^R/\cos\theta_1^L)(g_R\zeta/g_L) \quad (80)$$

$$\eta_{RL}^{(e)} \simeq -e^{-i\omega}(g_R\zeta/g_L)\sqrt{\tilde{v}_e} , \quad (81)$$

where m_1, m_2 are the masses of W_1, W_2; $\cos\theta_1^L = (U_L)_{ud}$ and $e^{i\alpha}\cos\theta_1^R = (U_R)_{ud}$. Note that for the phases φ_L and φ_R (see Section II.2.1) one has the relation $\varphi_R = -\varphi_L (= -\alpha - \omega)$.

A comprehensive analysis of [39] of the constraints on general $SU(2)_L \times SU(2)_R \times U(1)$ models led to the bound

$$(g_R^2 m_1^2 / g_L^2 m_2^2) \lesssim 7.5 \times 10^{-2} , \quad (82)$$

valid for any type of right-handed neutrinos. The limit (82) has been obtained from the $K_L - K_S$ mass difference Δm_K requiring that each individual contribution to Δm_K, corresponding to box diagrams with a given pair of internal quarks, is smaller than the experimental value of Δm_K, and assuming some reasonable restrictions on fine-tuned cancellations. In the presence of CP-violation there is also a constraint from ϵ [40]. The limit on $g_R^2 m_1^2/g_L^2 m_2^2$ would become then more stringent than (82) if the CP-violating phases are not small.

The limit (82) implies

$$|\eta_{RR}^{(e)}| \lesssim 7.5 \times 10^{-2} , \quad (83)$$

where we have used $u_e \simeq 1$, and $\cos\theta_1^L \simeq 1$.

A search [41] for the processes $W' \to e\nu$ and $W' \to \mu\nu$ in $\overline{p}p$ collisions, where W' is a heavy vector boson, led to the lower bound $m_{W'} > 520$ GeV/c^2 (95% c.l.) (and therefore $m_2 > 520$ GeV/c^2) for $m_{\nu_R} \lesssim 15$ GeV/c^2, assuming Standard Model-strength couplings to the three fermion families. To deduce from this result a limit for η_{RR} does not appear to be straightforward and has not yet been to our knowledge considered.

A limit on $\eta_{RL}^{(e)}$ is provided by muon decay. The constants g_{LL}^V and κ_{ij}^V ($ij = RR, LR, RL$) (see Eqs. (223)-(225) in Section III.2) involved in the muon decay Hamiltonian generated by the leptonic couplings in (76) are related to the beta decay constants (78)-(81) as

$$(G_F/\sqrt{2})g^V_{LL} = a_{LL}/\cos\theta^L_1 , \quad (84)$$
$$\kappa^V_{RR} = \eta_{RR}[(\cos\theta^R_1/\cos\theta^L_1)e^{i\alpha}]^{-1} , \quad (85)$$
$$\kappa^V_{LR} = \eta_{LR}[(\cos\theta^R_1/\cos\theta^L_1)e^{i\alpha}]^{-1} , \quad (86)$$
$$\kappa^V_{RL} = \eta_{RL} . \quad (87)$$

The best limit on $|\kappa^V_{RL}\sqrt{\tilde{v}_e}|$ comes from the experimental value of the ρ-parameter (see Section III.2), implying $|\kappa^V_{RL}\sqrt{\tilde{v}_e}| < 0.067$ (90% c.l.), and therefore

$$|\eta^{(e)}_{RL}| < 0.067 \qquad (90\% \text{ c.l.}). \quad (88)$$

Note that $\eta^{(e)}_{RR}$ is not constrained by muon decay data [42], since muon decay gives a bound on $\kappa^V_{RR}\sqrt{\tilde{v}_e\,\tilde{v}_\mu}$, and $\tilde{v}_\mu$ could be much smaller than $\tilde{v}_e$ [43].

Turning to the D-coefficient (Eq. (55)), the contribution from $Im\,\eta^{(e)*}_{RR}\,\eta^{(e)}_{RL}$ is relatively small [42] due to the relation $Im\,\eta^{(e)*}_{RR}\,\eta^{(e)}_{RL} = (g^2_R m^2_1/g^2_L m^2_2) Im\,\eta_{LR}$, and the limit (82). One has therefore [38]

$$D/a_D \simeq Im\,\eta_{LR} \simeq -(g_R\zeta/g_L)(\cos\theta^R_1/\cos\theta^L_1)\sin(\alpha+\omega) , \quad (89)$$

and thus from (56)

$$|Im\,\eta_{LR}| < 1.1 \times 10^{-3} . \quad (90)$$

The phase $\alpha+\omega$ is constrained also by the experimental limit on the neutron electric dipole moment D_n and on ϵ'/ϵ, which require $|Im\,\eta_{LR}| \lesssim 3\times10^{-5}$ [44]. These limits are however not as reliable as the limit (90), since the corresponding calculations may involve unknown uncertainties. Combining the bound (59) from P^F_L/P^{GT}_L with the limit on $Im\,\eta^{(e)*}_{RR}\,\eta^{(e)}_{RL}$ yields

$$|\eta^{(e)}_{RR}||\eta^{(e)}_{RL}| < 6.9\times10^{-4} . \quad (91)$$

Information on η_{LR} in $SU(2)_L\times SU(2)_R\times U(1)$ models can be obtained from data on inclusive $\overline{\nu}_\mu$- and ν_μ-scattering on nuclei [45], since the constant η_{LR} for the $\nu_\mu\,d \to \mu\,u$ interaction is the same as for $d \to u\,e^-\overline{\nu}_e$ [46]. The ratio of the antineutrino to neutrino cross-sections is sensitive to $|\eta_{LR}|$ for x and y both large. A recent experiment by the CCFR collaboration yielded [47]

$$|\eta_{LR}| < 0.039 \qquad (90\% \text{ c.l.}) . \quad (92)$$

For $Re\,\eta_{LR}$ one has also the limit [48]

$$|Re\,\eta_{LR}| < 6\times10^{-3} \quad (93)$$

from the requirement that the PCAC predictions for the $K\to3\pi$ amplitudes in terms of the $K\to2\pi$ amplitudes, which hold to an accuracy of $\sim10\%$, would be retained even in the presence of right-handed currents.

Combining (90) and (93) gives

$$|\eta_{LR}| < 6.1\times10^{-3} . \quad (94$$

We comment yet on a relation involving $Re\,\eta_{LR}$ which follows from charged current universality. The unitarity of the Kobayashi-Maskawa matrix for thre families leads to the relation [49]

$$(g_R\zeta/g_L)[(U_L)_{ud}\,Re\,e^{i\omega}(U_R)_{ud} + (U_L)_{us}\,Re\,e^{i\omega}(U_R)_{us}] \simeq \tfrac{1}{2}(S_u-1) , \quad (95)$$

where $S_u \equiv \sum_{i=d,s,b}|(\tilde{U}_L)_{ui}|^2$, and $|(\tilde{U}_L)_{ui}|$ is the apparent value of $|(U_L)_{ui}|$.

For U_R which yields the limit (82) one has $(U_R)_{ud}\neq0$, $(U_R)_{us}=0$, and therefore the relation (95) sets a bound on $Re\,\eta_{LR}$. Using for S_u the values obtained in various treatments of the superallowed $0^+\to0^+$ beta decays (see Refs. [3,5]), one obtains $|Re\,\eta_{LR}| < (1.6 \text{ to } 3.4)\times10^{-3}$. Combining this with the limit (90) yields $|\eta_{LR}| < 3.6\times10^{-3}$. For the cases when $(U_R)_{us}\neq0, (U_R)_{ud}=0$ there is no significant constraint from (95) on $(g_R\xi/g_L)Re\,e^{i\omega}(U_R)_{us}$, since the present experimental result [50] on the D-coefficient in hyperon decays sets only a weak limit on the imaginary part of this term [51].

Nonmanifest models with $\tilde{v}_e=\tilde{v}_\mu=1$. If in the models discussed above (referred to as models with nonmanifest left-right symmetry [52], which allow $g_R\neq g_L$ and $\theta^R_i\neq\theta^L_i$) $\tilde{v}_e=\tilde{v}_\mu=1$, there is a limit on $|\eta^{(e)}_{RR}|$ which is somewhat stronger than (83), from the experimental lower bound on the quantity $R = 1-\delta\,\xi\,P_\mu/\rho$ in muon decay (see Section III.2). The latter implies $|\kappa^V_{RR}| < 0.040$, and therefore (since $|\eta^{(e)}_{RR}| = |\eta_{RR}| \lesssim |\kappa^V_{RR}|$)

$$|\eta^{(e)}_{RR}| \lesssim 0.040 . \quad (96)$$

From the muon decay ρ-parameter one obtains in this case

$$|\eta^{(e)}_{RL}| \lesssim 0.047 . \quad (97)$$

Also, as $|\eta_{LR}| \lesssim |\eta_{RL}|$, and since $\eta_{RL}=\eta^{(e)}_{RL}$ for $\tilde{v}_e=1$, Eq. (97) implies the additional limit $|\eta_{LR}| \lesssim 0.047$ on $|\eta_{LR}|$. An example of models with $\tilde{v}_e=\tilde{v}_\mu=1$ is the class where U = V (such as $SU(2)_L\times SU(2)_R\times U(1)$ models with Dirac neutrinos and a discrete symmetry). One has $\tilde{v}_e=\tilde{v}_\mu=1$ also in models where all the neutrinos are sufficiently light to be produced in beta decay.

Models with pseudomanifest or manifest left-right symmetry. In such models $g_R=g_L$ and $\theta^R_i=\theta^L_i$ [52]. In manifestly left-right symmetric models [53] in addition the relative phases between the right-handed and the left-handed couplings vanish, and $\omega=0$ (so that $\sin(\alpha+\omega)=0$) [54].

In manifestly left-right symmetric models the K_L-K_S mass difference implies the bound $|\eta_{RR}| = m^2_1/m^2_2 \lesssim 6\times10^{-3}$ [55], and therefore

$$|\eta^{(e)}_{RR}| \lesssim 6\times10^{-3} . \quad (98)$$

In pseudomanifest models the constraint from ϵ has to be also considered [40]. Assuming that there are no fine-tuned cancellations among the contributions of the various CP-violating phases, the limit from ϵ is more stringent than (98) if the CP-violating phases are not small.

For $\eta^{(e)}_{RL}(= -\zeta\sqrt{\tilde{v}_e})$ the limit (97) holds. Applying the relation $|\zeta| \lesssim m_1^2/m_2^2$ [56], which fails to hold only if the Higgs sector of the model contains Higgs bosons in representations with $T_R \gg 1$ [39], one has also

$$|\eta^{(e)}_{RL}| \lesssim 6\times10^{-3} . \qquad (99)$$

For $|\eta_{LR}|(= |\zeta|)$ the limits (92) and (94) hold. In manifestly left-right symmetric models one has in addition the bound $|\zeta| < 3.4\times10^{-3}$ from charged-current universality (Eq. (95)).

II.2.3. Exotic Fermions

The interaction which gives rise to the $d \to u\,e^-\overline{\nu}_e$ transition can contain terms involving right-handed currents even if the electroweak gauge group is just the standard model SU(2)×U(1) group. This happens if new quarks and leptons exist whose right-handed components are in non-singlet representations of SU(2), and which mix with the usual quarks and leptons [57]. Fermions with noncanonical SU(2)×U(1) assignments are referred to as "exotic." Such fermions occur in many extensions of the SM. If the electric charge and color assignments of the new fermions are the standard ones, the only possibility for the non-singlet right-handed fermions is to be in doublet representations (i.e. they can only be mirror fermions or vector doublets) [58]. The new charged fermions have to be heavy (heavier than ~ 45 GeV) in view of limits on direct production at LEP [13].

The coupling of the W to charged currents involving the usual quarks and charged leptons and the light neutrinos is given by

$$\begin{aligned}\mathcal{L} = (g/\sqrt{2})[\overline{Q}^{(u)}_L\,\gamma_\lambda(A^{u\dagger}_L A^d_L)\,Q^d_L \;+\; \overline{n}_{\ell L}\,\gamma_\lambda(A^{\nu\dagger}_L A^e_L)E \\ +\; \overline{Q}^u_R\,\gamma_\lambda(F^{u\dagger}_R F^d_R)\,Q^{(d)}_R \\ +\; \overline{n}^c_{\ell R}\,\gamma_\lambda(F^{\nu\dagger}_R F^e_R)E]W^\lambda \;+\; \text{H.c.}\,,\end{aligned} \qquad (100)$$

where all the fields correspond to mass-eigenstates, $\overline{Q}^{(u)} \equiv (\overline{u},\overline{c},\overline{t})$, $\overline{Q}^{(d)} \equiv (\overline{d},\overline{s},\overline{b})$, $\overline{E} \equiv (e,\mu,\tau)$, $\overline{n}_{\ell L} \equiv (\nu_{1L},\nu_{2L},\ldots)$ and $\overline{n}^c_{\ell R} \equiv (\overline{\nu}^c_{1R},\overline{\nu}^c_{2R},\ldots) = C(\overline{n}_{\ell L})^T$. In Eq. (100) the matrices A^k_L and $F^k_R (k = u,d,e,\nu)$ relate, respectively, the ordinary and the exotic fermion weak eigenstates to the light fermion mass-eigenstates.

The mixing of the exotic fermions with the ordinary ones leads in general to flavor-changing neutral currents (FCNC) between ordinary fermions. For FCNC transitions among charged fermions there are stringent constraints on the strength of the corresponding interactions from experimental limits on processes such as for example $\mu \to 3e$ and $K_L \to \mu\mu$. Exotic-ordinary fermion mixing gives rise also to deviations from the SM predictions in flavor-conserving neutral current processes and in charged-current reactions. To analyze the constraints on the pertinent parameters one can work in the limit where the FCNC transitions are absent [58]. The matrices A^k_L and F^k_R $(k = u,d,e)$ have then the greatly simplified forms [58]

$$\begin{aligned}A^k_L &= \hat{A}^k_L\,c^k_L \qquad (k = u,d,e)\,,\\ F^k_R &= \hat{F}^k_R\,s^k_R \qquad (k = u,d,e)\,,\end{aligned} \qquad (101)$$

where the $\hat{A}^k_L$ are unitary matrices which describe usual intergenerational mixing, and the $\hat{F}^k_R$ are unitary matrices in the special case when the number of the exotic states is equal to the number of ordinary states. The c^k_L and s^k_R are diagonal matrices of $\cos\theta^i_L$ and $\sin\theta^i_R$, respectively (the matrix elements of c^u_L, for example, are $(c^u_L)_{ij} = \delta_{ij}\cos\theta^i_L$ $(i = u,c,t)$). The angles $\theta^i_{L,R}$ are the light-heavy mixing angles. The weak eigenstates of the charged leptons can be chosen so that each corresponds to a unique light mass-eigenstate [58]. Allowing for CP-violation in the mixing described by $F^{(e)}_R$, one has then $A^e_L = c^e_L$ and $F^e_R = s^e_R\,e^{i\varphi_{(e)}}$, where $e^{i\varphi_{(e)}}$ is a diagonal matrix of $e^{i\varphi_i}$ $(i = e,\mu,\tau)$.

It follows that the beta decay interaction arising from the couplings (100) is of the form (3) with

$$a^{(e)}_{LL} \simeq (g^2 U_{ud}/8m^2_W)\sqrt{u_e}\,, \qquad (102)$$

$$\eta_{LR} \simeq s^u_R\,s^d_R\,(\hat{V}_R)_{ud}\,, \qquad (103)$$

$$\eta^{(e)}_{RL} \simeq e^{i\varphi_e}\,s^e_R\,\sqrt{\tilde{v}_e}\,, \qquad (104)$$

$$\eta^{(e)}_{RR} \simeq \eta_{LR}\,\eta^{(e)}_{RL}\,, \qquad (105)$$

where in a given coupling constant we have kept only the terms lowest order in the light-heavy mixing. The quantities u_e and v_e are given here by $u_e = \sum_i |(A^\nu_L)_{ei}|^2$ and $v_e = \sum_i |(F^\nu_R)_{ei}|^2$ (denoted in Ref. [58] by $(c^{\nu_e}_L)^2$ and $(s^{\nu_e}_R)^2$, respectively). In Eq. (103) $\hat{V}_R$ is a matrix which is unitary for $n^u_R = n^d_R = m^u_R$, where n^u_R and n^d_R are, respectively, the number of the $Q(\equiv$ electric charge$) = 2/3$ and $Q = -1/3$ right-handed singlet states, and m^u_R is the number of the $Q = 2/3$ right-handed doublet states. The phase φ_e (which is the phase φ_{RL} here) has no detectable effect, as discussed in Section II.2.1.

A global analysis of the constraints on ordinary-exotic fermion mixings was made in Ref. [58]. The results for the parameters $\eta^{(e)}_{RL}$ and $Re\,\eta_{LR}$ are [59]

$$|\eta^{(e)}_{RL}| < 4.2\times10^{-2} \qquad (90\%\ c.l.)\,, \qquad (106)$$

$$|Re\,\eta_{LR}| < 6\times10^{-3} \qquad (90\%\ c.l.)\,. \qquad (107)$$

The limit (106) originates from muon decay data. $Re\,\eta_{LR}$ is constrained by the PCAC prediction for $K \to 3\pi$ decays (Eq. (93)), by $\overline{\nu}_\mu$ and ν_μ scattering on nuclei (Eq. (92)), and by charged current universality [58].

A new global analysis of the constraints on ordinary-exotic fermion mixings, which updates the analysis of Ref. [58], was carried out in Ref. [60]. The limits on $|\eta^{(e)}_{RL}|$ and $|Re\,\eta_{LR}|$ remain the same as in the previous analysis (Eqs. (106) and (107)).

The elements of $\hat{V}_R$ are complex. The phase in $(\hat{V}_R)_{ud}$ contributes to D_t (Ref. [61]; see also Ref. [62]). Note that the phases φ_R and φ_L (see Section II.2.1) are here related ($\varphi_R = -\varphi_L$). Writing $(\hat{V}_R)_{ud} = e^{i\phi}(\hat{V}_R)'_{ud}$, where $(\hat{V}_R)'_{ud}$ is real, we have (see Eqs. (55)).

$$D_t/a_D \simeq [1-(s^e_R\sqrt{\bar{v}_e})^2]\, Im\,\eta_{LR} \simeq Im\,\eta_{LR} \simeq s^u_R\, s^d_R (\hat{V}_R)'_{ud}\sin\phi\,. \quad (108)$$

From the experimental limit on D we have

$$|Im\,\eta_{LR}| < 1.1\times10^{-3}\,. \quad (109)$$

The phase ϕ contributes also to the electric dipole moment of the neutron D_n and to ϵ'/ϵ [61]. The upper limits on $|D_t/a_D|$ from these observables are $\sim 3\times10^{-5}$. However, as mentioned in Section II.2.2, the limits from D_n and ϵ'/ϵ are not as reliable as the limit (109).

Combining (107) and (109) one obtains

$$|\eta_{LR}| < 6.1\times10^{-3}\,. \quad (110)$$

From Eqs. (105), (106) and (110) one has

$$|\eta^{(e)}_{RR}| < 2.5\times10^{-4}\,. \quad (111)$$

Finally, Eq. (105) and the bounds (106), (107) and (109) yield

$$|Re\,\eta^{(e)*}_{RR}\,\eta^{(e)}_{RL}| < 1.1\times10^{-5}\,, \quad (112)$$

$$|Im\,\eta^{(e)*}_{RR}\,\eta^{(e)}_{RL}| < 1.9\times10^{-6}\,. \quad (113)$$

From (112) and (113) one obtains

$$|\eta^{(e)}_{RR}|\,|\eta^{(e)}_{RL}| < 1.1\times10^{-5}\,. \quad (114)$$

II.2.4. V,A Interactions from Leptoquark Exchange

Leptoquarks are bosons which couple to lepton-quark pairs. They appear in many extensions of the SM [63]. In some models they can be light enough to cause observable effects in low-energy processes. Both spin-one and spin-zero leptoquarks occur.

The $d\to u\,e^-\overline{\nu}^{(L,R)}_e$ transitions can be mediated by leptoquarks of electric charge $|Q|=2/3$ and $|Q|=1/3$ (see Refs. [64,62]). We shall denote the spin-one and spin-zero leptoquarks generically as $X_{|Q|}$ and $Y_{|Q|}$, respectively.

The exchange of $X_{|Q|}$ and $Y_{|Q|}$ leptoquark states which contribute to t[he] $d\to u\,e^-\overline{\nu}^{(L,R)}_e$ transitions generate four fermion interactions of the general form:

$$H_{X(2/3)} = \overline{u}\gamma_\lambda(1-\gamma_5)\nu^{(L)}_e[f_{LL}\,\overline{e}\gamma^\lambda(1-\gamma_5)d + f_{LR}\,\overline{e}\gamma^\lambda(1+\gamma_5)d] + \overline{u}\gamma_\lambda(1+\gamma_5)\nu^{(R)}_e[f_{RL}\,\overline{e}\gamma^\lambda(1-\gamma_5)d + f_{RR}\,\overline{e}\gamma^\lambda(1+\gamma_5)d] + \text{H.c.} \quad (115)$$

$$H_{X(1/3)} = \overline{d^c}\gamma_\lambda(1-\gamma_5)\nu^{(L)}_e[h_{LL}\,\overline{e}\gamma^\lambda(1-\gamma_5)u^c + h_{LR}\,\overline{e}\gamma^\lambda(1+\gamma_5)u^c] + \overline{d^c}\gamma_\lambda(1+\gamma_5)\nu^{(R)}_e[h_{RL}\,\overline{e}\gamma^\lambda(1-\gamma_5)u^c + h_{RR}\,\overline{e}\gamma^\lambda(1+\gamma_5)u^c] + \text{H.c.} \quad (116)$$

$$H_{Y(2/3)} = \overline{u}(1-\gamma_5)\nu^{(L)}_e[F_{LL}\,\overline{e}(1-\gamma_5)d + F_{LR}\,\overline{e}(1+\gamma_5)d] + \overline{u}(1+\gamma_5)\nu^{(R)}_e[F_{RL}\,\overline{e}(1-\gamma_5)d + F_{RR}\,\overline{e}(1+\gamma_5)d] + \text{H.c.} \quad (117)$$

$$H_{Y(1/3)} = \overline{d^c}(1-\gamma_5)\nu^{(L)}_e[H_{LL}\,\overline{e}(1-\gamma_5)u^c + H_{LR}\,\overline{e}(1+\gamma_5)u^c] + \overline{d^c}(1+\gamma_5)\nu^{(R)}_e[H_{RL}\,\overline{e}(1-\gamma_5)u^c + H_{RR}\,\overline{e}(1+\gamma_5)u^c] + \text{H.c.} \quad (118)$$

In Eqs. (115) - (118) the first superscript on the coupling constants denotes always the neutrino chirality, while the second subscript indicates the chirality of the fourth fermion in the coupling. The fields u, d and e in Eqs. (115) - (118) are mass-eigenstates. The constants f_{ij}, h_{ij}, F_{ij} and $H_{ij}(i=L,R;\,j=L,R)$ are sums of ratios of the form $x_{ik}x'_{jk}/m^2_{Y_k}$, where x_{ik}, x'_{jk} are the effective fermion-leptoquark coupling constants and m_{Y_k} are the leptoquark masses.

A Fierz transformation takes the Hamiltonians (115) and (116) into quark-lepton interactions of the form (2) with the tensor interaction absent, and the Hamiltonians (117) and (118) into interactions of the form (2) involving all the possible Lorentz covariants (i.e. V,A,S,P and T) [65]. The V,A and S,P (or S,P,T) parts are in general governed by different combinations of the coupling constants. In this section we shall consider the V,A part of the leptoquark-exchange interaction. The S,P component, and for spin-zero leptoquarks the S,P,T component, will be discussed in Section II.5.

The V,A components of the four-fermion interaction resulting from the exchange of $X_{|Q|}$ and $Y_{|Q|}$ ($|Q|=2/3,1/3$) are of the form (3) with (Ref. [64]; see also Ref. [62]).

$$a_{LL} = f_{LL},\quad a_{RR} = f_{RR},\quad a_{LR} = a_{RL} = 0 \qquad (X_{(2/3)}-\text{exchange}) \quad (119)$$

$$a_{LL} = a_{RR} = 0$$
$$a_{LR} = -h_{LL} \qquad (X_{(1/3)} - \text{exchange}) \qquad (120)$$
$$a_{RL} = -h_{RR}$$

$$a_{LL} = a_{RR} = 0$$
$$a_{LR} = -\tfrac{1}{2} F_{LR} \qquad (Y_{(2/3)} - \text{exchange}) \qquad (121)$$
$$a_{RL} = -\tfrac{1}{2} F_{RL}$$

and

$$a_{LL} = -\tfrac{1}{2} H_{LR}$$
$$a_{RR} = -\tfrac{1}{2} H_{RL} \qquad (Y_{(1/3)} - \text{exchange}) \qquad (122)$$
$$a_{LR} = a_{RL} = 0$$

To obtain the full V,A interaction we have to add to a_{LL} in each case the SM contribution $(a_{LL})_{SM} = g^2 U_{ud}/8m_W^2$.

A consequence of Eqs. (119) - (122) is that for all cases the product $a^*_{RR}a_{RL}$ vanishes. It follows that there is no contribution from the V,A part of the $X_{|Q|}$-exchange or the $Y_{|Q|}$-exchange interaction to P_L^F/P_L^{GT} (Ref. [64]; see also Ref. [62]), and also that the second term in Eq. (55) for the D-coefficient is absent. A nonzero value of $a^*_{RR}a_{RL}$ could arise however if leptoquarks of the same spin but different charges, or leptoquarks of the same charge but different spins simultaneously contribute. We note that in such cases the phases φ_R and φ_L (see Section II.2.1) are different in general (Ref. [61]; see also Ref. [66]).

In the presence of only one of the four leptoquark types $X_{|Q|}$ and $Y_{|Q|}$, D_t can be nonzero for $X_{(1/3)}$- and $Y_{(2/3)}$-exchange (Ref. [61]; see also Ref. [62]). We have

$$D_t = -\, a_D\, Im\, h_{LL}/(a_{LL})_{SM} \qquad (X_{(1/3)} - \text{exchange})\,, \qquad (123)$$

and

$$D_t = -\frac{1}{2}\, a_D\, Im\, F_{LR}/(a_{LL})_{SM} \qquad (Y_{(2/3)} - \text{exchange})\,. \qquad (124)$$

The Lagrangian describing the most general $SU(2)_L \times U(1) \times SU(3)_c$ invariant couplings of spin-zero leptoquarks to a fermion family of the SM contains 9 different leptoquark states (10 states if a right-handed neutrino is included in the family) characterized by a definite fermion number and definite quantum numbers with respect to the SM gauge group [67,68]. Similarly, the most general such Lagrangian for the spin-one leptoquarks has 9 distinct leptoquark states [68] (10 if a right-handed neutrino is included). Inspection shows that from the spin-zero leptoquarks the ones that can contribute to the $d \to u\, e^- \overline{\nu}_e^{(L,R)}$ transitions are (using the notation of Ref. [68]) the $|Q| = 1/3$ states S_1 and $(S_3)_{T_z=0}$, and the $|Q| = 2/3$ states $(R_2)_{T_z=-1/2}$ and $(\tilde{R}_2)_{T_z=+1/2}$. The spin-one leptoquarks contributing to $d \to u\, e^- \overline{\nu}_e^{(L,R)}$ are the $|Q| = 1/3$ states $(V_{2\mu})_{T_z=-1/2}$ and $(\tilde{V}_{2\mu})_{T_z=+1/2}$ and the $|Q| = 2/3$ states $U_{1\mu}$ and $(U_{3\mu})_{T_z=0}$. Terms of the a_{LL}-type can arise from $S_1, (S_3)_{T_z=0}, U_{1\mu}$ or $(U_{3\mu})_{T_z=0}$ exchange, while a_{RR}-type couplings can be generated only by S_1 and $U_{1\mu}$ exchange [64]. None of the leptoquark states of definite $SU(2)_L \times U(1)$ quantum numbers, contributing to $d \to u\, e^- \overline{\nu}_e^{(L,R)}$ gives rise to an a_{LR}- or a_{RL}-type interaction. Such couplings can however be generated if (as generally expected) the states $(R_2)_{T_z=-1/2}$ and $(\tilde{R}_2)_{T_z=+1/2}$ or the states $(V_{2\mu})_{T_z=-1/2}$ and $(\tilde{V}_{2\mu})_{T_z=+1/2}$ mix (Ref. [61]; see also Ref. [69]).

We shall consider as an example the interaction generated by the S_1 leptoquark [70]. The coupling of the S_1 to the first fermion family is given by

$$\mathcal{L} = [\tfrac{1}{2} g_{1L}(\overline{u}^{c\prime}(1-\gamma_5)\, e - \overline{d}^{c\prime}(1-\gamma_5)\, \nu_e^{(L)}) + \tfrac{1}{2} g_{1R}\, \overline{u}^{c\prime}(1+\gamma_5)\, e + \tfrac{1}{2} g_{1R}^{(\nu)} \overline{d}^{c\prime}(1+\gamma_5)\, \nu_e^{(R)}]\, S_1 + \text{H.c.} \qquad (125)$$

In Eq. (125) the primed fields and $\nu_e^{(L)}$, $\nu_e^{(R)}$ are weak eigenstates. Neglecting for simplicity generation mixing and neutrino mixing, the coupling constants for the beta decay interaction resulting from (125) are [71]

$$a_{LL} = |g_{1L}|^2/8m_1^2\,, \qquad (126)$$
$$a_{RR} = (-g^*_{1R}\, g_{1R}^{(\nu)}/8m_1^2)\,, \qquad (127)$$
$$a_{LR} = a_{RL} = 0\,, \qquad (128)$$
$$(C_S - C'_S)/g_S = g_{1L}\, g^*_{1R}/4m_1^2\,, \qquad (129)$$
$$(C_S + C'_S)/g_S = g^*_{1L}\, g_{1R}^{(\nu)}/4m_1^2 \qquad (130)$$
$$(C_T - C'_T)/g_T = -(C_S - C'_S)/g_S = -(C_P - C'_P)/g_P\,, \qquad (131)$$
$$(C_T + C'_T)/g_T = -(C_S + C'_S)/g_S = -(C_P - C'_P)/g_P\,, \qquad (132)$$

where m_1 is the mass of the S_1.

From the constraints we have considered for the V,A beta decay interactions only the model independent ones (Section II.2.1) apply for the general interaction from leptoquark exchange. The limit (66) on $Re\,\eta_{LR}$ holds only if, in addition to the conditions already given for (66), there are no accidental cancellations in R_π between the $Re\,\eta_{LR}$ contribution and the contributions from the a_{LL} and the $Re\,(C_P - C'_P)/g_P$-part of the interaction.

The existing limits on the masses and the couplings of spin-zero leptoquarks from constraints on direct production at accelerators still allow the strength of the leptoquark-exchange interaction to be comparable with the strength of the weak interaction, and even somewhat stronger than G [13]. The limits on the masses and the couplings of spin-one leptoquarks, which are not quoted in Ref. [13], are probably similar. The exchange of leptoquarks which couple to $Q = -1/3$ quarks can contribute to the decay $K_L \to \mu e$ (at the tree-level) and to the $K^o \to \overline{K}^o$ amplitude (through box diagrams involving two leptoquark propagators) [72]. However, the constraints from these observables (which in principal could be stringent) may not apply since the leptoquark-fermion coupling constants involved, which are not identical to those appearing in beta decay, could be suppressed. For muon decay there is no contribution from leptoquarks at the tree level. The size of the loop-level contributions is most likely considerably below the present experimental limits; moreover, the contributing diagrams contain coupling constants different from those relevant for beta decay.

II.3. Scalar Interactions

The general form of the effective $d \to u e^- \overline{\nu}_e$ Hamiltonian involving scalar and pseudoscalar currents is given in Eq. (4). For nuclear beta decay we can restrict our attention to the terms in (4) involving the scalar current $\overline{u}d$ (see Eq. (8)). This part of (4) can be written as

$$H_S = [a_{LS}\,\overline{e}(1-\gamma_5)\nu_e^{(L)} + a_{RS}\,\overline{e}(1+\gamma_5)\nu_e^{(R)}]\overline{u}d + \text{H.c.}, \qquad (133)$$

where $a_{LS} = A_{LL} + A_{LR}$, $a_{RS} = A_{RR} + A_{RL}$. The constants C_S and C'_S are given by $C_S - C'_S = 2g_S\, a_{LS}$, $C_S + C'_S = 2g_S\, a_{RS}$, where g_S has been defined in Eq. (22). We shall define also $\eta_{kS} \equiv a_{kS}/a_{LL} \simeq a_{kS}/(G\,U_{ud}/\sqrt{2})$ $(k = L, R)$ and $\eta^{(e)}_{RS} \equiv \eta_{RS}\sqrt{\overline{v}_e}$.

A possible source of scalar-type beta decay couplings is the exchange of charged Higgs bosons [73]. The bounds on charged Higgs masses and couplings from limits on direct production still allow the strength of the scalar contributions to be comparable and even somewhat stronger than the strength of the weak interactions [13]. Charged Higgs bosons are present in many extensions of the SM, and also in the standard $SU(2)_L \times U(1)$ model if the Higgs sector contains (for example) two Higgs doublets. In supersymmetric models the presence of two Higgs doublets is required. In multi-Higgs models the couplings of the Higgs bosons to the fermions is in general undetermined. It should be noted however that in models which use discrete symmetries to eliminate flavor changing neutral currents and thus allow light neutral Higgs bosons, the couplings of the charged Higgs bosons to the first family are usually small, suppressed by the small fermion masses, although some enhancement could come from ratios of Higgs vacuum expectation values. But in general scalar contributions from Higgs-exchange to the beta decay interaction of strength near the present experimental limits on scalar couplings are not ruled out.

In the supersymmetric SM with R-parity violation [74] scalar contributions to the $d \to u e^- \overline{\nu}_e$ transition can arise at the tree-level from the exchange of sleptons. Scalar beta decay interactions can also come from the exchange of leptoquarks. This class of scalar interactions will be discussed in Section II.5. The scalar-type interactions would show up in the allowed approximation only in Fermi (or mixed) transitions. Restricting attention, as before, only to coupling constant combinations which do not result from the interference of amplitudes involving neutrinos of different chirality, the observables in Fermi transitions can depend on

$$K^2_{V,S} \equiv |C_V|^2 + |C'_V|^2 + |C_S|^2 + |C'_S|^2 = 2[1 + g_S^2(|\eta_{LS}|^2 + |\eta^{(e)}_{RS}|^2](G\,U_{ud}/\sqrt{2})^2 \qquad (134)$$

$$Re\, C_S C'^*_S / K^2_{V,S} \simeq \frac{1}{2} g_S^2(-|\eta_{LS}|^2 + |\eta^{(e)}_{RS}|^2), \qquad (135)$$

$$Re(C_S C^*_V + C'_S C'^*_V)/K^2_{V,S} \simeq 2g_S\, Re\,\eta_{LS}, \qquad (136)$$

$$Re(C_S C'^*_V + C'_S C^*_V)/K^2_{V,S} \simeq -2g_S\, Re\,\eta_{LS}, \qquad (137)$$

$$Im(C_S C^*_V + C'_S C'^*_V)/K^2_{V,S} \simeq 2g_S\, Im\,\eta_{LS}, \qquad (138)$$

$$Im(C_S C'^*_V + C'_S C^*_V)/K^2_{V,S} \simeq -2g_S\, Im\,\eta_{LS}. \qquad (139)$$

In observables in mixed transitions one can have in addition combinations proportional to

$$Re(C_S C^*_A + C'_S C'^*_A)/K^2_{V,S} \simeq 2g_A g_S\, Re\,\eta_{LS}, \qquad (140)$$

$$Re(C_S C'^*_A + C'_S C^*_A)/K^2_{V,S} \simeq -2g_A g_S\, Re\,\eta_{LS}, \qquad (141)$$

$$Im(C_S C^*_A + C'_S C'^*_A)/K^2_{V,S} \simeq 2g_A g_S\, Im\,\eta_{LS}, \qquad (142)$$

$$Im(C_S C'^*_A + C'_S C^*_A)/K^2_{V,S} \simeq -2g_A g_S\, Im\,\eta_{LS}. \qquad (143)$$

We shall consider now the available constraints on $Re\,\eta_{kS}$ and $Im\,\eta_{kS}$ $(k = L, R)$, assuming that the Hamiltonian (133) is the only new beta decay interaction.

$Im\,\eta_{LS}$. A constraint on the CP-violating coupling constant $Im\,\eta_{LS}$ comes from the experimental limit on the parity and time reversal violating tensor-type electron-nucleon interaction $(G/\sqrt{2})C^{eN}_T\ \overline{e}\sigma_{\lambda\mu}e\,\overline{N}\gamma_5\sigma^{\lambda\mu}N$. The constant $Im\,\eta_{LS}$ contributes to C^{eN}_T through diagrams involving W-exchange in addition to the scalar interaction [75]. The present limit on C^{eN}_T is $|C^{eN}_T| < 4 \times 10^{-8}$ [76], which follows from the experimental bound on the electric dipole moment of the ^{199}Hg atom. This limit on C^{eN}_T implies [75]

$$|Im\,\eta_{LS}| \lesssim 8 \times 10^{-5}. \qquad (144)$$

Direct limits on $Im\,\eta_{LS}$ can be obtained from searches for the T-odd R-correlation (Eq. (33)). For a scalar-type interaction R_t is given by

$$R_t \simeq 2a_R g_S\, Im\,\eta_{LS}, \qquad (145)$$

where the quantity a_R contains the ratio $g_A M_{GT}/g_V M_F$. A measurement of R in ^{19}Ne-decay yielded $R = 0.079 \pm 0.053$ [77]. For ^{19}Ne-decay $a_R \simeq 0.26$ [77], so that one obtains

$$-0.015 < g_S\, Im\,\eta_{LS} < 0.32 \quad (90\%\ c.l.) \tag{146}$$

The contribution R_f of electromagnetic final-state interactions is small, about 10^{-3} [78].

Limits on $Im\,\eta_{LS}$ follow also from experimental results on time-reversal-even observables sensitive to the combination $|C_S|^2 + |C'_S|^2$. The most stringent limit on $g_S^2(|\eta_{LS}|^2 + |\eta_{RS}^{(e)}|^2)$ $(= \frac{1}{2}(|C_S|^2 + |C'_S|^2))$ among limits provided by individual observables is

$$g_S^2(|\eta_{LS}|^2 + |\eta_{RS}^{(e)}|^2) < 1.1 \times 10^{-2} \quad (90\%\ c.l.)\ , \tag{147}$$

deduced from data on the broadening of the delayed proton peak following the superallowed $1/2^+ \to 1/2^+$ decay of ^{32}Ar and ^{33}Ar [79]. The limit (147) yields

$$|g_S\, Im\,\eta_{LS}| < 0.105 \quad (90\%\ c.l.)\ . \tag{148}$$

Bounds on $Im\,\eta_{LS}$ can be obtained in principle also from experiments measuring the $e^{\pm}$-longitudinal polarization in Fermi transitions. The constant $Im\,\eta_{LS}$ enters P_L through the Coulomb corrections (see Eq. (29)). Limits on $Im\,\eta_{LS}$ obtained this way have not been so far reported.

$Re\,\eta_{LS}$. A source of constraints on $g_S\, Re\,\eta_{LS}$ are observables which are sensitive to the Fierz interference term in Fermi transitions or mixed transitions. One of these is the charged lepton longitudinal polarization P_L^F. The measured ratios P_L^F/P_L^{GT} imply [26] $Re(C_S - C'_S) = (0.0027 \pm 0.0109)(G\,U_{ud}/\sqrt{2})$, yielding

$$-7.7 \times 10^{-3} < g_S\, Re\,\eta_{LS} < 1.0 \times 10^{-2} \quad (90\%\ c.l.) \tag{149}$$

The Fierz interference term affects also the ft-values. For $g_S\, Re\,\eta_{LS}$ the bound

$$-3.5 \times 10^{-3} < g_S\, Re\,\eta_{LS} < 4.7 \times 10^{-3} \quad (90\%\ c.l.) \tag{150}$$

was deduced from experimental ft-values of superallowed beta decays [80]. It should be noted however, that in recent studies of superallowed beta transitions it has been observed that the uncertainties in the effects of charge-dependent nuclear forces on the ft-values are larger than previously considered [81]. Consequently, the constraint on $g_S\, Re\,\eta_{LS}$ may be weaker than that in Eq. (150) [82,3].

The limit (147) on $|C_S|^2 + |C'_S|^2$ yields

$$|g_S\, Re\,\eta_{LS}| < 0.105 \quad (90\%\ c.l.)\ . \tag{151}$$

$\eta_{RS}^{(e)}$. Neglecting the interference term between the SM and the η_{RS} amplitudes (which is proportional to neutrino mass), the parameter $\eta_{RS}^{(e)}$ enters the beta decay observables always quadratically. Since the contribution of $Im\,\eta_{RS}^{(e)}$ to the parity and time-reversal violating C_T^{eN}-interaction (through diagrams involving $Im\,\eta_{RS}^{(e)}$ and the weak interaction) is also proportional to the neutrino mass, the best limit on both $Re\,\eta_{RS}^{(e)}$ and $Im\,\eta_{RS}^{(e)}$ comes from bounds on $|C_S|^2+|C'_S|^2$. Eq. (147) implies

$$|g_S\, Re\,\eta_{RS}^{(e)}| < 0.105 \quad (90\%\ c.l.)\ , \tag{152}$$

$$|g_S\, Im\,\eta_{RS}^{(e)}| < 0.105 \quad (90\%\ c.l.)\ . \tag{153}$$

The Results of a Comprehensive Analysis of Beta Decay Data. A global analysis of beta decay data [3], which included experimental data on neutron decay, ^{19}Ne-decay, and the ft-values of superallowed $0^+ \to 0^+$ beta decays yielded the following 90% c.l. upper limits:

$$|g_S\,\eta_{LS}| < 0.035\ , \tag{154}$$

$$|g_S\, Re\,\eta_{LS}| < 0.007\ , \tag{155}$$

$$|g_S\,\eta_{RS}^{(e)}| < 0.072\ , \tag{156}$$

and [31]

$$g_S^2(|\eta_{LS}|^2 + |\eta_{RS}^{(e)}|^2) < 0.0055\ . \tag{157}$$

All the limits on η_{LS} and $\eta_{RS}^{(e)}$ from beta decay depend on the value of g_S (Eq. (22)). The constant g_S was calculated in Ref. [83] in connection with a study of neutral current interactions of a general Lorentz structure. Employing a quark model with spherically symmetric wave function, g_S can be expressed as $g_S = -\frac{1}{2} + \frac{9}{10}\, g_A \simeq 0.6$. The uncertainty in this prediction has been estimated to be about 30% to 60% [83]. Including an uncertainty of this size, one has

$$1/4 \lesssim g_S \lesssim 1\ . \tag{158}$$

II.4. Tensor Interactions

The general tensor interaction for $d \to u\, e^- \nu_e^{(L,R)}$ is given in Eq. (5). We shall write it in the form

$$H_T = \left(a_{LT}\, \bar{e}\, \frac{\sigma_{\lambda\mu}}{\sqrt{2}}\, (1-\gamma_5)\, \nu_e^{(L)} + a_{RT}\, \bar{e}\, \frac{\sigma_{\lambda\mu}}{\sqrt{2}}\, (1+\gamma_5)\, \nu_e^{(R)}\right) \bar{u}\, \frac{\sigma^{\lambda\mu}}{\sqrt{2}}\, d + \text{H.c.} \tag{159}$$

where $a_{LT} = 2\alpha_{LR}$, $a_{RT} = 2\alpha_{RL}$. Note that $C_T - C'_T = 2g_T\, a_{LT}$, $C_T + C'_T = 2g_T\, a_{RT}$, where the constant g_T has been defined in Eq. (23). We shall define also $\eta_{kT} \equiv \eta_{kT}/a_{LL} \simeq a_{kT}/(G\,U_{ud}/\sqrt{2})$ $(k = L, R)$ and $\eta_{RT}^{(e)} = \eta_{RT}\,\sqrt{\bar{v}_e}$.

One of the ways in which tensor interactions can arise in gauge theories is through loop corrections to the tree-level amplitudes. In the SM this mechanism is the only possibility, and yields an interaction of the form (159) with $a_{RT} = 0$ and $\eta_{LT} \simeq 10^{-8} - 10^{-9}$ [84]. In renormalizable gauge-theories the only mechanism that can generate tensor quark-lepton interactions at the tree-level is the exchange of spin-zero leptoquarks [65]. The constraints discussed in this section apply to tensor interactions of any origin. For the tensor interactions arising from leptoquarks exchange additional constraints apply. These will be discussed in the next Section.

In the allowed approximation tensor interactions can manifest themselves only in Gamow-Teller (or mixed) beta decays. The observables in pure Gamow-Teller transitions can depend on combinations of coupling constants, which can be obtained from Eqs. (134)-(139) by replacing $C_S, C'_S, C_V, C'_V, \eta_{LS}, \eta_{RS}, g_V$ and g_S with $C_T, C'_T, C_A, C'_A, \eta_{LT}, \eta_{RT}, g_A$ and g_T, respectively. In mixed transitions the observables can depend in addition on the coupling constant combinations (140)-(143) with $C_S, C'_S, C_A, C'_A, \eta_{LS}, \eta_{RS}$ and $g_A g_S$ replaced by $C_T, C'_T, C_V, C'_V, \eta_{LT}, \eta_{RT}$ and $g_V g_T$, respectively.

$Im\,\eta_{LT}$. Like $Im\,\eta_{LS}$, the parameter $Im\,\eta_{LT}$ is constrained by the experimental limit on the parity and time reversal violating tensor-type electron-nucleon interaction [75]. The limit $|C_T^{eN}| < 4 \times 10^{-8}$ (see Section II.3) implies [75]

$$|Im\,\eta_{LT}| \lesssim 2 \times 10^{-5} . \qquad (160)$$

$Im\,\eta_{LT}$ contributes to the T-odd R-correlation. R_t is given by

$$R_t = 2a'_R\, g_T\, Im\,\eta_{LT} , \qquad (161)$$

where the constant a'_R contains for mixed transitions the ratio of the nuclear matrix elements. For ^{19}Ne-decay $a'_R \simeq 0.18$, so that the experiment of Ref. [77] (see Section II.3.) sets the limit $|g_T\, Im\,\eta_{LT}| < 0.5$. A more stringent bound on $g_T\, Im\,\eta_{LT}$ comes from a recent measurement of R in the decay $^8Li \rightarrow {}^8Be$ (2.9 MeV) $+ e^- + \overline{\nu}_e$ [85]. R_t for this decay is given by Eq. (161) with $a'_R = (3g_A)^{-1} \simeq 0.26$. The experiment yielded $R = (0 \pm 4) \times 10^{-3}$, which implies [85]

$$|g_T\, Im\,\eta_{LT}| < 1.3 \times 10^{-2} \quad (90\%\ c.l.) . \qquad (162)$$

The contribution R_f of the electromagnetic final-state interactions is expected to show up at the level of $\sim 10^{-3}$ [86].

Limits on $g_T\, Im\,\eta_{LT}$ follow also from bounds on $|C_T|^2 + |C'_T|^2$. From the results of a comprehensive analysis of beta decay data [3], listed further on in this section, the limit on $g_T^2(|\eta_{LT}|^2 + |\eta_{RT}^{(e)}|^2)$ $(= \frac{1}{2}(|C_T|^2 + |C'_T|^2))$ is (Eq. (181))

$$g_T^2(|\eta_{LT}|^2 + |\eta_{RT}^{(e)}|^2) < 4.3 \times 10^{-3} \quad (90\%\ c.l.) . \qquad (163)$$

This implies

$$|g_T\, Im\,\eta_{LT}| < 6.6 \times 10^{-2} \quad (90\%\ c.l.) . \qquad (164)$$

Data on $e^{\pm}$-longitudinal polarizations yielded [87]

$$-4.5 \times 10^{-2} < g_T\, Im\,\eta_{LT} < 2.9 \times 10^{-2} \quad (90\%\ c.l.) . \qquad (165)$$

$Re\,\eta_{LT}$. It was pointed out in Ref. [88] that the tensor interaction (159) i constrained by the experimental value of the ratio R_π (Eq. (60)) of the $\pi \rightarrow e\nu$ to $\pi \rightarrow \mu\nu_\mu$ rates, since electromagnetic corrections to the tensor interaction (159 induce an effective interaction involving pseudoscalar quark currents [89].

The induced S, P interaction for the operator (159) is of the form [88]

$$H = \tfrac{1}{4}\, a_{LT}\, k_o\, \overline{e}(1-\gamma_5)\nu_e^{(L)}\, \overline{u}(1-\gamma_5)d + \tfrac{1}{4}\, a_{RT}\, k_o\, \overline{e}(1+\gamma_5)\nu_e^{(R)}\, \overline{u}(1+\gamma_5)d + \text{H.c.} , \qquad (166)$$

where $k_o \simeq -2.8 \times 10^{-2}$. The contribution of the Hamiltonian (159) to R_π is [27]

$$R_\pi = (R_\pi)_{SM}(u_e/u_\mu)[(1-\tfrac{1}{4}\, k_o\omega\, Re\,\eta_{LT})^2 + (\tfrac{1}{4}\, k_o\omega\, Im\,\eta_{LT})^2 + |\tfrac{1}{4}\, k_o\omega\,\eta_{RT}^{(e)}|^2] , \qquad (167)$$

where $\omega \equiv (m_\pi/m_e)(m_\pi/(m_u+m_d)) \simeq 3.3 \times 10^3$ [90],

Let us consider the case when $\eta_{RT}^{(e)} = 0$. Neglecting $Im\,\eta_{LT}$, we have from the experimental result (62)

$$0.9932\,(u_\mu/u_e)^{1/2} < \left|1 - \tfrac{1}{4}\, k_o\omega\, Re\,\eta_{LT}\right| < 1.0007\,(u_\mu/u_e)^{1/2} \qquad (168)$$

Allowing for $(u_e/u_\mu)^{1/2}$ the range $0.98 < (u_e/u_\mu)^{1/2} < 1.02$ (see Section II.2.1) $(R_\pi)_{expt}$ (Eq. (30)) requires [30] $Re\,\eta_{LT}$ to be either in the range

$$-1.2 \times 10^{-3} < Re\,\eta_{LT} < 9.1 \times 10^{-4} , \qquad (169)$$

or in the range

$$-8.86 \times 10^{-2} < Re\,\eta_{LT} < -8.75 \times 10^{-2} . \qquad (170)$$

Since it seems highly unlikely that $Re\,\eta_{LT}$ would have a nonzero value in a fine-tuned interval like (170), we have to conclude that most probably $|Re\,\eta_{LT}|$ obeys

$$|Re\,\eta_{LT}| < 1.2 \times 10^{-3} . \qquad (171)$$

The bounds on $Re\,\eta_{LT}$ from $(R_\pi)_{expt}$ could be weaker if there is also a tensor contribution to $d\overline{u} \rightarrow \mu\nu_\mu$. Note that $R_\pi = (R_\pi)_{SM}$ if $u_e = u_\mu$ and the ratio $(Re\,\eta_{LT})_e/(Re\,\eta_{LT})_\mu$ of the tensor coupling constants for the electron and the muon family is m_e/m_μ. There is however no reason for such a scenario.

The range of values of $Re\,\eta_{LT}$ allowed by $(R_\pi)_{expt}$ could be wider if both the a_{LT} and the a_{RT} terms in (159) are present [91]. R_π is then given by Eq. (167). Since the present upper limit (Eqs. (180), (181)) on $|\eta_{RT}^{(e)}|^2$ allows $|\frac{1}{4}\, k_o\omega\,\eta_{RT}^{(e)}|^2$ to be as large as ~ 5, the lower bound on $[R_\pi/(R_\pi)_{SM}]$ from $(R_\pi)_{expt}$ can be satisfied even with $|1 - \frac{1}{4}\, k_o\omega\, Re\,\eta_{LT}| = 0$. It follows that the values of $Re\,\eta_{LT}$ allowed by $(R_\pi)_{expt}$ span now the whole interval

$$-8.9 \times 10^{-2} < Re\,\eta_{LT} < 9.1 \times 10^{-4} \,. \qquad (172)$$

A limit $Re(C_T - C_T') = g_A(0.0027 \pm 0.0109)(GU_{ud}/\sqrt{2})$ is provided by the experimental results on the ratio P_L^F/P_L^{GT} of longitudinal polarization in beta decays [26] yielding

$$-9.6 \times 10^{-3} < g_T\,Re\,\eta_{LT} < 1.3 \times 10^{-2} \quad (90\%\ c.l.) \,. \qquad (173)$$

$\eta_{RT}^{(e)}$. The best limits on $Re\,\eta_{RT}^{(e)}$ and $Im\,\eta_{RT}^{(e)}$ come from bounds on $|C_T|^2+|C_T'|^2$. The upper bound (181) implies

$$|g_T\,Re\,\eta_{RT}^{(e)}| < 6.6 \times 10^{-2} \quad (90\%\ c.l.) \,, \qquad (174)$$

$$|g_T\,Im\,\eta_{RT}^{(e)}| < 6.6 \times 10^{-2} \quad (90\%\ c.l.) \,. \qquad (175)$$

The constant $|\eta_{RT}^{(e)}|$ is constrained also by $(R_\pi)_{expt}$. From Eq. (167) and (62) we obtain

$$|\eta_{RT}^{(e)}| < 4.5 \times 10^{-2} \qquad (176)$$

($|\eta_{RT}^{(e)}| < 1.7 \times 10^{-3}$ if $Re\,\eta_{LT}$ is absent). The same limit follows from (167) and (62) for $|Im\,\eta_{LT}|$.

The Results of a Comprehensive Analysis of Beta Decay Data. For the Hamiltonian consisting of the SM Hamiltonian and the Hamiltonian (159), the global analysis of beta decay data in Ref. [3] yielded the following 90% c.l. limits:

$$|g_T\,\eta_{LT}| < 0.026 \qquad (177)$$

$$|g_T\,Re\,\eta_{LT}| < 0.007 \qquad (178)$$

$$|g_T\,Im\,\eta_{LT}| < 0.026 \qquad (179)$$

$$|g_T\,\eta_{RT}^{(e)}| < 0.050 \qquad (180)$$

and [31]

$$g_T^2(|\eta_{LT}|^2 + |\eta_{RT}^{(e)}|^2) < 0.0043 \,. \qquad (181)$$

The limits on η_{LT} and η_{RT} from beta decay depend on the constant g_T. This was estimated in Ref. [83] in the same framework as g_S, with the result $g_T = \frac{5}{3}(\frac{1}{2} + \frac{3}{10}\,g_A) \simeq 1.46$. Allowing for the same uncertainty in this prediction as in the prediction for g_S, we have

$$0.6 \lesssim g_T \lesssim 2.3 \,. \qquad (182$$

Data from a recent experiment on $\pi \to e\,\nu_e\gamma$ decay [92] appear to be in dis agreement with the SM description of this decay. It has been suggested [93] tha the discrepancy may be due to the presence of an a_{LT}-type tensor interaction. Thi interpretation has been investigated further in Refs. [84], [88], [94], [95], and [91] To account for the discrepancy, the value of $|Re\,\eta_{LT}|$ is required to be in the rang [96]

$$1.5 \times 10^{-2} < |Re\,\eta_{LT}| < 4.2 \times 10^{-2} \,. \qquad (183)$$

For this range to have an overlap with the values of $|Re\,\eta_{LT}|$ allowed by the limit (178) from beta decay, the tensor constant $|g_T|$ cannot be larger than ~ 0.5 [97]. Not considering the solution (170), $(R_\pi)_{expt}$ allows values of $|Re\,\eta_{LT}|$ in the range (183) only if there is a sufficiently large additional contribution to R_π from new interactions. This could be an a_{RT}-tensor interaction as described around Eq. (172).

II.5. S,T Interactions from Leptoquark-Exchange

In Section II.2.4. we have considered the leptoquark states that can mediate the $d \to u\,e^-\overline{\nu}_e^{L,R}$ transition, and discussed the V,A part of the resulting interactions. Here we shall discuss the S,P (or S,P,T) part of these interactions. As mentioned in Section II.2.4, the coupling constants of the S,P (or S,P,T) part of the leptoquark-exchange interactions are in general independent of the coupling constants in the V,A parts.

Spin-one leptoquarks. The S,P part of the $d \to u\,e^-\overline{\nu}_e^{(L,R)}$ interaction from $X_{(2/3)}$ and $X_{(1/3)}$ exchange is of the form (4) with (Ref. [64]; see also Ref. [62])

$$a_{LS} = a_{LP} = -2f_{LR} \qquad (184)$$

$$(X_{(2/3)} - exchange)$$

$$a_{RS} = -\,a_{RP} = -2f_{RL} \qquad (185)$$

and

$$a_{LS} = a_{LP} = -2h_{LR} \qquad (186)$$

$$(X_{(1/3)} - exchange)$$

$$a_{RS} = -\,a_{RP} = 2h_{RL} \qquad (187)$$

where a_{LS} and a_{RS} are defined in Eq. (133), and the coupling constants $a_{LP} = A_{LR} - A_{LL}$ and $a_{RP} = A_{RR} - A_{RL}$ describe the part of the interaction (4) involving the pseudoscalar current $\overline{u}\gamma_5 d$; the constants f_{LR}, f_{RL} and h_{LR}, h_{RL} have been defined in Eqs. (115)-(116).

Since the a_{LP} and a_{RP} terms contribute to R_π, the scalar interaction from $X_{(2/3)}$ and $X_{(1/3)}$ exchange is constrained by $(R_\pi)_{expt}$. Using the relations (184), (185) and (186), (187) R_π can be expressed as

$$R_\pi = (R_\pi)_{SM}(u_e/u_\mu)[(1+\omega\,Re\,\eta_{LS})^2 + (\omega\,Im\,\eta_{LS})^2 + |\omega\,\eta_{RS}^{(e)}|^2] \,. \qquad (188)$$

Eq. (188) with $0.98 \le (u_e/u_\mu)^{1/2} \le 1.02$, and the experimental result (62) imply, assuming that both the a_{LP} and the a_{RP} terms are present (see the discussion around Eq. (172)),

$$-6.2\times10^{-4} < Re\,\eta_{LS} < 6.4\times10^{-6} \qquad (90\%\,c.l.)\,. \tag{189}$$

In Eq. (188) we have left out the contribution of the A-component of the interaction, assuming that if it is not small (note that this contribution is not enhanced by ω), there is no accidental cancellation in R_π between the $\omega\, Re\,\eta_{LS}$-term and the terms from the A-component involving the left-handed neutrino.

For an S,P interaction the tensor electron-nucleon coupling constant C_T^{eN} constrains the combination $Im(\eta_{LS}-\eta_{LP})$ [75], which vanishes due to the relations (184) and (186). A limit on $Im\,\eta_{LS}$ comes in this case from the experimental limit on the scalar-type parity and time-reversal violating electron-nucleon interaction $(G/\sqrt{2})\,C_S^{eN}\,\bar{e}\,i\gamma_5 e\,\overline{N}N$. The present limit on C_S^{eN} is $|C_S^{eN}| < 1.4\times10^{-6}$ (90% c.l.) [99], obtained from the experimental bound on the electric dipole moment of the Tl atom. This implies $|Im\,\eta_{LP}| \lesssim 10^{-4}$ [75], and therefore

$$|Im\,\eta_{LS}| \lesssim 10^{-4}\,. \tag{190}$$

The limit on $|Im\,\eta_{LS}|$ from $(R_\pi)_{expt}$ is $|Im\,\eta_{LS}| < 3.1\times10^{-4}$ (6.3×10^{-5} if $Re\,\eta_{LS}$ is absent). For $|\eta_{RS}^{(e)}|$ the same limits are obtained. Thus

$$|\eta_{RS}^{(e)}| < 3.1\times10^{-4} \tag{191}$$

($|\eta_{RS}^{(e)}| < 6.3\times10^{-5}$ if $Re\,\eta_{LS}$ is absent). For $u_e = u_\mu$ one would have $|\eta_{RS}^{(e)}| < 3.1\times10^{-4}$ ($|\eta_{RS}^{(e)}| < 1.2\times10^{-5}$ if $Re\,\eta_{LS}$ is absent).

The further constraints on η_{LS} and $\eta_{RS}^{(e)}$ are those considered in Section II.3.

Spin-zero leptoquarks. The general form of S,P,T, part of the $d \to u\,e^-\overline{\nu}_e^{(L,R)}$ interaction mediated by $Y_{(2/3)}$ and $Y_{(1/3)}$ leptoquarks is the sum of the Hamiltonians (4) and (5), with (Ref. [64]; see also Ref. [62])

$$a_{LS} = -a_{LP} = -\tfrac{1}{2}F_{LL} \tag{192}$$
$$a_{RS} = a_{RP} = -\tfrac{1}{2}F_{RR} \tag{193}$$
$$(Y_{(2/3)} - exchange)$$
$$a_{LT} = a_{LS} \tag{194}$$
$$a_{RT} = a_{RS}\,, \tag{195}$$

and

$$a_{LS} = -a_{LP} = -\tfrac{1}{2}H_{LL} \tag{196}$$
$$a_{RS} = a_{RP} = -\tfrac{1}{2}H_{RR} \tag{197}$$
$$(Y_{(1/3)} - exchange)$$
$$a_{LT} = -a_{LS} \tag{198}$$
$$a_{RT} = -a_{RS}\,, \tag{199}$$

respectively. The constants a_{LT}, a_{RT} have been defined in Eq. (159), and the constants $F_{LL}, F_{RR}, H_{LL}, H_{RR}$ in Eqs. (117) and (118).

The $\overline{u}\gamma_5 d$-component of the S,P,T part of the interaction contributes to R_π. Leaving out (as in Eq. (188)) the contribution of the axial-vector interaction, R_π is given by

$$R_\pi = (R_\pi)_{SM}\,(u_e/u_\mu)[(1+\omega\,Re\,\eta_{LP}^{(|Q|)})^2 + (\omega\,Im\,\eta_{LP}^{(|Q|)})^2 + |\omega\,\eta_{RP}^{(e)(|Q|)}|^2] \tag{200}$$

($|Q|$) = 2/3, 1/3), where $\eta_{LP}^{|Q|} = a_{LP}^{(|Q|)}/a_{LL}$, $\eta_{RP}^{(e)(|Q|)} = a_{RP}^{(e)(|Q|)}/a_{LL}$. In Eq. (200) we have neglected the small contribution of the P-interaction generated from the tensor part of the interaction by electromagnetic corrections. Using the relations (192) - (195) and (196) - (199) and allowing $0.98 < (u_e/u_\mu)^{1/2} < 1.02$, one has from $(R_\pi)_{expt}$

$$-6.4\times10^{-6} < Re\,\eta_{LT}^{(2/3)} < 6.2\times10^{-4}\,, \tag{201}$$

$$-6.2\times10^{-4} < Re\,\eta_{LT}^{(1/3)} < 6.4\times10^{-6} \tag{202}$$

The same limits hold also for $Re\,\eta_{LS}^{|Q|}$ ($|Q| = 2/3,\,1/3$).

For S,P and T interactions one has from C_T^{eN} (see Section II.3) the limit $|Im(6\eta_{LT}+\eta_{LS}-\eta_{LP})| \lesssim 8\times10^{-5}$ [75]. Using the relations (192), (194) and (196), (198) this yields

$$|Im\,\eta_{LT}^{(2/3)}| \lesssim 1\times10^{-5}\,, \tag{203}$$

$$|Im\,\eta_{LT}^{(1/3)}| \lesssim 2\times10^{-5}\,. \tag{204}$$

The limit on $|Im\,\eta_{LS}|$ (= $|Im\,\eta_{LT}|$) from $(R_\pi)_{expt}$ is the same as in the case of spin-one leptoquark exchange. The same limit holds also for $|\eta_{RS}^{(e)(|Q|)}|$ (= $|\eta_{RT}^{(e)(|Q|)}|$) ($Q = 2/3, 1/3$). Thus

$$|\eta_{RT}^{(e)(|Q|)}| = |\eta_{RS}^{(e)(|Q|)}| < 3.1\times10^{-4}\,. \tag{205}$$

In the presence of both scalar and tensor couplings the constraint from the ratio P_L^F/P_L^{GT} of positron polarizations is given by [98] (recall that since $\eta_{RR}^{(e)*}\,\eta_{RL}^{(e)} = 0$, the V,A part of the interaction does not contribute to P_L^F/P_L^{GT})

$$Re(C_S - C_S') + g_A^{-1}\,Re(C_T - C_T') = (0.0027 \pm 0.0109)(G\,U_{ud}/\sqrt{2})\,. \tag{206}$$

Let us denote the η_{LT} from the $Y_{(|Q|)}$-exchange interaction by $\eta_{LT}^{(|Q|)}$. Using the relations (194) and (198), Eq. (206) gives

$$-9.6\times10^{-3} \;<\; (g_T + g_S\, g_A)\, Re\,\eta_{LT}^{(2/3)} \;<\; 1.3\times10^{-2} \qquad (90\%\ c.l.)\ , \quad (207)$$

and

$$-9.6\times10^{-3} \;<\; (g_T - g_S\, g_A)\, Re\,\eta_{LT}^{(1/3)} \;<\; 1.3\times10^{-2} \qquad (90\%\ c.l.)\ . \quad (208)$$

The constraints (207) and (208) are the same as (173), except for the factors $(g_T \pm g_S\, g_A)$, which replace g_T in Eq. (173).

The bounds from the ft-values of superallowed beta decays considered in Section II.3 hold here not only for $g_S\, Re\,\eta_{LS}$, but also for $g_S\, \eta_{LT}^{(2/3)}$ and $g_S\,(-\eta_{LT}^{(1/3)})$.

In the presence of both S- and T-couplings the D-coefficient contains a term proportional to $Im(C_S C_T^* + C_S' C_T'^*) = 2 g_S g_T Im(a_{LS}\, a_{LT}^* + a_{RS}^{(e)}\, a_{RT}^{(e)*})$ [14]. This term vanishes however for leptoquark interactions due to the relations (194), (195) and (198), (199) [100].

Interaction terms of the a_{LS}-type can arise from S_1, $(R_2)_{T_z=-1/2}$, $U_{1\mu}$ and $(V_{2\mu})_{T_z=-1/2}$ exchange, while a_{RS}-type couplings can be generated by S_1, $(\tilde{R}_2)_{T_z=+1/2}$, $U_{1\mu}$ and $(\tilde{V}_{2\mu})_{T_z=1/2}$ exchange [64]. As discussed earlier, the exchange of spin-one leptoquark states will generate simultaneously P-type couplings, and the exchange of spin-zero states simultaneously P,T couplings.

The limits (201) and (202) make $Re\,\eta_{LT}$ from $Y_{(2/3)}$ or $Y_{(1/3)}$ exchange too small to be able to account for the discrepancy between theory and experiment in $\pi \to e\nu_e\gamma$ decay [91], mentioned at the end of Section II.4. The constraints on S,T couplings from $(R_\pi)_{expt}$ could be weaker if there is a cancellation in R_π between leptoquark contributions to $\pi \to e\nu_e$ and $\pi \to \mu\nu_\mu$, but there is no known model in which this could happen naturally. The constraints from $(R_\pi)_{expt}$ on the tensor couplings could be weaker also if $Y_{(2/3)}$ and $Y_{(1/3)}$ contribute simultaneously [66]. Then $(R_\pi)_{expt}$ constrains only $Re(\eta_{LT}^{(2/3)} - \eta_{LT}^{(1/3)})$, while $Re\,\eta_{LT} = Re(\eta_{LT}^{(2/3)} + \eta_{LT}^{(1/3)})$ could in principal be sufficiently large. Similarly, the limits on the scalar couplings could be weaker than (189) and (191) if spin-one and spin-zero leptoquarks contribute simultaneously [66]. But again, there is no known model in which such cancellations could take place in a way other than accidentally.

III. MUON DECAY

III.1. Introduction

The main decay mode of the muon is the decay into two neutrinos: $\mu^+ \to e^+ + n + n'$ [101]. In the SM $n = \nu_{eL}$ and $n' = \overline{\nu}_{\mu L}$, where ν_{eL} and $\nu_{\mu L}$ are massless two-component neutrinos which are, respectively, the $T_z = 1/2$ states of the $SU(2)_L$ doublets involving the electron and the muon. The interaction responsible for this decay is due to W-exchange and has the $V - A$ form

$$H_{SM}^{(\mu)} = (G/\sqrt{2})\, \overline{e}\, \gamma_\lambda (1-\gamma_5)\, \nu_e\, \overline{\nu}_\mu\, \gamma^\lambda (1-\gamma_5)\mu \;+\; \text{H.c.} \qquad (209)$$

where $G = (g^2/8M_W^2)(1+\Delta r)$; Δr represents radiative corrections [102].

In extensions of the SM there may be new interactions contributing to the decay mode $\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$, and also new interactions giving rise to other decays of the type $\mu^+ \to e^+ + n + n'$. In the presence of the new interactions the neutrinos are expected to be massive, and the neutrino gauge group eigenstates are not expected to coincide with the mass eigenstates.

In models where lepton family numbers are conserved, or conserved to a good approximation, it is sufficient to consider for muon decay the most general lepton family number conserving (LC) four-fermion interaction. This can be written in the helicity projection form [103] as

$$\begin{aligned} H_{LC}^{(\mu)} \;=\; & 4(G_F/\sqrt{2})(g_{LL}^V\, \overline{e}_L\, \gamma_\lambda\, \nu_{eL}\, \overline{\nu}_{\mu L}\, \gamma^\lambda\, \mu_L \\ & +\; g_{RR}^V\, \overline{e}_R\, \gamma_\lambda\, \nu_{eR}\, \overline{\nu}_{\mu R}\, \gamma^\lambda\, \mu_R \;+\; g_{LR}^V\, \overline{e}_L\, \gamma_\lambda\, \nu_{eL}\, \overline{\nu}_{\mu R}\, \gamma^\lambda\, \mu_R \\ & +\; g_{RL}^V\, \overline{e}_R\, \gamma_\lambda\, \nu_{eR}\, \overline{\nu}_{\mu L}\, \gamma^\lambda\, \mu_L \;+\; g_{LL}^S\, \overline{e}_L\, \nu_{eR}\, \overline{\nu}_{\mu R}\, \mu_L \;+\; g_{RR}^S\, \overline{e}_R\, \nu_{eL}\, \overline{\nu}_{\mu L}\, \mu_R \\ & +\; g_{LR}^S\, \overline{e}_L\, \nu_{eR}\, \overline{\nu}_{\mu L}\, \mu_R \;+\; g_{RL}^S\, \overline{e}_R\, \nu_{eL}\, \overline{\nu}_{\mu R}\, \mu_L \\ & +\; g_{LR}^T\, \overline{e}_L\, t_{\alpha\beta}\, \nu_{eR}\, \overline{\nu}_{\mu L}\, t^{\alpha\beta}\, \mu_R \;+\; g_{RL}^T\, \overline{e}_R\, t_{\alpha\beta}\, \nu_{eL}\, \overline{\nu}_{\mu R}\, t^{\alpha\beta}\, \mu_L) \;+\; \text{H.c.} \end{aligned} \qquad (210)$$

In Eq. (210) we have followed the notation and the normalization of the coupling constants of Ref. [104] (used also in Refs. [105] and [4]). Thus the first and the second subscript on the coupling constants indicates the handedness of the electron and of the muon, respectively. G_F is the Fermi constant, and $t_{\alpha\beta} = (1/\sqrt{2})\,\sigma_{\alpha\beta}$. The neutrino states $\nu_{eL} (= \frac{1}{2}(1-\gamma_5)\nu_e)$ and $\nu_{eR} (= \frac{1}{2}(1+\gamma_5)\nu_e)$ are the left-handed and the right-handed components of the mass-eigenstate ν_e.

The Hamiltonian (210) contains 19 real parameters (10 complex coupling constants minus an overall phase). Neglecting neutrino masses, the $e^\pm$-observables depend on 10 real constants [106], denoted in the literature by $a, a', b, b', c, c', \alpha, \alpha', \beta$ and β'. The constants $a, a', \ldots$ are bilinear combinations of the coupling constants. The transverse polarization parameters α, α', β and β' are measured directly, while the remaining six constants are determined through measurements of the muon lifetime, the spectrum parameters ρ, ξ, δ and the longitudinal polarization parameters ξ' and ξ''.

In Ref. [104] (see also Ref. [4]) limits have been set on all the coupling constants of the Hamiltonian (210) using the experimental results on the muon decay parameters and on the inverse muon decay cross-section. One of the results is the lower bound

$$Q_{LL} \equiv (\tfrac{1}{4}|g_{LL}^S|^2 + |g_{LL}^V|^2) \;>\; 0.949 \qquad (90\%\ c.l.)\ , \qquad (211)$$

obtained from muon decay data alone, on the quantity Q_{LL} which contains the SM contribution. To obtain limits on the individual constants g_{LL}^V and g_{LL}^S in (211) one needs additional information. This is provided by the inverse muon decay process $\nu_\mu e^- \to \mu^- \nu_e$ [104]. Taking into account the constraints on the coupling constants from muon decay data, and using the result $1 + h < 0.00318$ for the

ν_μ-helicity h (deduced [107] from the experimental lower bound on $P_\mu \xi \delta/\rho$ [108], where ($-P_\mu$) is the degree of longitudinal polarization of the μ^+ at the instant of μ^+-decay), one obtains [104] for the ratio of the $\nu_\mu e^- \to \mu^- \nu_e$ cross-section and the $\nu_\mu e^- \to \mu^- \nu_e$ cross section predicted by the SM

$$S \simeq |g^V_{LL}|^2 \, . \tag{212}$$

The cross-section for inverse muon decay has been measured recently by the CHARM II collaboration [109] and by the CCFR collaboration [110], obtaining

$$S = 1.054 \pm 0.079 \qquad \text{(CHARM II)}\,, \tag{213}$$

$$S = 0.981 \pm 0.057 \qquad \text{(CCFR)}\,. \tag{214}$$

The result (214) yields (Ref. [110]; see also Ref. [105])

$$|g^V_{LL}| > 0.96 \qquad (90\%\ c.l.)\,, \tag{215}$$

and since $Q_{LL} \leq 1$, one has also (Ref. [110]; see also Ref. [105])

$$|g^S_{LL}| < 0.55 \qquad (90\%\ c.l.)\,. \tag{216}$$

The limits from (213) are only slightly weaker.

Thus the conclusion for the Hamiltonian (210) is that the only term in (210) which we know to be nonzero is the SM term. Moreover, this term is responsible for at least 92.5% of the observed muon decay rate. For the absolute values of the non-standard coupling constants the upper limits obtained are in the range 0.033 to 0.55 (Refs. [110, 111]; see also Ref. [105]).

The most general Hamiltonian [112], which allows for lepton family number violation and total lepton number violation can be obtained from the Hamiltonian (210) by the replacements

$$g^V_{LL}\, \overline{e}_L \gamma_\lambda \nu_{eL} \overline{\nu}_{\mu L} \gamma^\lambda \mu_L \;\to\; \sum_{i,j} (g^V_{LL})_{ij}\, \overline{e}_L \gamma_\lambda n_{iL} \overline{n}_{jL} \gamma^\lambda \mu_L \,, \tag{217}$$

$$g^V_{RR}\, \overline{e}_R \gamma_\lambda \nu_{eR} \overline{\nu}_{\mu R} \gamma^\lambda \mu_R \;\to\; \sum_{i,j} (g^V_{RR})_{ij}\, \overline{e}_R \gamma_\lambda n^c_{iR} n^c_{jR} \gamma_\lambda \mu_R \,,$$

and analogous replacements for all the other terms in (210). As in (210) the fermion fields in (217) are mass eigenstates. The indices i, j run over all the neutrino states that can be emitted in the decay. The set n_{iL} includes all the left-handed neutrinos ($n_{1L} \equiv \nu_{eL}$, $n_{2L} \equiv \nu^c_{eL}$, $n_{3L} \equiv \nu_{\mu L}$, etc.), and the set n^c_{iR} all the right-handed ones ($n^c_{1R} \equiv \nu^c_{eR}$, $n^c_{2R} \equiv \nu_{eR}$, $n^c_{3R} \equiv \nu^c_{\mu R}$, etc.).

For the general case the $e^\pm$-spectrum and polarizations can again be described (assuming that the masses of the neutrinos that can be emitted in the decay can be neglected) by the constants $a, a', b, b', c, c', \alpha, \alpha', \beta$ and β', which have the same physical meaning as in the LC case [112]. The constants $a, a', \ldots$ are obtained by replacing the set of coupling constant combinations appearing in the LC case by a new set involving the coupling constants of the general Hamiltonian. There is a one-to-one correspondence between the coupling constant combinations pertinent to the LC case and the coupling constant combinations for the general case [112]. Consequently, the limits set on the coupling constant combinations of the LC case apply for the corresponding combinations of the general case. Thus, since [112]

$$\tfrac{1}{4}\, |g^S_{LL}|^2 \leftrightarrow \sum_{i>j} |(g^V_{LL})_{ij} + \tfrac{1}{2}(g^S_{LL})_{ji}|^2 \,,$$

$$|g^V_{LL}|^2 \leftrightarrow \sum_{i\leq j} |(g^V_{LL})_{ij} + \tfrac{1}{2}(g^S_{LL})_{ji}|^2 \,, \tag{218}$$

the constraint (211) becomes

$$Q_{LL} \equiv \sum_{i,j} |(g^V_{LL})_{ij} + \tfrac{1}{2}(g^S_{LL})_{ji}|^2 > 0.949 \qquad (90\%\ c.l.) \tag{219}$$

The inverse muon decay process is now $\nu_\pi e^- \to \mu^- n_i$, where $\nu_\pi = \sum_j c_j n_{jL}$ is the neutrino state emitted in $\pi^+ \to \mu^+ \nu_\pi$ decay, and n_i are some neutrino states. The ratio S is given by [112]

$$S \simeq \sum_i |(g^V_{LL})_{i3} + \tfrac{1}{2}(g^S_{LL})_{3i}|^2 \,, \tag{220}$$

where the neutrino state ν_π has been denoted as n_{3L}.

The limits corresponding to (215) and (216) are

$$\sum_i |(g^V_{LL})_{i3} + \tfrac{1}{2}(g^S_{LL})_{3i}|^2 > 0.925 \qquad (90\%\ c.l.)\,, \tag{221}$$

$$\sum_{\substack{i,j\\ j\neq 3}} |(g^V_{LL})_{ij} + \tfrac{1}{2}(g^S_{LL})_{ji}|^2 < 0.075 \qquad (90\%\ c.l.)\,. \tag{222}$$

The sum in Eq. (221) contains the SM contribution, but the contributions of the scalar and the vector coupling constants in (221) cannot be separated. What one learns in this case from the bound (221) is that at least one of the μ^+ decay modes which involves the neutrino $\overline{\nu}_\pi$ produced in $\pi^- \to \mu^- \overline{\nu}_\pi$ decay dominates the μ^+ decay rate [112].

We note yet that there is some information also on the second neutrino in muon decay [113]. This follows from the experiment of Ref. [114], where neutrinos

(n_e) from μ^+ decay have been observed through the reaction $n_e D \to ppe^-$. The good agreement of the measured $n_e D \to ppe^-$ cross section with the calculated one in the SM indicates that the total muon decay rate contains a substantial contribution from μ^+ decay into a final state in which one of the neutrinos is the one accompanying the positron in nuclear beta decay. From a search [115] for $e^\pm$ production by ν_π on nucleons one has in addition some evidence that $\nu_\pi \neq n_e$ and $\nu_\pi \neq \bar{n}_e$.

In the subsequent two sections we shall discuss new contributions to muon decay in left-right symmetric models and in models with exotic fermions. Tree-level contributions to $\mu^+ \to e^+ + n + n'$ decays from new interactions arise also in many other theoretical schemes. These include models involving charged Higgs bosons [116], models with neutral flavor-changing gauge bosons or Higgs bosons [117], R-parity violating supersymmetric models [74], models with dileptonic gauge bosons [118], and models with dileptonic Higgs bosons [119]. New contributions to $\mu^+ \to e^+ + n + n'$ are present also in models with composite leptons, generated by constituent exchange [120]. The strength of these interactions is of the order of g^2/Λ_c^2 where g is an effective strong coupling constant and Λ_c is the compositeness scale. Assuming $g^2/4\pi \simeq 1$, muon decay provides a lower bound of a few TeV on Λ_c for some types of couplings.

We note yet that the undetected weakly interacting particles in the process $\mu^+ \to e^+ + missing\ neutrals$ may include also light particles other than neutrinos [121].

III.2. Left-Right Symmetric Models

Neglecting mixing in the leptonic sector, the leptonic couplings in the Lagrangian (76) give rise to a V,A muon decay interaction of the form included in Eq. (210) with [122]

$$(G_F/\sqrt{2})\, g^V_{LL} \simeq g_L^2/8m_1^2 , \tag{223}$$

$$\kappa^V_{RR} \simeq g_R^2 m_1^2/g_L^2 m_2^2 , \tag{224}$$

$$\kappa^V_{LR} = \kappa^{V*}_{RL} \simeq -e^{i\omega}\,(g_R\zeta/g_L) , \tag{225}$$

where $\kappa^V_{ik} = g^V_{ik}/g^V_{LL}$ $(ik = RR, LR, RL)$. Note that

$$|g^V_{LL}|^2 = (1+|\kappa^V_{RR}|^2 + |\kappa^V_{LR}|^2 + |\kappa^V_{RL}|^2)^{-1} , \tag{226}$$

$$|g^V_{ij}|^2 = |\kappa^V_{ij}|^2\,(1+|\kappa^V_{RR}|^2 + |\kappa^V_{LR}|^2 + |\kappa^V_{RL}|^2)^{-1}$$
$$\simeq |\kappa^V_{ij}|^2 \qquad (ij = RR, LR, RL) . \tag{227}$$

In $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models the neutrinos are massive in general, and the neutrinos are therefore expected to mix.

Let us consider the case when the neutrinos are Dirac fermions. Neglecting the kinematic effects of the neutrino masses (including interference terms proportional to neutrino mass) the mixing can be taken into account by making in observables the substitutions

$$g^V_{LL} \to g^V_{LL}\,\sqrt{u_e u_\mu} , \tag{228}$$

$$\kappa^V_{RR} \to \kappa^V_{RR}\,\sqrt{\tilde{v}_e \tilde{v}_\mu} , \tag{229}$$

$$\kappa^V_{LR} \to \kappa^V_{LR}\,\sqrt{\tilde{v}_\mu} , \tag{230}$$

$$\kappa^V_{RL} \to \kappa^V_{RL}\,\sqrt{\tilde{v}_e} , \tag{231}$$

where

$$u_l = \sum_i{}' \,|U_{li}|^2 \qquad (l = e, \mu) , \tag{232}$$

$$v_l = \sum_i{}' \,|V_{li}|^2 \qquad (l = e, \mu) , \tag{233}$$

and $\tilde{v}_l = v_l/u_l$ $(l = e, \mu)$; for n generations U and V are $n \times n$ unitary matrices. The prime on the summation signs in Eqs. (232) and (233) indicates that the sum is over the neutrinos that can be emitted in muon decay. Note that the u_e and v_e in Eqs. (232) and (233) are not equal in general to the analogous quantities for beta decay, but we shall assume here for simplicity that they are the same, and use the same symbol for them. Some of the muon decay observables involve the muon polarization. The polarization of the μ^+ from $\pi^+ \to \mu^+ \nu_\mu$ decay is $(-P_\mu)$, where

$$P_\mu \simeq 1 - 2|\eta_{RR} - \eta_{RL}|^2\,\tilde{v}_\mu . \tag{234}$$

The coupling constants η_{RR} and η_{RL} in Eq. (234) are $\eta_{RR} = \eta^{(e)}_{RR}/\sqrt{\tilde{v}_e}$ and $\eta_{RL} = \eta^{(e)}_{RL}/\sqrt{\tilde{v}_e}$, where $\eta^{(e)}_{RR}$ and $\eta^{(e)}_{RL}$ are given in Eqs. (79) and (81).

It follows that the $e^\pm$ spectrum and polarizations are described by four parameters: $\kappa^V_{RR}\sqrt{\tilde{v}_e\tilde{v}_\mu}$, $|\kappa^V_{LR}|\sqrt{\tilde{v}_\mu}$, $|\kappa^V_{LR}|\sqrt{\tilde{v}_e}$ (where we have used $|\kappa^V_{RL}| = |\kappa^V_{LR}|$) and $|\eta_{RR} - \eta_{RL}|\sqrt{\tilde{v}_\mu}$.

Among limits provided by individual observables the best one on $|\kappa^V_{LR}|\sqrt{\tilde{v}_\mu}$ and $|\kappa^V_{LR}|\sqrt{\tilde{v}_e}$ comes from the parameter ρ, and the best limit on $\kappa_{RR}\sqrt{\tilde{v}_e\tilde{v}_\mu}$ from measurements of the quantity $R_{(\mu)} \equiv 1 - (\delta\xi/\rho)P_\mu$ [108,4].

The parameter ρ is given by

$$\rho \simeq \tfrac{3}{4}(1 - |\kappa^V_{LR}|^2\,\tilde{v}_\mu - |\kappa^V_{LR}|^2\,\tilde{v}_e) . \tag{235}$$

The experimental value $\rho = 0.7518 \pm 0.0026$ [13] implies [123]

$$|\kappa^V_{LR}|\,\sqrt{\tilde{v}_\mu} \;<\; 6.7\times 10^{-2} \qquad (90\%\ c.l.)\ , \tag{236}$$

$$|\kappa^V_{LR}|\,\sqrt{\tilde{v}_e} \;<\; 6.7\times 10^{-2} \qquad (90\%\ c.l.)\ . \tag{237}$$

$R_{(\mu)}$ is given by

$$R_{(\mu)} \;\simeq\; 2|\kappa^V_{RR}|^2\,\tilde{v}_e\,\tilde{v}_\mu \;+\; 2|\eta_{RR}-\eta_{RL}|^2\,\tilde{v}_\mu\ . \tag{238}$$

The experimental result $R_{(\mu)} < 0.00323$ [108] yields

$$|\kappa^V_{RR}|\,\sqrt{\tilde{v}_e\,\tilde{v}_\mu} \;<\; 0.040 \qquad (90\%\ c.l.)\ , \tag{239}$$

and also

$$|\eta_{RR}-\eta_{RL}|\,\sqrt{\tilde{v}_\mu} \;<\; 0.040 \qquad (90\%\ c.l.)\ . \tag{240}$$

Combining (240) and (236) (noting that $|\eta_{RL}| = |\kappa^V_{LR}|$), we obtain

$$|\eta_{RR}|\,\sqrt{\tilde{v}_\mu} \;<\; 0.11\ . \tag{241}$$

A slightly better bound than (241) follows from the limit (82).

The remaining spectrum parameters and the polarization parameters are given by

$$\delta \;\simeq\; \tfrac{3}{4}(1 - 3|\kappa^V_{LR}|^2\,\tilde{v}_\mu \;+\; 3|\kappa^V_{LR}|^2\,\tilde{v}_e)\ , \tag{242}$$

$$\xi \;\simeq\; 1 - 2|\kappa^V_{RR}|^2\,\tilde{v}_e\,\tilde{v}_\mu \;+\; 2|\kappa_{LR}|^2\,\tilde{v}_\mu - 4|\kappa^V_{LR}|^2\,\tilde{v}_e\ , \tag{243}$$

$$\xi' \;\simeq\; 1 - 2|\kappa^V_{RR}|^2\,\tilde{v}_e\,\tilde{v}_\mu \;-\; 2|\kappa^V_{LR}|^2\,\tilde{v}_e\ , \tag{244}$$

$$\xi'' \;\simeq\; 1 + 2|\kappa_{LR}|^2\,\tilde{v}_e \;+\; 2|\kappa^V_{RL}|^2\,\tilde{v}_\mu\ , \tag{245}$$

$$\alpha = \alpha' = \beta = \beta' = 0\ . \tag{246}$$

In models where $\tilde{v}_e = \tilde{v}_\mu = 1$ one obtains from the experimental limit on $R_{(\mu)}$

$$|\kappa^V_{RR}| \;<\; 0.040 \qquad (90\%\ c.l.) \tag{247}$$

(which is a slightly better limit than the bound (82)), and (using $|\eta_{RR}| \lesssim \kappa^V_{RR}$)

$$4|\eta_{RR}|^2 \;+\; 2|\eta_{RL}|^2 \;+\; 4Re\,\eta_{RR}\,\eta^*_{RL} \;\lesssim\; 0.00323\ . \tag{248}$$

Eq. (248) yields $|\eta_{RR}| \lesssim 0.040$ and $|\eta_{LR}| = |\kappa^V_{LR}| \lesssim 0.057$. From the ρ-parameter and from the relation $|\kappa^V_{LR}| \lesssim |\kappa^V_{RR}|$ [56] (which is valid unless Higgs boson with $T_R \gg 1$ are present) one has the better limits

$$|\kappa^V_{LR}| \;<\; 0.047 \qquad (90\%\ c.l.) \tag{249}$$

and

$$|\kappa^V_{LR}| \;\lesssim\; 0.040, \tag{250}$$

respectively.

In manifestly left-right symmetric models (where $\kappa^V_{RR} \simeq m_1^2/m_2^2$ and $\kappa^V_{LR} = \kappa^V_{RL} = -\zeta$) the best limits are $|\kappa^V_{RR}| \lesssim 6\times 10^{-3}$ (from the $K_L - K_S$ mass difference and $|\kappa^V_{LR}| \lesssim 3.4\times 10^{-3}$ (from charged-current universality) (see Section II.2.2). For pseudomanifest models one has again $|\kappa^V_{RR}| \lesssim 6\times 10^{-3}$ (see the text after Eq. (98) and, barring a cancellation in Eq. (95), $|\kappa^V_{LR}| < 3.6\times 10^{-3}$ (obtained by combining the limits from (95) and (90)).

So far we have been dealing with the case when the neutrinos are Dirac fermions For Majorana neutrinos $\nu^{(L)}_{\ell} - (\nu^{(R)}_{\ell'})^c$ ($\ell' = \ell$ or $\ell' \neq \ell$) mixing can also take place (U and V are then $n\times 2n$ matrices; $(U, V^*)^T$ is unitary [11]), and as a consequence new terms appear in the muon decay spectrum [124]. The new terms affect the parameters α, α', β and β', which no longer vanish, and the coupling constants $|\kappa^V_{LR}|^2\,\tilde{v}_\mu$ and $|\kappa^V_{LR}|^2\,\tilde{v}_e$, which have to be replaced by $|\kappa^V_{LR}|^2\,(\tilde{v}_\mu + |\omega_{\mu e}|^2/u_e\,u_\mu)$ and $|\kappa^V_{LR}|^2\,(\tilde{v}_e + |\omega_{e\mu}|^2/u_e\,u_\mu)$, where $\omega_{\ell\ell'} = \sum'_k V_{\ell k}\,U_{\ell' k}$ [124]. The upper limits in Eqs. (236) and (237) remain valid, and the same limits apply for $|\omega_{\mu e}|^2/u_e\,u_\mu$ and $|\omega_{e\mu}|^2/u_e\,u_\mu$. For α, α', β and β' one has, considering for simplicity only terms first order in the new interactions, $\alpha \simeq 0$, $\alpha' \simeq 0$, and [124]

$$\beta \;\simeq\; -8\,\kappa^V_{RR}\;Re\,\omega_{e\mu}\,\omega^*_{\mu e}/u_e\,u_\mu\ , \tag{251}$$

$$\beta' \;\simeq\; -8\,\kappa^V_{RR}\;Im\,\omega_{e\mu}\,\omega^*_{\mu e}/u_e\,u_\mu\ . \tag{252}$$

The experimental values $\beta/A = (3.9\pm 6.2)\times 10^{-3}$ and $\beta'/A = (1.5\pm 6.3)\times 10^{-3}$ yield

$$-2.8\times 10^{-2} \;<\; \kappa^V_{RR}\;Re\,\omega_{e\mu}\,\omega^*_{\mu e}/u_e\,u_\mu \;<\; 1.3\times 10^{-2} \qquad (90\%\ c.l.)\ , \tag{253}$$

$$-2.4\times 10^{-2} \;<\; \kappa^V_{RR}\;Im\,\omega_{e\mu}\,\omega^*_{\mu e}/u_e\,u_\mu \;<\; 1.8\times 10^{-2} \qquad (90\%\ c.l.)\ . \tag{254}$$

In addition to new contributions from the gauge bosons, muon decay receives in $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ models also contributions from the exchange of charged Higgs bosons. One such contribution is from the physical charged Higgs bosons of the bidoublet field $\phi(2T_L+1 = 2, 2T_R+1 = 2, Y = 2)$, which generates the Dirac masses of fermions [35]. The size of this contribution depends among others on the allowed range of the right-handed scale in the model and on unknown Yukawa couplings.

A further Higgs contribution is present in an attractive class of $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ models [125], which employs the triplet Higgs boson Δ_R (1,3,2) to induce part of the symmetry breaking. These models provide a framework for the understanding of the smallness of the masses of the usual neutrinos. The Δ_R couples to the right-handed leptons, and generates a large Majorana mass term for the right-handed neutrinos. The models contain also the triplet Higgs field Δ_L (3,1,2), required by the discrete left-right symmetry included in the model. The singly charged Higgs boson Δ_L^+ mediates the exotic muon decay $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$ [126]. The corresponding Hamiltonian is of the form

$$\begin{aligned} H &= 2(G_\mu^{(e)}/\sqrt{2})\, \overline{\nu}_e^c\, (1-\gamma_5)\, e\, \overline{\mu}(1+\gamma_5)\, \nu_\mu^c \;+\; \text{H.c.} \\ &= (G_\mu^{(e)}/\sqrt{2})\, \overline{\mu}\, \gamma_\lambda (1-\gamma_5)\, e\, \overline{\nu}_\mu \gamma^\lambda\, (1-\gamma_5)\, \nu_e \;+\; \text{H.c.}\,, \end{aligned} \tag{255}$$

where $G_\mu^{(e)} = \sqrt{2} f_{ee} f_{\mu\mu}^* / 4m_+^2$; the $f_{\ell\ell}$ ($\ell = e, \mu$) are lepton-Δ_L Yukawa couplings and m_+ is the mass of the Δ_L^+. It can be shown [127] that $|G_\mu^{(e)}| \gtrsim 2 \times 10^{-4}\, G_F$ for muon neutrino masses in the range 35 keV $\lesssim m_{\nu_\mu} \lesssim$ 270 keV (= the present experimental upper limit for m_{ν_μ}), which in the model is the allowed range of m_{ν_μ} for which ν_μ is required by cosmological considerations to be unstable. The lower bound on $|G_\mu^{(e)}|$ in the range 35 keV $\lesssim m_{\nu_\mu} \lesssim A$ ($<$ 270 keV) increases with decreasing A. For $A = 35$ keV one has $|G_\mu^{(e)}| \gtrsim 2 \times 10^{-2}\, G_F$.

The best limit on $|G_\mu^{(e)}|$ is [128]

$$|G_\mu^{(e)}| \;<\; 0.16\, G_F \qquad (90\%\ \text{c.l.})\,, \tag{256}$$

obtained in an experiment searching for the decay $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$. $|G_\mu^{(e)}|$ can be as large as the upper limit (256) [129]. The Δ_L^+ contributes also to other two-neutrino muon decay modes, but the amplitudes for these are proportional to leptonic mixing angles. The non-standard contributions to muon decay from the gauge bosons are expected to be small, since in this class of models the limit on $g_R^2 m_1^2 / g_L^2 m_2^2$ is $\sim$ 0.015 [39] and the contribution of the light mass eigenstates in $\nu_e^{(R)}$ is suppressed by the small light-heavy mixing angles.

III.3. Models with Exotic Fermions

The Langrangian (100) (Section II.2.3) generates a muon decay interaction of V,A structure. In the case of Dirac neutrinos muon decay is described by the effective coupling constants $g_{LL}^V \sqrt{u_e u_\mu}$, $\kappa_{RR}^V \sqrt{\tilde{v}_e \tilde{v}_\mu}$, $\kappa_{LR}^V \sqrt{\tilde{v}_\mu}$, and $\kappa_{RL}^V \sqrt{\tilde{v}_e}$, which are given by

$$g_{LL}^V \sqrt{u_e u_\mu} \;=\; (g^2/8M_W^2)\, \sqrt{u_e u_\mu}\,, \tag{257}$$

$$\kappa_{LR}^V \sqrt{\tilde{v}_\mu} \;\simeq\; s_R^\mu \sqrt{\tilde{v}_\mu}\,, \tag{258}$$

$$\kappa_{RL}^V \sqrt{\tilde{v}_e} \;\simeq\; s_R^e \sqrt{\tilde{v}_e}\,, \tag{259}$$

$$\kappa_{RR}^V \sqrt{\tilde{v}_e \tilde{v}_\mu} \;\simeq\; s_R^e\, s_R^\mu \sqrt{\tilde{v}_e \tilde{v}_\mu} \;\simeq\; (\kappa_{LR}^V \sqrt{\tilde{v}_\mu})\, (\kappa_{RL}^V \sqrt{\tilde{v}_e})\,. \tag{260}$$

The quantities u_ℓ and $v_\ell = \tilde{v}_\ell u_\ell$ ($\ell = e, \mu$) are here given by $u_\ell = \sum_i |(A_L^\nu)_{\ell i}|^2$ and $v_\ell = \sum_i |(F_R^\nu)_{\ell i}|^2$ (see Section II.2.3). The muon decay parameters are given by expressions identical to those in Eqs. (235), (238), and (242)-(246), except that $|\kappa_{LR}^V| \sqrt{\tilde{v}_e}$ has to be replaced everywhere by $|\kappa_{RL}^V| \sqrt{\tilde{v}_e}$, since here $|\kappa_{RL}^V| \neq |\kappa_{LR}^V|$.

The global analysis in Ref. [58] of the contraints on ordinary-exotic fermion mixings yielded

$$|\kappa_{LR}^V| \sqrt{\tilde{v}_\mu} \;<\; 0.039 \qquad (90\%\ \text{c.l.})\,, \tag{261}$$

$$|\kappa_{RL}^V| \sqrt{\tilde{v}_e} \;<\; 0.042 \qquad (90\%\ \text{c.l.})\,. \tag{262}$$

The sources of these limits are data on muon decay [58]. For $\kappa_{RR}^V \sqrt{\tilde{v}_\mu}$ the relation (260) and the limits (261) and (262) imply

$$|\kappa_{RR}^V| \sqrt{\tilde{v}_e \tilde{v}_\mu} \;<\; 1.7 \times 10^{-3}\,. \tag{263}$$

As $|s_R^{(e)} \sqrt{\tilde{v}_e}| \lesssim |s_R^{(e)}|$ and $|s_R^{(\mu)} \sqrt{\tilde{v}_\mu}| \lesssim |s_R^{(\mu)}|$, the coupling constants $|\kappa_{LR}^V| \sqrt{\tilde{v}_\mu}$ and $|\kappa_{RL}^V \sqrt{\tilde{v}_e}|$ are constrained also by limits on $|s_R^e|$ and $|s_R^\mu|$, which come predominantly from neutral current data (both low energy and at the Z-peak). The present limits on $|s_R^e|$ and $|s_R^\mu|$ are $|s_R^e| < 0.10$ and $|s_R^\mu| < 0.12$ [60], which give weaker limits on $|\kappa_{LR}^V| \sqrt{\tilde{v}_\mu}$ and $|\kappa_{RL}^V| \sqrt{\tilde{v}_e}$ than (261) and (262).

For Majorana neutrinos $\nu_e^{(L)} - (\nu_e^{(R)})^c$ and $\nu_\mu^{(L)} - (\nu_e^{(R)})^c$ mixing can be present, induced by mixing between the ordinary and the exotic doublet neutrinos. The muon decay parameters are obtained in this case formally in the same way as in left-right symmetry models. The quantity $\omega_{\ell\ell'}$ (see Section III.2) is given here by $\omega_{\ell\ell'} = (A_L^\nu F_L^{\nu\dagger})_{\ell'\ell}$ [58], where $F_L^{\nu*} = F_R^\nu$.

IV. CONCLUSIONS

In this article we have reviewed and discussed possible new interactions in beta decay and muon decay, and the available constraints on the associated coupling constants. We can summarize the main points as follows:

Beta Decay

For new V,A interactions beta decay provides information on the coupling constants $\eta_{RR}^{(e)}$ and $\eta_{RL}^{(e)}$ (including the T-violating combination $Im\, \eta_{RR}^{(e)*}\, \eta_{RL}^{(e)}$ sensitive to their relative phase) describing the interactions involving right-handed neutrinos, and on the T-violating constant $Im\, \eta_{LR}$ which is sensitive to the relative phase of the a_{LR} and the SM interaction.

New V,A interactions involving right-handed currents can arise at the tree-level through the exchange of new gauge bosons with right-handed couplings to the fermions (such as the W_R in left-right symmetric models), as a result of mixing of the usual fermions with exotic ones with right-handed couplings to the W, and from the exchange of leptoquarks.

All the above mechanisms can give a T-violating contribution to the D-coefficient. In models involving leptoquarks a nonzero $Im\,\eta_{LR}$ can be generated by the exchange of $X_{(1/3)}$ or $Y_{(2/3)}$ leptoquarks. A nonzero $Im\,\eta_{RR}^{(e)*}\,\eta_{RL}^{(e)}$ term can arise from leptoquark exchange only if leptoquarks of the same spin but different charges or leptoquarks of the same charge but different spins contribute simultaneously. Both $|Im\,\eta_{LR}|$ and $|Im\,\eta_{RR}^{(e)*}\,\eta_{RL}^{(e)}|$ can be as large as the present experimental limit (10^{-3}) on $|Im\,(\eta_{LR}+\eta_{RR}^{(e)*}\,\eta_{RL}^{(e)})|$. In left-right symmetric models and in models with exotic fermions the D-coefficient is dominated by the $Im\,\eta_{LR}$ term. In these models more stringent limits ($\sim 10^{-4}$) on $|Im\,\eta_{LR}|$ than from the D-coefficient follow from the experimental bounds on the electric dipole moment of the neutron D_n and on ϵ'/ϵ. However in view of the uncertainties in the calculations of D_n and ϵ'/ϵ these limits are not as reliable as the direct limit.

For leptoquark models the best limits on $|\eta_{RR}^{(e)}|$ and $|\eta_{RL}^{(e)}|$ are those from beta decay ($|\eta_{RR}^{(e)}| < 0.10$, $|\eta_{RL}^{(e)}| < 0.04$). Already in the most general version of $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ models $|\eta_{RL}^{(e)}|$ is constrained also by muon decay data. In $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ models with manifest or pseudomanifest left-right symmetry there is a stringent limit on $|\eta_{RR}^{(e)}|$ from the $K^\circ - \overline{K}^\circ$ amplitude, and on $|\eta_{RR}^{(e)}|$ from $K \to 3\pi$ decays, from charged-current universality, and also from the relation $|\zeta| \lesssim m_1^2/m_2^2$. In models with exotic fermions limits for $\eta_{RL}^{(e)}$ and $\eta_{RR}^{(e)}$ follow from muon decay data and also from neutral current experiments. At present the best limit on $|\eta_{RL}^{(e)}|$ comes from muon decay, and the best limit on $|\eta_{RR}^{(e)}|$ from combining the constraints from $K \to 3\pi$ and muon decay.

Scalar beta decay interactions can arise at the tree level through the exchange of charged Higgs bosons, the exchange of leptoquarks, and in R-parity violating supersymmetric models also by the exchange of sleptons. Beta decay is sensitive to the coupling constants $g_S\,\eta_{LS}$ and $g_S\,\eta_{RS}$, and with the exception of $g_S\,Im\,\eta_{LS}$ provides the most stringent limits on them ($|g_S\,Re\,\eta_{LS}| < 7\times 10^{-3}$, $|g_S\,\eta_{RS}^{(e)}| < 7\times 10^{-2}$). $Im\,\eta_{LS}$ can be probed in beta decay through the T-odd R-coefficient and also through some T-even observables. The most stringent limit on $Im\,\eta_{LS}$ ($|Im\,\eta_{LS}| \lesssim 10^{-4}$, which is three orders of magnitude smaller than the upper limit from beta decay) has been deduced from the experimental bound on the P,T-violating tensor electron-nucleon interaction. However this limit may involve unknown theoretical uncertainties and therefore a limit from beta decay, even an order of magnitude weaker, would be valuable.

Tensor beta decay interactions are described by the coupling constants $g_T\,\eta_{LT}$ and $g_T\,\eta_{RT}$. Except for $Im\,\eta_{LT}$, for a tensor interaction of unspecified origin the best limits on these constants are from beta decay ($|g_T\,Re\,\eta_{LT}| < 7\times 10^{-3}$, $|g_T\,\eta_{RT}| < 5\times 10^{-2}$). The most stringent limit on $Im\,\eta_{LT}$ ($|Im\,\eta_{LT}| \lesssim 2\times 10^{-5}$) comes from the experimental bound on the tensor electron-nucleon interaction. The next best limit ($|g_T\,Im\,\eta_{LT}| \lesssim 10^{-2}$) is from a search for the R-correlation. Again, improved limits from beta decay would be valuable.

The exchange of spin-one and spin-zero leptoquarks generates S,P and S,P,T beta decay interactions (in addition to V,A couplings), respectively. In renormalizable gauge theories the exchange of spin-zero leptoquarks is the only possible source of tensor interactions at the tree level. For the S and T interactions from leptoquark exchange stringent limits on η_{LS}, $\eta_{RS}^{(e)}$, η_{LT} and $\eta_{RT}^{(e)}$ follow from the $\pi \to e\nu_e/\pi \to \mu\nu_\mu$ ratio ($|Re\,\eta_{Lk}| < 6\times 10^{-4}$, $|Im\,\eta_{Lk}| < 3\times 10^{-4}$, $|\eta_{Rk}^{(e)}| < 3\times 10^{-4}$ ($k = S,T$). These limits would be weaker if there is some cancellation between the electronic and the muonic pseudoscalar contribution to R_π. But there is no known model in which this could happen naturally. The limits from $(R_\pi)_{expt}$ for the tensor couplings could also be weaker if there is a pseudoscalar contribution from both $Y_{(2/3)}$ and $Y_{(1/3)}$ leptoquarks, and for the scalar couplings if there is such a contribution from both spin-one and spin-zero leptoquarks. But again, there is no known model in which the required cancellations could take place in a way other than accidentally.

New beta decay interactions could arise also in composite models (which we mentioned here only in connection with muon decay), as a result of constituent exchange. The strength of such interactions is of the order of $4\pi/\Lambda_c^2$, where Λ_c is the compositeness scale. With $\Lambda_c \simeq$ a few TeV (the current lower bound from other processes), the corresponding beta decay coupling constants would have values near their present upper bounds.

Muon Decay

Experiment indicates that among the decays $\mu^+ \to e^+ + n + n'$ the decay mode which dominates the total $\mu^+ \to e^+ + n + n'$ rate involves the neutrino species of the Standard Model scenario. The combination of the coupling constants which dominates the rate includes the coupling constant of the Standard Model interaction.

Muon decay gives information on left-right symmetric models and on models with exotic fermions which is complementary to the information provided by beta decay.

In left-right symmetric models the gauge bosons generate a V,A interaction described for Dirac neutrinos by the four effective coupling constants $g_{LL}^V\,\sqrt{u_e\,u_\mu}$, $\kappa_{RR}^V\,\sqrt{\bar{v}_e\,\bar{v}_\mu}$, $\kappa_{LR}^V\,\sqrt{\bar{v}_\mu}$ and $\kappa_{LR}^V\,\sqrt{\bar{v}_e}$. For the most general version of $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ models the best limits on $\kappa_{RR}^V\,\sqrt{\bar{v}_e\,\bar{v}_\mu}$, $\kappa_{LR}^V\,\sqrt{\bar{v}_\mu}$ and $\kappa_{LR}^V\,\sqrt{\bar{v}_e}$ come from muon decay itself ($\kappa_{RR}^V\,\sqrt{\bar{v}_e\,\bar{v}_\mu} < 4\times 10^{-2}$, $|\kappa_{LR}^V|\,\sqrt{\bar{v}_\mu} < 7\times 10^{-2}$, $|\kappa_{LR}|\,\sqrt{\bar{v}_e} < 7\times 10^{-2}$). For Majorana neutrinos some additional parameters are needed to account for the effects of neutrino mixing, but these do not affect the above limits. In $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models with triplet Higgs bosons the new contributions from the gauge bosons are expected to be small. A new effect for muon decay is the presence of the exotic decay mode $\mu^+ \to e^+\,\overline{\nu}_e\nu_\mu$, mediated by the Δ_L^+ Higgs boson. This decay can probe the mass of the muon neutrino in the range where cosmological considerations require ν_μ to be unstable.

In models with exotic fermions the V,A interaction is described formally by the same four effective coupling constants and the same additional parameters for the Majorana neutrino case as in left-right symmetric models, except that now $|\kappa_{RL}^V| \neq |\kappa_{LR}^V|$. At present the best limits on $|\kappa_{LR}^V|\,\sqrt{\bar{v}_\mu}$, $|\kappa_{RL}^V|\,\sqrt{\bar{v}_e}$ and $\kappa_{RR}^V\,\sqrt{\bar{v}_e\bar{v}_\mu}$ ($|\kappa_{LR}^V|\,\sqrt{\bar{v}_\mu} < 4\times 10^{-2}$, $|\kappa_{RL}^V|\,\sqrt{\bar{v}_e} < 4\times 10^{-2}$, $\kappa_{RR}^V\,\sqrt{\bar{v}_e\bar{v}_\mu} < 2\times 10^{-3}$) come from muon decay. These coupling constants are constrained also by neutral current experiments.

Muon decay probes new leptonic interactions also in many other extensions of the Standard Model.

Our overall conclusion is that beta decay and muon decay gives unique information on some classes of possible interactions beyond the Standard Model. It is important to improve the sensitivity of the pertinent experiments by as much as possible.

Note Added. After most of this article was completed I learned that for the $\pi \to e\nu_e/\pi \to \mu\nu_\mu$ ratio R_π a new experimental result [130] has appeared. The constraint on $R_\pi/(R_\pi)_{SM}$ from this experiment is $0.9967 \; (u_\mu/u_e)^{1/2} < R_\pi/(R_\pi)_{SM} < 1.0033 \; (u_\mu/u_e)^{1/2}$, to be compared with the bound in Eq. (63). Inspection shows that the bound corresponding to Eq. (64) is $(-3.3 \times 10^{-2}) < Re\eta_{LR} < 3.3 \times 10^{-2}$ and that the difference between the bounds (65), (66), (169), (176), (189), and (191) and the corresponding bounds implied by the result of Ref. [130] is negligible.

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[129] Indirect empirical limits on $G_\mu^{(e)}$ come from the bound (222), from the W-mass, and from charged-current universality (Ref. [127]); for a review see P. Herczeg, Ref. [113]. We would like to warn the reader that in the latter reference we called G_F the constant $(g^2/8M_W^2)(1+\Delta r)$ and G_μ the constant whose value is deduced from the muon decay rate ($G_\mu = 1.6639(2) \times 10^{-5} GeV^{-2}$). This can be confusing, since Ref. [13] defines G_F as $G_F = 1.16639(2) \times 10^{-5}\ GeV^{-2}$. In the present article we define G_F as in Ref. [13] and use the symbol G for $(g^2/8M_W^2)(1+\Delta r)$.

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