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Bounds on the mass of W_R and the W_L-W_R mixing angle ζ in general $SU(2)_L \times SU(2)_R \times U(1)$ models

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We consider the phenomenological constraints on the mass M_R and the W_L-W_R mixing angle ζ in a very general class of $SU(2)_L \times SU(2)_R \times U(1)$ models. In particular, almost no model-dependent assumptions are made concerning left-right symmetry or the Higgs structure of the theory, which means that U^R, the mixing matrix for right-handed quarks, is unrelated to the left-handed Cabibbo-Kobayashi-Maskawa matrix U^L. We consider a number of possibilities for the neutrinos occurring in right-handed currents, including (a) heavy Majorana neutrinos, (b) heavy Dirac neutrinos, (c) intermediate-mass (10–100 MeV) neutrinos, and (d) light neutrinos (e.g., the Dirac partners of the ordinary left-handed neutrinos). For each case we utilize relevant constraints from the K_L-K_S mass difference, $B_d\bar{B}_d$ oscillations, the b semileptonic branching ratio and decay rate, neutrinoless double-beta decay, theoretical relations between mass and mixing, universality, nonleptonic kaon decays, muon decay, and astrophysical constraints from nucleosynthesis and SN 1987A. As is to be expected the limits on M_R are considerably weaker than for the special case of manifest or pseudomanifest left-right symmetry ($M_R > 1.4$ TeV). In fact, if extreme fine-tuning is allowed the W_R could be as light as the ordinary W_L. However, with reasonable restrictions on fine-tuning one obtains $M_R > 300$ GeV for $g_R = g_L$, with more stringent limits holding for most of parameter space. If CP-violating phases in U^R are small the limit on mixing ($|\zeta| < 0.0025$ for $g_R = g_L$) is almost as stringent as for the case of left-right symmetry. For large phases $|\zeta|$ could be as large as ~ 0.013.

I. INTRODUCTION

Soon after the discovery of parity violation[1] it was established that to first approximation the weak charged currents have $V - A$ structure.[2] This is incorporated into the standard model[3] by having only the left-handed fermions transform nontrivially under the SU(2) group. The question then naturally arises as to whether the right-handed fermions take part in charged-current weak interactions at all, and, if they do, with what strength. One can easily introduce charged-current interactions for the right-handed fermions by extending the gauge group.[4] The simplest example is the $SU(2)_L \times SU(2)_R \times U(1)$ model,[5] in which the left-handed fermions transform as doublets under $SU(2)_L$ and are invariant under $SU(2)_R$, with the situation reversed for the right-handed fermions. The U(1) factor is also different from the standard model U(1): for the ordinary quarks and leptons it couples to $B - L$. [One sometimes denotes the gauge group as $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.]

The addition of a new SU(2) to the gauge group implies the existence of three new gauge bosons: two charged and one neutral. There are thus two sets of charged gauge bosons: the $W_L^\pm$ belonging to $SU(2)_L$ are the same as the $W^\pm$ of the standard model, while the $W_R^\pm$ of $SU(2)_R$ are new. In general these gauge group eigenstates mix with each other to form mass eigenstates $W_{1,2}^\pm$ of mass $M_{1,2}$:

$$W_L^+ = \cos\zeta W_1^+ - \sin\zeta W_2^+ ,$$
$$W_R^+ = e^{i\omega}(\sin\zeta W_1^+ + \cos\zeta W_2^+) , \tag{1}$$

where ζ is a mixing angle and ω is a CP-violating phase.[6] If ζ is small (which turns out to be the case) then W_R and W_L approximately coincide with W_2 and W_1, respectively. In this case, $M_2 \simeq M_R$ and $M_1 \simeq M_L$, where M_R and M_L are the masses of W_R and W_L in the absence of mixing. There is also a second Z boson Z'. Limits on the Z' mass (>325 GeV) and mixing angle with the ordinary Z ($|\theta| < 0.05$) can be obtained from weak neutral-current analyses[7] and will not be discussed here.

There have been many theoretical and phenomenological studies of $SU(2)_L \times SU(2)_R \times U(1)$ models,[8] and many limits have been presented on M_R and ζ (Ref. 9). However, almost all of these analyses have involved extra assumptions, especially on the Higgs structure of the theory. Most authors have assumed that the Lagrangian is invariant under a discrete left-right (L-R) symmetry in which the left- and right-handed fermions are interchanged. These models imply that the gauge couplings g_L and g_R of the $SU(2)_L$ and $SU(2)_R$ subgroups, respectively, are equal, as well as restrictions on the Yukawa couplings of the theory. They are attractive because they imply that the original Lagrangian is parity conserving; i.e., parity violation occurs because of spontaneous symmetry breaking, which yields different masses for the $SU(2)_L$ and $SU(2)_R$ gauge bosons. Such models are viable when considered in isolation, but run into serious difficulties when embedded in grand unified theories or when their cosmological implications are considered. In particular, they lead to much too high a prediction for $\sin^2\theta_W$, have severe difficulties in accounting for the cosmological baryon asymmetry, and may lead to cosmological domain-wall problems.[10] For these reasons, most recent authors[11] have assumed that the discrete *L-R* symmetry is not a good symmetry at low (TeV) energies, i.e., that it is broken at much higher scale than the $SU(2)_L \times SU(2)_R \times U(1)$-breaking scale. This allows $g_L \neq g_R$.

In addition to *L-R* symmetry, almost all limits have been derived under the further assumption that the symmetry is either manifest (see Ref. 12 for a detailed discussion) or pseudomanifest.[13] Manifest *L-R* symmetry follows from the unrealistic assumption that *CP* violation is generated by complex Yukawa couplings, but that the vacuum expectations values (VEV's) of the Higgs field which generate the fermion masses are real. It implies $U^L = U^R$, where U^L is the ordinary Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix for the left-handed currents,[14] and U^R is the analogous mixing matrix for the right-handed currents. Pseudomanifest *L-R* symmetry requires that both *CP* and *P* violation are spontaneous, i.e., that the Yukawa couplings are real. It implies that the quark mixing matrices are related by $U^R = U^{L*}K$, where K is a diagonal phase matrix.[8] While reasonable on particle-physics grounds, spontaneous *CP* breaking runs into the aforementioned difficulties with the cosmological baryon asymmetry and domain walls.

The usual ansatz $|U_{ij}^L| = |U_{ij}^R|$ is therefore dependent on questionable assumptions concerning the origin of *CP*-breaking phases. Such phases could well be large, leading to a large violation. (*CP* violation in the standard model is small because of small intergenerational mixing angles, *not* because of a small phase.) The ansatz also depends on the specific Higgs content of the theory: it can be evaded for the more general realizations of *L-R* symmetry allowed if there are extra Higgs representations.[8] Finally, it depends on the assumption of a discrete left-right symmetry which restricts the form of the Yukawa couplings. This *L-R* symmetry is not required by either the $SU(2)_L \times SU(2)_R \times U(1)$ gauge symmetry or by its possible extension to SO(10) (Ref. 15). Despite these caveats, we consider $|U^L| = |U^R|$ to be the simplest and most likely possibility. However, alternatives should be investigated.

Similarly, most analyses have assumed that the neutrinos involved in right-handed currents are either very light or else are heavy Majorana neutrinos. This is again a model-dependent (i.e., Higgs- and fermion-representation-dependent) ansatz.

We consider the $SU(2)_L \times SU(2)_R \times U(1)$ gauge structure to be more fundamental than the Higgs/Yukawa/fermion-representation structure, so it seems worthwhile to reexamine the phenomenological limits without assuming manifest or pseudomanifest *L-R* symmetry, for a variety of assumptions concerning the right-handed neutrinos, and allowing $g_L \neq g_R$ (we do assume that g_L and g_R are the same order of magnitude). In particular, we investigate the existing constraints and limits on M_R and ζ allowing a completely arbitrary U^R for several classes of $SU(2)_L \times SU(2)_R \times U(1)$ models: (a) those involving heavy ($> m_\mu$) Majorana neutrinos in the right-handed currents, (b) those with heavy Dirac neutrinos (whose left-handed partners are distinct from the ordinary left-handed neutrinos), (c) intermediate-mass (10–100 MeV) neutrinos, and (d) models involving light right-handed neutrinos (e.g., Dirac partners of the ordinary left-handed neutrinos). The implications of these general models for high-energy colliders[16] and for the rare decay $K_{L \to \mu e}$ (Ref. 17) are considered elsewhere.

The plan of this paper is the following. In Sec. II we briefly review the relevant formalism of general $SU(2)_L \times SU(2)_R \times U(1)$ theories and of specific models. Section III is devoted to the various experimental and theoretical constraints. Most are generalizations of constraints that have been obtained previously in specific models. The most important are the following.

(i) For light (< 1–10 MeV) right-handed neutrinos there are extremely stringent constraints from nucleosynthesis[18] and from the energetics of Supernova 1987A (Ref. 19).

(ii) Limits on deviations of muon decay parameters from the $V-A$ predictions yield (Refs. 20–22)

$$M_R > 406 \text{ GeV}, \quad -0.04 < \zeta < 0.056 \tag{2}$$

and the correlated allowed region in Fig. 1. [We have translated these results into our sign convention for ζ (Ref. 23).] These limits apply only if the right-handed neutrinos are light enough to be produced without kinematic suppression in μ decay.

(iii) The K_L-K_S mass difference Δm_K can receive an important contribution from the box diagrams in Fig. 2 involving both W_L and W_R exchange, which have a strongly enhanced matrix element.[24] For the cases of manifest or pseudomanifest *L-R* symmetry, this yields a very stringent bound[24,25] $M_R > 1.4$–2.5 TeV, with the exact value depending on certain theoretical assumptions. However, this limit is strongly dependent on the assumed

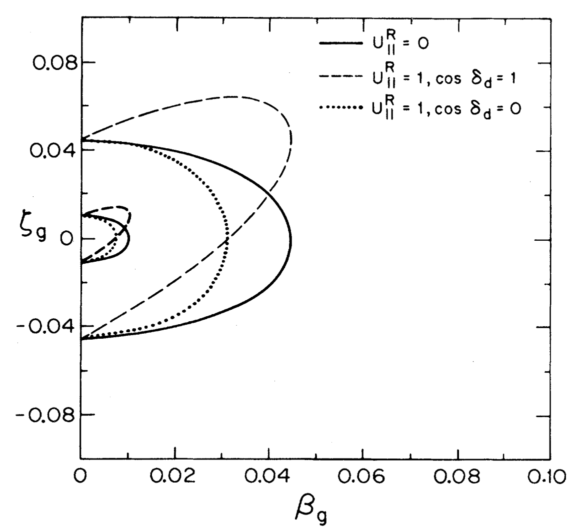

FIG. 1. 90%-C.L. regions in β_g and ζ_g from muon decay for forms $U_{(III)}^R$ or $U_{(IV)}^R$ (region between solid lines); forms $U_{(I)}^R$ or $U_{(II)}^R$ with $\cos\delta_d = 1$ (dashed lines); forms $U_{(I)}^R$ or $U_{(II)}^R$ with $\cos\delta_d = 0$ (dotted lines). The constraints for PMLRS are almost identical to $U_{(I)}^R$ and $U_{(II)}^R$. The standard model corresponds to $\beta_g = \zeta_g = 0$.

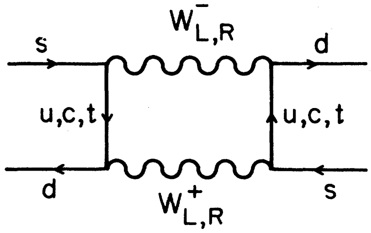

FIG. 2. Box diagrams for K^0-$\bar{K}^0$ mixing, ignoring W_L-W_R mixing (which gives negligible corrections for allowed parameters). The W_L-W_L diagram is the standard-model contribution.

(pseudo)manifest L-R symmetry (PMLRS). For arbitrary U^R the limits are much weaker.[26] In fact, there are certain fine-tuned values for the elements of U^R for which Δm_K yields no useful constraint on M_R. In Sec. III we will formulate reasonable criteria forbidding extreme fine-tuning. In that case there are two small (but not excessively fine-tuned) regions of parameter space which yield the weakest constraints. These are centered around

$$U^R_{(A)}(\alpha)=\begin{bmatrix}1 & 0 & 0\\ 0 & c_\alpha & \pm s_\alpha\\ 0 & s_\alpha & \mp c_\alpha\end{bmatrix},\qquad U^R_{(B)}(\alpha)=\begin{bmatrix}0 & 1 & 0\\ c_\alpha & 0 & \pm s_\alpha\\ s_\alpha & 0 & \mp c_\alpha\end{bmatrix}, \tag{3}$$

where $c_\alpha \equiv \cos_\alpha$, $s_\alpha \equiv \sin\alpha$, and α is an arbitrary angle. We will concentrate on the special cases $\alpha=0$ and $\alpha=\pi/2$, which yield the four special forms $U^R_{(\mathrm{I})}$-$U^R_{(\mathrm{IV})}$ listed in Table I. The results for other values of α smoothly interpolate between these limits. It should be noted that given the observed hierarchy of quark mass eigenvalues one expects small mixings between families in $U^{L,R}$ except for fine-tuned values of the quark mass matrices. Only form $U^R_{(\mathrm{I})} \sim I$ satisfies this criterion. We

TABLE I. Special forms for U^R allowed by three-family unitarity. The constraints on M_R are weakest for U^R in the vicinity of $U^R_{(A)}(\alpha)$ and $U^R_{(B)}(\alpha)$ in (3). $U^R_{(A)}(\alpha)$ interpolates smoothly between $U^R_{(\mathrm{I})}$ and $U^R_{(\mathrm{II})}$ as α varies between 0 and $\pi/2$, while $U^R_{(B)}$ interpolates between $U^R_{(\mathrm{III})}$ and $U^R_{(\mathrm{IV})}$. (Only the absolute values of the matrix elements are relevant.) $U^R_{(LR)}$ can represent either the case of manifest ($U^R_{(LR)}=U^L$) or pseudomanifest ($U^R_{(LR)}=U^{L*}K$) left-right symmetry.

$U^R_{(\mathrm{I})}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$U^R_{(\mathrm{III})}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$
$U^R_{(\mathrm{II})}=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$	$U^R_{(\mathrm{IV})}=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$
$\lvert U^R_{(LR)_{ij}}\rvert=\lvert U^L_{ij}\rvert$	

will, however, consider the phenomenological implications for all four cases. The weakest limit from Δm_K is for $U^R_{(\mathrm{I})} \sim I$, which yields $M_R > 300$ GeV for $g_R \approx g_L$, independent of the properties of the right-handed neutrinos. We emphasize, however, that most of the volume of parameter space yields stronger constraints, closer to those for PMLRS.

(iv) $B_d\bar{B}_d$ mixing places a stringent constraint on M_R for $U^B_{(\mathrm{IV})}$. [The W_R-exchange diagrams are unimportant for PMLRS (Ref. 27).] Neither $B_s\bar{B}_s$ mixing (expected to be near maximal in the standard model) nor $D\bar{D}$ mixing yield important constraints.

(v) For the case of very heavy ($>m_b$) neutrinos the right-handed current does not contribute to leptonic or semileptonic decays of the known fermions.[28] In this case the b-quark semileptonic branching ratio is modified from the standard-model prediction. This constraint improves the bounds obtained from Δm_K alone (from around 350 GeV to $\sim$450 GeV) for U^R near $U^R_{(\mathrm{II})}$ or $U^R_{(\mathrm{IV})}$. Plausible arguments involving the total b lifetime[29] and the b semileptonic decay spectrum can be used to extend these limits to neutrino masses smaller than m_b.

(vi) Mohapatra has argued[30] that the combination of limits on neutrinoless double-beta decay[31] ($\beta\beta_{0\nu}$—the relevant diagram is shown in Fig. 3) and vacuum stability imply strong constraints on models with PMLRS for the case of heavy Majorana neutrinos. We generalize the argument to the case of arbitrary U^R and show that it considerably strengthens the limits on M_R for forms I and II.

There are several stringent bounds on the W_L-W_R mixing angle ζ.

(a) Masso[32] has derived the important bound

$$|\zeta| \le \beta \equiv \frac{M_1^2}{M_2^2}, \tag{4}$$

for the case of L-R symmetry. In fact, L-R symmetry only enters in the justification of using relatively small Higgs representations of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)$ and for taking $g_L = g_R$. We show that (4) generalizes to

$$|\zeta_g| \le C\beta_g, \tag{5}$$

where

$$\zeta_g \equiv \frac{g_R}{g_L}\zeta,\quad \beta_g \equiv \frac{g_R^2}{g_L^2}\beta = \frac{g_R^2}{g_L^2}\frac{M_1^2}{M_2^2}, \tag{6}$$

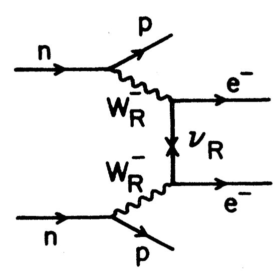

FIG. 3. New contribution to neutrinoless double-beta decay ($\beta\beta_{0\nu}$) for the case of a heavy Majorana neutrino. The effects of W_L-W_R mixing are unimportant (Ref. 30).

and C is a constant that is of order unity for all reasonable Higgs representations. Equation (5) is mainly important for very heavy (TeV) W_R.

(b) Wolfenstein[33] has derived a stringent limit ($|\zeta_g|<0.002$ using current data) from weak universality for the case of heavy neutrinos and PMLRS. We generalize the constraint to arbitrary U^R and show that it actually applies (to leading order in small quantities) to the case of light neutrinos as well. The universality constraint is very stringent for all U^R as long as CP-violating phases in $e^{i\omega}U^R$ are small. For maximal phases, however, only a much weaker second-order constraint (which applies only to heavy neutrinos) survives.

(c) Demanding that the PCAC (partial conservation of axial-vector-current) relations between $K\to 2\pi$ and $K\to 3\pi$ be valid to an accuracy of 10%, Donoghue and Holstein obtained a bound on ζ_g comparable to that from universality for the case of PMLRS, independent of the neutrino masses.[34] Its generalization has a somewhat different dependence on U^R_{ud} and U^R_{us} than the universality constraint, and they are therefore complementary. Again, the constraints are very stringent for small phases in $e^{i\omega}U^R$. For large phases, second-order constraints from PCAC are weaker by a factor ~ 3.5, but nevertheless lead to the best limits on mixing for this case. The theoretical assumptions in the K decay limit are somewhat more questionable than the other constraints—the sensitivity of the results (for small phases) to this particular input can be read off from Figs. 5–7 below.

(d) W_L-W_R mixing reduces the mass M_1 of the lighter gauge boson from the standard-model value M_L (Ref. 35). Current data sets a limit of a few percent on $|\zeta|$ for M_R in the several hundred GeV range. Future data should improve this constraint considerably.

Several other constraints on M_2 and ζ are not used because they are weaker than those considered here. These include direct production limits,[36] β-decay asymmetries and distributions,[37] and the y distribution in deep-inelastic charged-current neutrino scattering.[38] There are many limits[39] on the mixing between the ordinary neutrinos and possible new heavy neutrinos in the SU(2)×U(1) model, particularly for heavy neutrinos in the 10 MeV–10 GeV range. Some of these could possibly be translated into useful constraints on M_2 and ζ for narrow ranges of right-handed neutrino masses, but we have not attempted to do so. We have not included any constraints from CP violation.[40] That is an entirely different dimension involving many new phases. (CP-violating effects could be important when $e^{i\omega}U^R$ has large phases—the subject merits further investigation.) Similarly, we do not consider flavor-changing neutral currents induced by Higgs bosons.[8]

In Sec. IV we present the limits on M_2 and ζ. One cannot effectively separate g_R/g_L, so limits are presented for $M_{2g}\equiv g_L M_2/g_R$, ζ_g, and β_g. The most important single constraint is Δm_K, which severely limits the value of M_{2g} except in the vicinity of the special values of U^R listed in (3) and in Table I. (We have carried out fits in which the elements of U^R were completely arbitrary except for unitarity constraints, but have found no weaker limits or interesting cases other than these special forms.) Essentially all of the constraints are ineffective for certain cases for U^R and the neutrino masses, but collectively they set reasonably strong limits on M_2 and ζ in all cases. The weakest limit (barring extreme fine-tuning) turns out to be $\beta_g<0.075$ ($M_{2g}>300$ GeV) at 90% C.L., which occurs for a heavy Dirac neutrino for U^R near $U^R_{(\mathrm{I})}$ (the identity); the limits are much stronger in most other cases. (These results assume three-family unitarity, but would be essentially unchanged if there are additional families with small mixing with the first three.) The limits on M_{2g} for all cases are summarized in Table II, and estimates of possible production limits at the Superconducting Super Collider (SSC) and Fermilab Tevatron are given in Fig. 4.

Limits on ζ_g are summarized in Table III and in Figs. 5–7. The limits depend critically on the phases $\delta_{d,s}$ of $e^{i\omega}U^R_{ud}$ and $e^{i\omega}U^R_{us}$. We therefore consider the extreme cases $\cos\delta_{d,s}=1$ (small CP violation) and $\cos\delta_{d,s}=0$ (maximum CP phases). The limits on $|\zeta_g|$ are always very stringent for $\cos\delta=1$, the weakest being

TABLE II. 90%-C.L. limits on $\beta_g\equiv g_R^2M_1^2/g_L^2M_2^2$ and on $M_{2g}\equiv g_LM_2/g_R$ (GeV) for forms $U^R_{(\mathrm{I})}-U^R_{(\mathrm{IV})}$ and PMLRS and for various assumptions concerning the neutrinos. The $B_d\bar{B}_d$ constraints (relevant to case IV only) are for $M_t=50$ GeV. The limits in square brackets for case IV are obtained by omitting the $B_d\bar{B}_d$ mixing constraint. The limits listed for light neutrinos are from SN 1987A or nucleosynthesis only. The intermediate-mass limits also apply where they are more stringent. For light and intermediate-mass Majorana ν_{eR} the stringent restrictions in (55) and (56) from $\beta\beta_{0\nu}$ also apply.

Case	β_g	M_{2g}
Heavy Majorana neutrino ($\Delta m_K+B_d\bar{B}_d+b+\beta\beta_{0\nu}$)		
$U^R_{(\mathrm{I})}$	0.0099	810
$U^R_{(\mathrm{II})}$	0.010	800
$U^R_{(\mathrm{III})}$	0.015	670
$U^R_{(\mathrm{IV})}$	0.012 [0.032]	740 [450] GeV
$U^R_{(LR)}$	0.0036	1.4 TeV
Heavy Dirac neutrino ($\Delta m_K+B_d\bar{B}_d+b$)		
$U^R_{(\mathrm{I})}$	0.075	300
$U^R_{(\mathrm{II})}$	0.032	460
$U^R_{(\mathrm{III})}$	0.015	670
$U^R_{(\mathrm{IV})}$	0.012 [0.032]	740 [450] GeV
$U^R_{(LR)}$	0.0036	1.4 TeV
Intermediate-mass neutrino ($\Delta m_K+B_d\bar{B}_d+\mu$ decay)		
$U^R_{(\mathrm{I})}$	0.027	500
$U^R_{(\mathrm{II})}$	0.027	500
$U^R_{(\mathrm{III})}$	0.021	560
$U^R_{(\mathrm{IV})}$	0.012 [0.038]	740 [420] GeV
$U^R_{(LR)}$	0.0039	1.3 TeV
Light neutrino $m_{\nu_{1R}}<10$ MeV [Supernova 1987A (Ref. 19)]		
$U^R_{(\mathrm{III})}, U^R_{(\mathrm{IV})}$	0.013	720
$U^R_{(\mathrm{I})}, U^R_{(\mathrm{II})}, U^R_{(LR)}$	2.5×10^{-5}	16.2 TeV
Light neutrinos $m_{\nu_{iR}}<1$ MeV [nucleosynthesis (Ref. 18)]		
All		O(1 TeV)

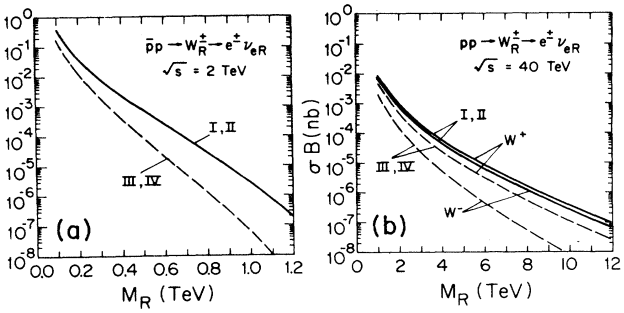

FIG. 4. (a) Production cross section times branching ratio for $\bar{p}p \to W_R^\pm \to e^\pm \nu_{eR}(\overline{\nu_{eR}})$ as a function of M_R at Tevatron energies ($\sqrt{s}=2$ TeV). They are computed as in Ref. 61 with $g_R=g_L$, for forms $U^R_{(\mathrm{I,II},LR)}$ (dominated by $u\bar{d}$ or $\bar{u}d$, solid lines) and $U^R_{(\mathrm{III,IV})}$ (dominated by $u\bar{s}$ or $\bar{u}s$, dashed lines). $\sigma B=10^{-4}$ nb (ten events for an integrated luminosity of 10^{38} cm^{-2}) corresponds to $M_R=680$ or 490 GeV, respectively. (b) Same as (a), for $pp \to W_R^+ \to e^+\nu_{eR}$ or $W_R^- \to e^-\overline{\nu}_{eR}$ at SSC energies ($\sqrt{s}=40$ TeV). $\sigma B=10^{-6}$ nb (ten events for an integrated luminosity of 10^{40} cm^{-2}) corresponds to $M_R=8.6$ or 8.2 TeV for W_R^+, or 7.3 or 5.3 TeV for W_R^-.

$|\zeta_g|<0.0025$. For $\cos\delta=0$, $|\zeta_g|$ could be as large as 0.013.

We also discuss briefly the implications for the right-handed neutrinos. The astrophysical constraints effectively preclude light neutrinos (e.g., Dirac partners of the ordinary neutrinos) unless M_{2g} is very large ($>1-16$ TeV). Neutrinoless double-beta decay eliminates the possibility of Majorana neutrinos in the range 1 MeV–10 GeV unless M_{2g} is *extremely* large, and leads to strong constraints (>670 GeV) on M_{2g} for larger neutrino masses. The weakest limits are for Dirac neutrinos heavier than around 10 MeV.

It should be emphasized that some of the constraints utilized have theoretical uncertainties and somewhat arbitrary assumptions, such as in the magnitude that is considered tolerable for new contributions to Δm_K. Also, we have imposed plausible but nevertheless arbitrary restrictions on possible fine-tuned cancellations between different contributions to Δm_K. Our results therefore cannot be considered as rigorous, but only as a rough guide to which domains of the many-dimensional param-

TABLE III. 90%-C.L. limits on $\zeta_g \equiv g_R\zeta/g_L$ for various forms for U^R. The ζ_g limits are almost independent of the nature of the neutrinos, except for the additional supernova constraint $|\zeta_g|<3\times10^{-5}$ for light neutrinos (Ref. 19). The universality constraint disappears (except for small second-order effects) for $\cos\delta_d=0$ (cases I, II, LR) or for $\cos\delta_s=0$ (cases III, IV). The nonleptonic kaon constraint becomes of second order for $\cos\delta_i=0$.

	$\cos\delta_d=1$
$U^R_{(\mathrm{I})}$	$-0.0024<\zeta_g<0.0008$
$U^R_{(\mathrm{II})}$	$-0.0025<\zeta_g<0.0007$
$U^R_{(\mathrm{III})}$	$-0.0014<\zeta_g<0.0012$
$U^R_{(\mathrm{IV})}$	$-0.0014<\zeta_g<0.0011$
$U^R_{(LR)}$	$-0.0020<\zeta_g<0.0007$
	$\cos\delta_d=0$
$U^R_{(\mathrm{I})}$	$\lvert\zeta_g\rvert<0.013$
$U^R_{(\mathrm{II})}$	$\lvert\zeta_g\rvert<0.013$
$U^R_{(\mathrm{III})}$	$\lvert\zeta_g\rvert<0.0045$
$U^R_{(\mathrm{IV})}$	$\lvert\zeta_g\rvert<0.0045$
$U^R_{(LR)}$	$\lvert\zeta_g\rvert<0.0030$

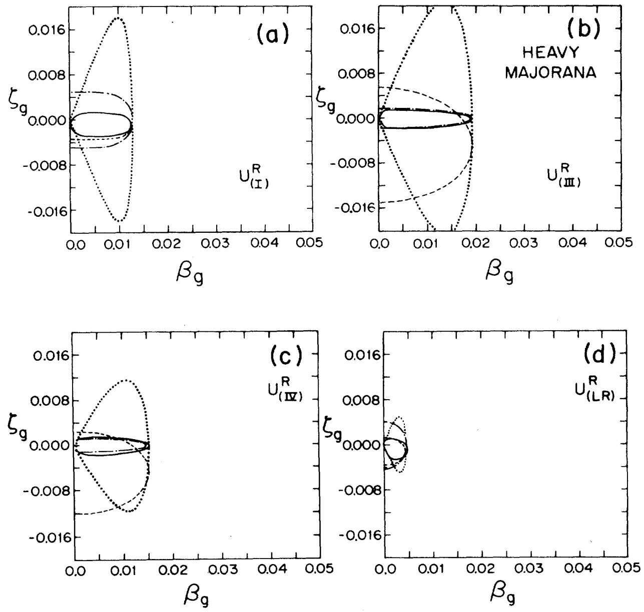

FIG. 5. (a) 90%-C.L. allowed regions for β_g and ζ_g for heavy Majorana neutrinos, assuming form $U^R_{(\mathrm{I})}$. The curves are $|\zeta_g|\le\beta_g$ (dotted), universality (dashed), $K_{\pi3}$ (dotted-dashed), where in each case these are combined with Δm_K, $B_d\bar{B}_d$ oscillations, b decay, and $\beta\beta_{0\nu}$. The solid line is the combined fit to all of these constraints. The contours for $U^R_{(\mathrm{II})}$ are almost identical. (b) Same, for $U^R_{(\mathrm{III})}$. (c) Same, for $U^R_{(\mathrm{IV})}$. (d) Same, for $U^R_{(LR)}$. All of the curves assume $\cos\delta_i=1$, where $i=d$ for (I,II,LR) and $i=s$ for (III,IV).

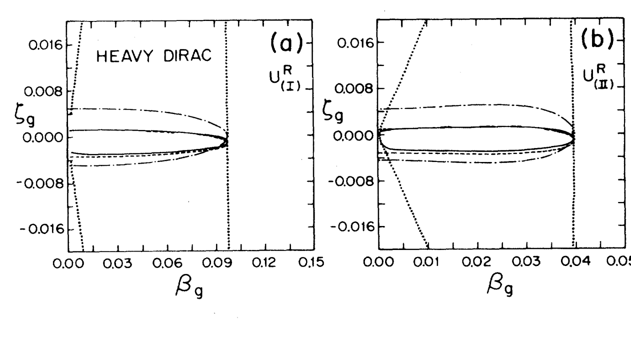

FIG. 6. Same as Fig. 5, only for heavy Dirac neutrinos (no $\beta\beta_{0\nu}$ constraint). (a) For $U^R_{(\mathrm{I})}$ (note the expanded β_g scale). (b) For $U^R_{(\mathrm{II})}$. Forms $U^R_{(\mathrm{III})}$, $U^R_{(\mathrm{IV})}$, and $U^R_{(LR)}$ are almost identical to the corresponding contours for heavy Majorana neutrinos (Fig. 5).

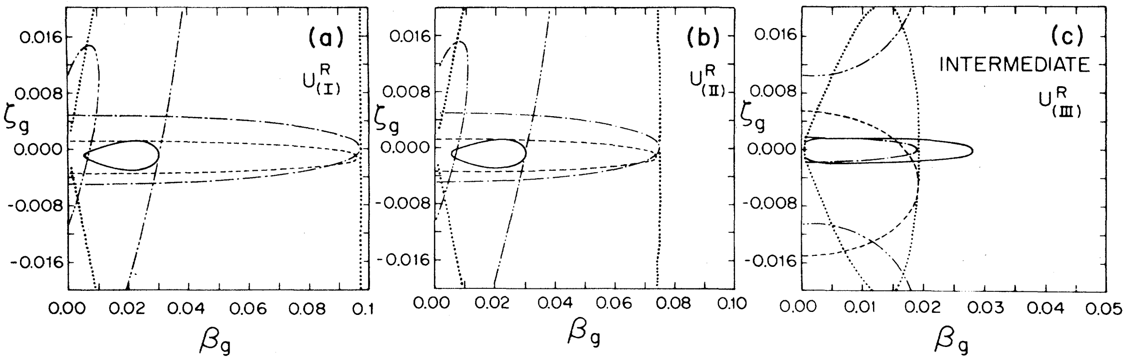

FIG. 7. Same as Fig. 5, only for intermediate-mass neutrinos (no b decay or $\beta\beta_{0\nu}$). The muon decay constraints (combined with Δm_K) are shown as dashed-double-dotted lines. For case III the allowed regions from μ decay, in the upper and lower left corners, are in mild conflict with the other constraints. Similar statements holds for $U^R_{(\mathrm{IV})}$ and $U^R_{(\mathrm{LR})}$. (a) For $U^R_{(\mathrm{I})}$. (b) For $U^R_{(\mathrm{II})}$. (c) For $U^R_{(\mathrm{III})}$. Note the expanded β_g scales for I and II. The cases of $U^R_{(\mathrm{IV})}$ and $U^R_{(LR)}$ are nearly identical to Fig. 5.

eter space are excluded and which are not.

We also comment briefly in Sec. IV on the possibility of an extremely light W_R (e.g., in the range between M_1 and 300 GeV), which is allowed by Δm_K and the other constraints if we relax our prohibitions on fine-tuning. In Sec. V we summarize our conclusions.

II. $SU(2)_L \times SU(2)_R \times U(1)$ MODELS

As described in the Introduction, the left- and right-handed fermions transform as doublets under $SU(2)_L$ and $SU(2)_R$, respectively. Defining the U(1) generator Y by

$$Q=T_{3L}+T_{3R}+\frac{Y}{2}\ , \tag{7}$$

where Q is the electric charge, the quark and lepton (T_L, T_R, Y) assignments are

$$\begin{bmatrix} u' \\ d' \end{bmatrix}_{iL}=(\tfrac{1}{2},0,\tfrac{1}{3}),\quad \begin{bmatrix} u' \\ d' \end{bmatrix}_{iR}=(0,\tfrac{1}{2},\tfrac{1}{3}) \tag{8}$$

and

$$\begin{bmatrix} \nu' \\ l' \end{bmatrix}_{iL}=(\tfrac{1}{2},0,-1),\quad \begin{bmatrix} \nu' \\ l' \end{bmatrix}_{iR}=(0,\tfrac{1}{2},-1)\ , \tag{9}$$

respectively. The primes indicate that the fermions are gauge group rather than mass eigenstates. In (9) the ν_{iR} are the right-handed neutrinos that must be introduced as the $SU(2)_R$ partners of the right-handed charged leptons.

In order to generate masses for the quarks and charged leptons one requires at least one Higgs multiplet Φ of the form

$$\Phi=\begin{bmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{bmatrix}=(\tfrac{1}{2},\tfrac{1}{2}^*,0)\ . \tag{10}$$

The most general form of the vacuum expectation value (VEV) of Φ that is invariant under the electromagnetic U_1^Q is

$$\langle\Phi\rangle=\begin{bmatrix} k & 0 \\ 0 & k' \end{bmatrix}\ , \tag{11}$$

where k and k', in general, are complex. Of course, one can have more than one Φ-type multiplet.[41]

Additional Higgs multiplets with $Y\neq 0$ are needed to break the symmetry down to $U(1)_Q$. Also, one requires an $SU(2)_L$-singlet, $SU(2)_R$-nonsinglet Higgs multiplet with a large VEV in order to generate $M_R \gg M_L$ (assuming that g_R is not much larger than g_L). The simplest choice is to introduce the doublets

$$\delta_L=\begin{bmatrix} \delta_L^+ \\ \delta_L^0 \end{bmatrix}=(\tfrac{1}{2},0,1),\quad \delta_R=\begin{bmatrix} \delta_R^+ \\ \delta_R^0 \end{bmatrix}=(0,\tfrac{1}{2},1)\ . \tag{12}$$

δ_R can generate a large M_R if $v_{\delta R} \gg (k,k',v_{\delta L})$, where $v_{\delta L,R}\equiv\langle\delta^0_{L,R}\rangle$. It can also generate a large Dirac mass for the ν_R if appropriate left-handed partners (distinct from the ordinary ν_L) with $(T_L,T_R,Y)=(0,0,0)$ are introduced into the theory. δ_L is not really required unless one imposes an L-R symmetry on the theory.

A popular alternative is to introduce Higgs triplets instead:[42]

$$\Delta_L=\begin{bmatrix} \Delta_L^{++} \\ \Delta_L^+ \\ \Delta_L^0 \end{bmatrix}=(1,0,2)\ ,\qquad \Delta_R=\begin{bmatrix} \Delta_R^{++} \\ \Delta_R^+ \\ \Delta_R^0 \end{bmatrix}=(0,1,2)\ , \tag{13}$$

with $v_{\Delta L,R}=\langle\Delta^0_{L,R}\rangle$. $v_{\Delta R}$ can produce a large M_R and can also generate a Majorana mass for ν_R. $v_{\Delta L}$, which can generate a Majorana mass for ν_L, must be much smaller than k or k' ($<8.1\%$) because the neutral-current ρ parameter.[7] We will allow both $\delta_{L,R}$ and $\Delta_{L,R}$ type multiplets, as well as arbitrary generalizations.

The gauge-covariant derivatives for the left- and right-handed fermions are given by

$$D^\mu f'_{L,R}=\partial^\mu f'_{L,R}+\frac{i}{2}(g_{L,R}\tau^a A^{\mu a}_{L,R}+g'YB^\mu)f'_{L,R}\ , \qquad (14)$$

where τ^a are the Pauli matrices, g' is the U(1) gauge coupling, and A^a_L, A^a_R, and B are the $SU(2)_L$, $SU(2)_R$, and U(1) gauge bosons, respectively. The covariant derivatives of the Higgs $\delta_{L,R}$ or $\Delta_{L,R}$, are defined in a similar way, with the τ^a being replaced by matrices of appropriate dimension. Similarly,

$$D_\mu\Phi=\partial_\mu\Phi+\frac{i}{2}(g_L\tau^a A^a_{\mu L}\Phi-g_R\Phi\tau^a A^a_{\mu R})\ . \qquad (15)$$

We will also need the Yukawa couplings

$$-L_{\text{Yukawa}}=\sum_i\sum_j[\bar{f}'_{iL}(r_{ij}\Phi+s_{ij}\tilde{\Phi})f'_{jR}+\text{h.c.}]\ , \qquad (16)$$

where $\tilde{\Phi}=\tau^2\Phi^*\tau^2$. (Additional neutrino mass terms may be added, as described above.) From (11) and (16) the quark mass matrices are

$$M^u=rk+sk'^*,\quad M^d=rk'+sk^*\ , \qquad (17)$$

where r and s are the quark Yukawa matrices with elements r_{ij} and s_{ij}, respectively. In general, these matrices are completely arbitrary.

The simplest form of an additional L-R symmetry that can optionally be imposed on the Lagrangian is

$$\begin{aligned}&A^a_L\leftrightarrow A^a_R,\quad B\leftrightarrow B\ ,\\ &f_L\leftrightarrow f_R,\quad \Phi\leftrightarrow\Phi^\dagger\ ,\\ &\delta_L\leftrightarrow\delta_R,\quad \Delta_L\leftrightarrow\Delta_R\ .\end{aligned} \qquad (18)$$

This symmetry would imply $g_L=g_R$ and that the Yukawa matrices r and s are Hermitian. Additional restrictions would apply to the Higgs potential and to the Yukawa couplings involving $\delta_{L,R}$ or $\Delta_{L,R}$. From (17) we see that L-R symmetry alone is not sufficient to make M^u or M^d Hermitian or symmetric. However, if k and k' are real (which is not natural if there are explicit CP-violating phases in r and s), then $M^{u,d}$ are Hermitian (manifest L-R symmetry). Similarly, if r and s are real but k and/or k' are complex (spontaneous CP violation) then $M^{u,d}$ are complex symmetric matrices (pseudomanifest L-R symmetry).

For the Higgs fields described above the charged-boson mass matrix is[43]

$$M_W^2=\begin{bmatrix}\frac{1}{2}g_L^2(|v_L|^2+|k|^2+|k'|^2) & -g_Lg_Rk'k^*\\ -g_Lg_Rk'^*k & \frac{1}{2}g_R^2(|v_R|^2+|k|^2+|k'|^2)\end{bmatrix}=\begin{bmatrix}M_L^2 & M_{LR}^2e^{i\alpha}\\ M_{LR}^2e^{-i\alpha} & M_R^2\end{bmatrix}\ , \qquad (19)$$

where $|v_{L,R}|^2=|v_{\delta L,R}|^2+2|v_{\Delta L,R}|^2$ and α is the phase of $k'k^*$. M_W^2 is a Hermitian matrix. It can be diagonalized by a unitary transformation, which can be written in terms of one angle and three phases. Two of these phases can be absorbed in the definition of the mass eigenstates $W_{1,2}$, so the gauge eigenstates $W_{L,R}$ can be written as

$$\begin{bmatrix}W_L^+\\ W_R^+\end{bmatrix}=\begin{bmatrix}\cos\zeta & -\sin\zeta\\ e^{i\omega}\sin\zeta & e^{i\omega}\cos\zeta\end{bmatrix}\begin{bmatrix}W_1^+\\ W_2^+\end{bmatrix}\ , \qquad (20)$$

where ζ is a mixing angle and ω is a phase. The mass eigenvalues are

$$M_{1,2}^2=\tfrac{1}{2}\{M_L^2+M_R^2\mp[(M_R^2-M_L^2)^2+4|M_{LR}^2|^2]^{1/2}\}\ . \qquad (21)$$

ζ and ω are given by

$$\tan 2\zeta=\frac{\mp 2M_{LR}^2}{M_R^2-M_L^2},\quad e^{i\omega}=\pm e^{i\alpha}\ . \qquad (22)$$

The two signs represent two physically equivalent phase conventions for W_2, as will be discussed below. For $|v_R|^2\gg|k|^2$, $|k'|^2$, $|v_L|^2$, we have

$$\begin{aligned}&M_1^2\simeq\tfrac{1}{2}g_L^2(|v_L|^2+|k|^2+|k'|^2)\ ,\\ &M_2^2\simeq\tfrac{1}{2}g_R^2|v_R|^2\ ,\end{aligned} \qquad (23)$$

and

$$\zeta\simeq\pm\frac{g_L}{g_R}\frac{2|kk'|}{|v_R|^2}\ . \qquad (24)$$

Equations (23) and (24) immediately lead to the inequality

$$|\zeta_g|\le\beta_g\ . \qquad (25)$$

This result continues to hold for multiple representations with the quantum numbers of Φ, $\delta_{L,R}$, and $\Delta_{L,R}$. Further generalizations are discussed in Sec. III.

The gauge and mass eigenstates of the quarks are related to each other by the unitary transformations

$$u'_{L,R}=C_{L,R}u_{L,R},\quad d'_{L,R}=D_{L,R}d_{L,R}\ , \qquad (26)$$

Where the unprimed fields denote mass eigenstates. In terms of these the charged-current interactions are

$$\begin{aligned}-L_{\text{CC}}=&\frac{g_L}{\sqrt{2}}\bar{u}_{iL}\gamma_\mu U^L_{ij}d_{jL}W_L^{\mu+}\\ &+\frac{g_R}{\sqrt{2}}\bar{u}_{iR}\gamma_\mu U^R_{ij}d_{jR}W_R^{\mu+}+\text{H.c.}\ ,\end{aligned} \qquad (27)$$

where

$$U^L = C_L^\dagger D_L , \quad U^R = C_R^\dagger D_R . \tag{28}$$

In general U^R is unrelated to U^L. With appropriate redefinitions of the quark phases the CKM matrix U^L can be parametrized in terms of three angles and one phase. U^R is a function of three angles and six phases all of which are observable. For the special case of manifest L-R symmetry $M^{u,d}$ are Hermitian, so that $U^R = U^L$. For pseudomanifest symmetry, $M^{u,d}$ are symmetric, implying $U^R = U^{L*}K$, where K is a diagonal phase matrix.

Transforming the gauge bosons into mass eigenstates, the charged-current Lagrangian in Eq. (27) can be written as

$$-L_{\rm CC} = \frac{\cos\zeta}{\sqrt{2}} \bar{u}\gamma_u[(g_L U^L \gamma_L + \tan\zeta g_R e^{i\omega} U^R \gamma_R) W_1^{\mu +} + (-\tan\zeta g_L U^L \gamma_L + g_R e^{i\omega} U^R \gamma_R) W_2^{\mu +}] d + \text{H.c.} , \tag{29}$$

where $\gamma_{L,R} \equiv (1 \mp \gamma^5)/2$. (One can, if desired, absorb the phase $e^{i\omega}$ into the quark mixing matrix U^R.) From (22) and (24) it is apparent that the sign of ζ is fixed if one picks a definite convention for the phase of W_2 (assuming $g_R/g_L > 0$). With such a convention one must allow for an arbitrary phase for $e^{i\omega}U_{11}^R$. We will use a more convenient convention in which the phase of W_2 is chosen so that $\text{Re}(e^{i\omega}U_{ij}^R)$, ($ij$=11 or 12, depending on the context), is positive definite. Accordingly, we must allow for an arbitrary sign for ζ.

The leptonic charged-current interaction is analogous to (29), with $u \to \nu$, $d \to e$, and $U^{L,R} \to V^{L,R}$, where $V^{L,R}$ are the leptonic analogues of $U^{L,R}$ (Ref. 17). For massless or very light ν_L (and neglecting small light-heavy mixings), one can choose a basis such that V^L is the identity.

From (29) one has the four-Fermi interaction

$$H = \frac{4\hat{G}_F}{\sqrt{2}} (a J_{L\mu}^\dagger J_L^\mu + b J_{L\mu}^\dagger J_R^\mu + c J_{R\mu}^\dagger J_L^\mu + d J_{R\mu}^\dagger J_R^\mu) , \tag{30}$$

where

$$\frac{\hat{G}_F}{\sqrt{2}} = \frac{g_L^2 \cos^2\zeta}{8M_1^2} , \tag{31}$$

$$J_{L,R\mu}^\dagger = \bar{u}_{L,R}\gamma_\mu U^{L,R} d_{L,R} + \bar{\nu}_{L,R}\gamma_\mu V^{L,R} e_{L,R} , \tag{32}$$

and

$$a = 1 + \beta\tan^2\zeta ,$$
$$b^* = c = e^{i\omega}\frac{g_R}{g_L}\tan\zeta(1-\beta) , \tag{33}$$
$$d = \frac{g_R^2}{g_L^2}(\tan^2\zeta + \beta) ,$$

where $\beta \equiv M_1^2/M_2^2$.

III. CONSTRAINTS

In this section we describe the constraints on the W_2 mass and W_L-W_R mixing, which turn out to be largely decoupled. We first consider the mass constraints on M_{2g}, then the mixing constraints on ζ_g, and finally some mixed constraints relevant to light and intermediate-mass neutrinos.

A. Bounds on $M_{2g} \equiv g_L M_2/g_R$

1. The K_L-K_S mass difference

The most important single constraint on M_2 is from the K_L-K_S mass difference Δm_K. Since this is a purely hadronic quantity, it is independent of the right-handed neutrino masses. Δm_K is measured to be

$$\Delta m_K|_{\text{expt}} = 0.35 \times 10^{-14} \text{ GeV} . \tag{34}$$

Theoretically the short-distance part of Δm_K is given by

$$\Delta m_K = 2\langle K^0 | H_{\Delta S=2}^{\text{eff}} | \bar{K}^0 \rangle , \tag{35}$$

where $H_{\Delta S=2}^{\text{eff}}$ is the effective Hamiltonian for $|\Delta S| = 2$ transitions. In the standard model this is calculated from the box diagram shown in Fig. 2. In the approximation of neglecting m_u compared to m_c one has the well-known result[44]

$$H_{\text{SM}}^{\text{eff}} = \eta^{LL} \frac{G_F^2 m_c^2}{4\pi^2} \sin^2\theta_c \bar{d}\gamma_\mu\gamma_L s \bar{d}\gamma^\mu\gamma_L s + \text{H.c.} , \tag{36}$$

where η^{LL} is the short distance QCD correction factor, which is close to unity.[45] The vacuum-saturation estimate

$$\langle K^0 | \bar{d}\gamma_\mu\gamma_L s \bar{d}\gamma^\mu\gamma_L s | \bar{K}^0 \rangle = \tfrac{1}{3} f_K^2 m_K \tag{37}$$

yields a value of Δm_K that is close to the experimental value. Of course, vacuum saturation may not be reliable,[9] and there may also be important long-distance contributions to Δm_K that have not been taken into account.[46] Nevertheless, it seems reasonable to require that any new contributions to Δm_K should not be larger than the experimental value, or to the standard-model prediction using (37).

In the $SU(2)_L \times SU(2)_R \times U(1)$ model the box diagrams in Fig. 2 contribute to $\Delta S = 2$ transitions. We ignore W_L-W_R mixing for Δm_K. Terms proportional to $\tan\zeta$ are also proportional to the external momenta and hence can be neglected, while higher-order terms are negligible. The W_R-W_R diagram is suppressed by β_g^2 with respect to the standard model and can be neglected. However, Beall, Bander, and Soni pointed out[24] that the two diagrams with one W_L and one W_R can be important because of enhanced matrix elements, despite a factor of β_g.

Neglecting mixing, the W_L-W_R diagrams yield an effective Hamiltonian[47]

$$H_{LR}^{\rm eff}=\frac{2G_F^2}{\pi^2}\beta_g \bar{d}\gamma_L s \bar{d}\gamma_R s \sum_{i=u,c,t} m_i U_{id}^{L*} U_{is}^{R} \sum_{j=u,c,t} m_j U_{jd}^{R*} U_{js}^{L} \eta_{ij}^{LR} J_0(x_i,x_j,\beta)+\text{H.c.} , \quad (38)$$

where $x_i=m_i^2/M_1^2$, $\beta=M_1^2/M_2^2$, $\beta_g=g_R^2\beta/g_L^2$, and the η's are short-distance QCD corrections.[25] The function J_0 is

$$J_0(x_i,x_j,\beta)=\left[\frac{x_i\ln x_i}{(x_j-x_i)(1-x_i)(1-\beta x_i)}+\frac{x_j\ln x_j}{(x_i-x_j)(1-x_j)(1-\beta x_j)}+\frac{\beta\ln\beta}{(1-\beta)(1-\beta x_i)(1-\beta x_j)}\right] . \quad (39)$$

In the vacuum-saturation approximation

$$\langle K^0|\bar{d}\gamma_L s\bar{d}\gamma_R s|\bar{K}^0\rangle=\left[\left[\frac{m_K}{m_s+m_d}\right]^2+\frac{1}{6}\right]\tfrac{1}{4}f_K^2 m_K , \quad (40)$$

where $[m_K/(m_s+m_d)]^2\simeq 6$ for $m_s\simeq 200$ MeV (Ref. 48). Hence, the ratio R of new to standard-model contributions to Δm_K is

$$R=8\times\frac{3}{4}\left[\left[\frac{m_K}{m_s+m_d}\right]^2+\frac{1}{6}\right]\beta_g \times\sum_{i,j}\frac{\eta_{ij}^{LR}m_i m_j U_{id}^{L*}U_{is}^{R}U_{jd}^{R*}U_{js}^{L}J_0(x_i,x_j,\beta)}{\sin^2\theta_c m_c^2} . \quad (41)$$

Thus, the W_L-W_R diagrams are enhanced by matrix-element and various numerical factors. Also, the $i=j=c$ term has a large $\eta_{cc}^{LR}J_0$ factor of order 10. For the case of PMLRS the cc term dominates, and the requirement $|R|<1$ implies the stringent limit $\beta<0.0021$ ($M_R>1.8$ TeV) (Ref. 49).

For a general U^R the terms in R involving t, and to a much lesser extent u, exchange can be important. One complication is that there could be cancellations between different terms in R. (Recall that U^R involves six unknown phases.) However, to obtain $|R|<1$ from a cancellation of much larger terms would require extreme fine-tuning. Our interest is in allowed regions of parameter space that do not require extreme fine-tuning (except for a brief discussion in the next section), so we require that each individual contribution to Δm_K is smaller than the standard model:[50]

$$|R_{ij}|=6\left[\left[\frac{m_K}{m_s+m_d}\right]^2+\frac{1}{6}\right]\beta_g\frac{|\eta_{ij}^{LR}m_i m_j U_{id}^{L*}U_{is}^{R}U_{jd}^{R*}J_{js}^{L}J_0(x_i,x_j,\beta)|}{\sin^2\theta_c m_c^2}\leq 1 . \quad (42)$$

The constraints in (42) are difficult to apply directly because they depend on many variables. In principle they depend on both the explicit factor of β_g and separately on β (in J_0). However, as long as g_R/g_L is not drastically different from unity (which we assume), the β dependence is actually rather weak for all values of $M_2>100$ GeV. Since we are interested in lower bounds on M_R we only keep the explicit β_g factor and evaluate J_0 using $M_2=100$ GeV. Similarly, the dependence on m_t is very weak (the dependence of J_0 on m_t compensates the explicit factors very well) for the allowed range of $\sim$40–200 GeV (Ref. 7), with the coefficients in (42) changing by at most 30%. Hence, we will fix $m_t=50$ GeV ($m_t=200$ GeV would allow values for M_{2g} at most 15% smaller). Finally, we use the values for the ordinary CKM matrix elements determined in standard-model analyses (Ref. 14); i.e., we assume that the new interactions in $SU(2)_L\times SU(2)_R\times U(1)$ are perturbations that do not significantly affect the determinations of U^L. The only questionable process is b decay, which is strongly suppressed in the standard model and is therefore sensitive to perturbations. However, we will argue below that the b semileptonic spectrum and branching ratio indicate that the decays are mediated by W_L rather than W_R, so the use of canonical U^L is justified.

Another complication involves the value of m_u. The small current-quark mass $m_u^{\rm cur}\sim 5.6$ MeV is suitable at short distances (Ref. 48), while a constituent mass $m_u^{\rm con}\sim 300$ MeV is appropriate at long distances. The latter should be interpreted as setting the scale for a running mass which falls as $1/Q^2$ (up to logarithms) at large Q^2. Since both short and long distances are relevant, we will use an effective running mass

$$m_u(Q^2)=m_u^{\rm cur}+m_u^{\rm con}\frac{\lambda^2}{\lambda^2+Q^2} , \quad (43)$$

where $\lambda\sim 1$ GeV is the strong-interaction scale. Of course, the use of a running mass modifies the analytic expression for $H_{LR}^{\rm eff}$. We do not display the explicit expression, but incorporate it in our numerical results. The form in (43) and other long-distance corrections to the u-exchange diagrams are considerably more uncertain than the short-distance diagrams involving c and t only. We have therefore done fits in which $m_u^{\rm cur}$ is used instead or in which the u-exchange constraints are omitted. Fortunately, the u diagrams only affect one of the regions of U^R in Table I significantly, and hardly affect the overall limits on β_g.

For the assumptions described above, (42) yields nine constraints of the form

$$\beta_g|U_{is}^R||U_{jd}^R|<a_{ij} , \quad (44)$$

where the a_{ij} are listed in Table IV. Clearly, the four constraints derived from c and t exchange are very much more stringent than the others. These four can be evaded entirely if U^R takes one of the special forms in (3), and by far the weakest limits on β_g are obtained from U^R in the vicinity of one of these. (We are assuming three-family unitarity. If there were more families with significant mixing with the first three there would be more solutions allowing a small M_2.) In particular, we concentrate on the four special cases $U^R_{(\mathrm{I})}-U^R_{(\mathrm{IV})}$ in Table I. Other values for α in (3) interpolate smoothly between cases I and II or III and IV.

However, these special forms are themselves highly fine-tuned. Any small change in the zero elements can lead to a significant contribution to Δm_K. We therefore impose a second restriction on fine-tuning, viz., that the constraints in (44) should continue to hold as each U^R_{ij} is varied by ϵ from its central value in Table I. This implies the new set of constraints

$$\beta_g |U^R_{ij}| < b_{ij}/\epsilon \ . \tag{45}$$

The values of the b_{ij} are listed in Table IV. We consider this restriction to be quite reasonable, but of course the value chosen for ϵ is arbitrary. Based on our experience with U^L we choose $\epsilon=0.01$. Because of this and other assumptions the limits cannot be considered rigorous—we are only trying to map out the most likely region in a multidimensional parameter space.

The upper limits on β_g and corresponding lower limits on M_{2g} obtained from Δm_K are listed in Table V for forms $U^R_{(\mathrm{I})}-U^R_{(\mathrm{IV})}$ and for the case of PMLRS. It is seen that even with our fine-tuning assumptions a much lighter M_2 is allowed than in the case of PMLRS. Altogether, the combined assumptions of Δm_K, approximate three-family unitarity, and no fine-tuning imply

$$\beta_g \le 0.075, \quad M_{2g} \ge 300 \text{ GeV} \ . \tag{46}$$

This would only be slightly weakened if the u-exchange diagrams were omitted.

2. $B_d\bar{B}_d$ mixing

The W_L-W_R box diagram does not have an important effect of $B_d\bar{B}_d$ mixing for PMLRS (Ref. 27). However, the tc diagram can be important for U^R near $U^R_{(\mathrm{IV})}$. (The tt diagram is smaller for allowed m_t.) From formulas analogous to (42) and theoretical assumptions as in Ref. 27 we obtain[50]

$$|U^R_{td}||U^R_{cb}|\beta_g < 6.7\times 10^{-3}\left[\frac{50\text{ GeV}}{m_t}\right] , \tag{47}$$

where we have required that the new contributions be no larger than the experimental mixing.[51] For $U^R_{(\mathrm{IV})}$ (47) implies $\beta_g<0.012$ (50 GeV$/m_t$) [$M_{2g}>740$ GeV $(m_t/50\text{ GeV})^{1/2}$] at 90% C.L. There are no significant constraints from $B_s\bar{B}_s$ mixing (predicted to be near maximal in the standard model) or $D\bar{D}$ mixing.

TABLE IV. Constraints from Δm_K from Eqs. (44) and (45). We choose $\epsilon=0.01$ in the numerical fits. The upper limits are interpreted as 1σ errors.

Quantity	Limit	Quantity	Limit
$\beta_g\|U^R_{us}\|\|U^R_{ud}\|$	<0.018	$\beta_g\|U^R_{ud}\|$	$<0.018/\epsilon$
$\beta_g\|U^R_{us}\|\|U^R_{cd}\|$	<0.0091	$\beta_g\|U^R_{cd}\|$	$<0.00047/\epsilon$
$\beta_g\|U^R_{us}\|\|U^R_{td}\|$	<0.10	$\beta_g\|U^R_{td}\|$	$<0.00035/\epsilon$
$\beta_g\|U^R_{cs}\|\|U^R_{ud}\|$	<0.18	$\beta_g\|U^R_{us}\|$	$<0.0091/\epsilon$
$\beta_g\|U^R_{cs}\|\|U^R_{cd}\|$	<0.00047	$\beta_g\|U^R_{cs}\|$	$<0.00047/\epsilon$
$\beta_g\|U^R_{cs}\|\|U^R_{td}\|$	<0.0010	$\beta_g\|U^R_{ts}\|$	$<0.00035/\epsilon$
$\beta_g\|U^R_{ts}\|\|U^R_{ud}\|$	<0.98		
$\beta_g\|U^R_{ts}\|\|U^R_{cd}\|$	<0.00049		
$\beta_g\|U^R_{ts}\|\|U^R_{td}\|$	<0.00035		

TABLE V. 90%-C.L. upper limits on β_g and lower limits on M_{2g} (in GeV) obtained from Δm_K for forms $U^R_{(\mathrm{I})}$-$U^R_{(\mathrm{IV})}$ and for PMLRS. The second and third columns assume m_u from (43). The fourth column displays the limits on β_g if m_u^{cur} is used instead, while the last column lists the β_g limits if the u-exchange diagrams are omitted. Dropping the u-exchange diagrams has little effect except for forms $U^R_{(\mathrm{III})}$. The overall limit ($M_{2g}>290$ GeV) is hardly changed.

Case	β_g $[m_u(Q^2)]$	M_{2g} $[m_u(Q^2)]$	β_g $[m_u^{\mathrm{cur}}]$	β_g [no u]
$U^R_{(\mathrm{I})}$	0.075	300	0.077	0.077
$U^R_{(\mathrm{II})}$	0.057	340	0.057	0.057
$U^R_{(\mathrm{III})}$	0.015	670	0.048	0.071
$U^R_{(\mathrm{IV})}$	0.054	350	0.054	0.057
$U^R_{(\mathrm{LR})}$	0.0036	1350	0.0036	0.0036

3. b decays

If the right-handed neutrinos are sufficiently heavy (either Dirac or Majorana) then the right-handed currents cannot contribute to normal leptonic and semileptonic decays. In particular, for $m_{\nu_R}>m_b-m_c\simeq 3.5$ GeV (since it is known that $b\to c$ predominates over $b\to u$) W_R can contribute only to nonleptonic b decays (ignoring mixing). Since the special forms $U^R_{(\mathrm{II})}$ and $U^R_{(\mathrm{IV})}$ have $U^R_{cb}=1$, a light W_R could significantly affect the b semileptonic branching ratio. The experimental branching ratio[52]

$$B(b\to e\nu_e X)|_{\mathrm{expt}}=11.5\pm 0.5\% \tag{48}$$

is consistent with the standard-model prediction[53]

$$B(b\to e\nu_e X)|_{\mathrm{SM}}=13.3\pm 1.6\% \ . \tag{49}$$

This constrains any new contribution $\Gamma^{\mathrm{new}}_{\mathrm{NL}}$ to the nonleptonic width, viz.,

$$\begin{aligned}\frac{\Gamma^{\mathrm{new}}_{\mathrm{NL}}}{\Gamma^0} &= \left[1-\frac{B(b\to e\nu_e X)|_{\mathrm{expt}}}{B(b\to e\nu_e X)|_{\mathrm{SM}}}\right]\frac{\hbar}{\Gamma^0\tau_b^{\mathrm{expt}}}\\ &=(0.13\pm 0.11)(7.8\pm 1.8)\times 10^{-3}\\ &=(1.05\pm 0.90)\times 10^{-3}<0.27(7.8\pm 1.8)\times 10^{-3} ,\end{aligned} \tag{50}$$

where $\Gamma^0\equiv G_F^2 m_b^5/192\pi^3$. In the $SU(2)_L\times SU(2)_R\times U(1)$ model with a heavy neutrino one has

$$\frac{\Gamma_{\rm NL}^{\rm new}}{\Gamma^0}=\eta^{\rm NL}\sum_i\sum_j\left[|U_{cb}^R|^2|U_{ij}^R|^2\beta_g^2 f(m_c,m_j,m_i)-2\beta_g|U_{cb}^L||U_{ij}^L||U_{cb}^R||U_{ij}^R|h(m_c,m_j,m_i)+(c\to u)\right], \tag{51}$$

where $\eta^{\rm NL}\sim 3.5$ is a QCD correction and $f(m_i,m_j,m_k)$ and $h(m_i,m_j,m_k)$ are three-body phase-space factors.[54] The signs of the L-R interference terms are unknown, so we have chosen -1 to give the weakest constraints. (In fact, the interference terms are unimportant in the parameter regions of interest.) All the terms are proportional to U_{ub}^R or U_{cb}^R. Therefore the constraint in (50) is trivially satisfied by forms $U_{\rm (I)}^R$ and $U_{\rm (III)}^R$. For forms $U_{\rm (II)}^R$ and $U_{\rm (IV)}^R$, for which $U_{cb}^R\simeq 1$, the limits from Δm_K are strengthened to

$$\beta_g\le 0.032 \ . \tag{52}$$

This only applies for right-handed neutrinos heavier than m_b-m_c. There are constraints from muon decay that are comparable to (52) for neutrinos light enough to be produced (we assume for simplicity that the right-handed neutrinos are degenerate; the remarks can easily be generalized to the nondegenerate case). For the intermediate range between $m_\mu/2$ and m_b-m_c one still has constraints from the total b decay rate. Since both W_L and W_R contribute to the decay one has

$$(|\beta_g|^2+|U_{bc}^L|^2)^{1/2}\equiv|\tilde U_{bc}^L|\simeq 0.043 \ , \tag{53}$$

for forms $U_{\rm (II)}^R$ and $U_{\rm (IV)}^R$, where $\tilde U_{bc}^L$ is the apparent value of U_{bc}^L extracted from the data assuming the validity of the standard model. This can be strengthened, however, because the semileptonic b decay spectrum clearly indicates that the decay is dominated by $V-A$ rather than $V+A$ (Ref. 55). It is hard to quantify this argument, but it is reasonable to assume $\beta_g<|U_{bc}^L|$, implying $\beta_g<\tilde U_{bc}^L/\sqrt{2}\simeq 0.030$. We will therefore reinterpret (52) as a constraint from all aspects of b decay, applying for forms $U_{\rm (II)}^R$ and $U_{\rm (IV)}^R$ down to small neutrino masses.

4. *Neutrinoless double-beta decay* ($\beta\beta_{0\nu}$)

The current limit $\tau_{1/2}>10^{24}$ yr for $^{76}{\rm Ge}\to{}^{76}{\rm Se}\,e^-e^-$ implies (Ref. 31) an upper limit of around 1 eV on the ordinary ν_{eL} mass if it is Majorana. Mohapatra emphasized (Ref. 30) that the W_R-W_R diagram in Fig. 3 can also give a significant contribution to $\beta\beta_{0\nu}$ for PMLRS and heavy Majorana ν_R. Generalizing his results one obtains

$$\beta_g^2|U_{ud}^R|^2 m_R F(A,m_R)<1\ {\rm eV}(10^{24}\ {\rm yr}/\tau_{1/2})^{1/2} \ , \tag{54}$$

where m_R is the ν_{eR} mass (assuming it is Majorana). $F(A,m_R)\equiv\langle e^{-m_R r}/r\rangle/\langle 1/r\rangle$ is a nucleus-dependent propagator factor; it is ~ 1 for $m_R\ll 40$ MeV, while $F\simeq m_0^2/m_R^2$ for $m_R\gg 40$ MeV, where (Ref. 31) $m_0\sim 95$ MeV for ^{76}Ge. Hence, for a light Majorana ν_{eR} one has

$$\beta_g|U_{ud}^R|<\left[\frac{1\ {\rm eV}}{m_R}\right]^{1/2}(10^{24}\ {\rm yr}/\tau_{1/2})^{1/4} \ , \tag{55}$$

while for a heavy Majorana neutrino,

$$\beta_g|U_{ud}^R|<3.3\times 10^{-4}\left[\frac{m_R}{1\ {\rm GeV}}\right]^{1/2}(10^{24}\ {\rm yr}/\tau_{1/2})^{1/4} \ . \tag{56}$$

(55) and (56) together imply that the W_R is probably too heavy to observe directly if $|U_{ud}^R|\sim 1$ and m_R is in the 1 MeV–10 GeV range.

For heavy Majorana neutrinos it is useful to rewrite (56) as

$$\beta_g^{5/2}|U_{ud}^R|^2<8.9\times 10^{-6}\left[\frac{g_R m_R}{g_L M_R}\right](10^{24}\ {\rm yr}/\tau_{1/2})^{1/2} \ . \tag{57}$$

Mohapatra has also argued (Ref. 28) that vacuum stability requires $m_R<M_R$ (independent of U^R). Hence, the weakest limit for heavy Majorana neutrinos occurs for $m_R\sim M_R$ (we also take $g_R\sim g_L$ on the right-hand side) (Ref. 50). Equation (57) implies the considerably strengthened bound $\beta_g<0.01$ for cases I or II if the ν_R is Majorana.

B. Bounds on the mixing angle $\zeta_g\equiv g_R\zeta/g_L$

1. *Theoretical bound*

The same Higgs fields that lead to W_L-W_R mixing also contribute to M_L^2. As we will now show this leads to a generalization of (25) for arbitrary Higgs representations. The charged-boson mass matrix can be parametrized as in (19), with

$$\begin{aligned}
M_L^2&=g_L^2\sum[t_L(t_L+1)-t_{L3}^2]|v(t_L,t_{L3},t_R,t_{R3})|^2 \ ,\\
M_R^2&=g_R^2\sum[t_R(t_R+1)-t_{R3}^2]|v(t_L,t_{L3},t_R,t_{R3})|^2 \ ,\\
M_{LR}^2e^{-\alpha}&=g_Lg_R\sum c^+(t_R,t_{R3})c^-(t_L,t_{L3})\\
&\quad\times v^*(t_L,t_{L3}-1,t_R,t_{R3}+1)\\
&\quad\times v(t_L,t_{L3},t_R,t_{R3}) \ ,
\end{aligned} \tag{58}$$

where the sum extends over the neutral Higgs fields with quantum numbers (t_L,t_{L3},t_R,t_{R3}) and VEV v, and

$$c^{\pm}(t,t_3)=[(t\mp t_3)(t\pm t_3+1)]^{1/2} \ . \tag{59}$$

The assumption $M_2\gg M_1$ implies that either (a) $M_L\sim M_R\sim|M_{LR}|$, with $|\zeta|\sim 45°$, or (b)

$$M_R\gg M_L,M_{LR} \ . \tag{60}$$

Rejecting case (a) as unphysical, (60) implies

$$M_1^2\simeq M_L^2,\quad M_2^2\simeq M_R^2,\quad |\zeta|\simeq\frac{M_{LR}^2}{M_R^2} \ . \tag{61}$$

Hence,

$$|\zeta_g|\simeq\frac{g_L M_{LR}^2}{g_R M_L^2}\beta_g \tag{62}$$

Applying the Schwarz inequality and simple manipulations to (58) this implies

$$|\zeta_g| \leq C\beta_g \ , \tag{63}$$

where

$$C \equiv \tfrac{1}{2} \max_{t_L \neq 0} \left| \frac{c^+(t_R,t_{R3})c^-(t_L,t_{L3}) + c^-(t_R,t_{R3})c^+(t_L,t_{L3})}{t_L(t_L+1) - t_{L3}^2} \right| . \tag{64}$$

In (64) the maximum is with respect to all Higgs representations with $t_L \geq \frac{1}{2}$ which have significant VEV's. For the Higgs representations in Sec. II we have $C=1$, recovering (25). In general, C will be order 1 except for ridiculous representations with $t_R \gg 1$. In the likely case that only Higgs with $t_L = \frac{1}{2}$ are relevant (as is supported by the weak neutral current data[7]), then

$$C = \max_{t_L = 1/2} c^{\pm}(t_R, t_{R3}) \ . \tag{65}$$

We will always take $C=1$.

The constraints from universality and kaon decay are much more important than (63) for small M_{2g} and the special forms in Table I (unless $\cos\delta_{d,s} \simeq 0$). Equation (63) is mainly important for precluding the possibility of a large mixing when M_{2g} is in the several TeV range, for which Δm_K is unimportant and U^R is completely arbitrary.

2. *Universality*

Wolfenstein pointed out (Ref. 33) that W_L-W_R mixing could modify the hadronic vector currents in beta and kaon decay, leading to an apparent violation of universality. If the right-handed neutrinos are heavy then from (30) the effective Hamiltonian for semileptonic decay is

$$H^{\text{eff}} = \frac{4\hat{G}_F}{\sqrt{2}} a(\bar{l}_L \gamma_\mu \nu_L) \left[\bar{u}\gamma^\mu \left[\gamma_L U^L + \frac{c}{a}\gamma_R U^R \right] d \right] + \text{H.c.} \ , \tag{66}$$

where a and c are defined in (33). The apparent (measured) values of the CKM matrix elements[14]

$$\begin{aligned} \tilde{U}^L_{ud} &= 0.9744 \pm 0.0010 \ , \\ \tilde{U}^L_{us} &= 0.220 \pm 0.002 \ , \end{aligned} \tag{67}$$

are obtained by dividing the observed coefficients of the hadronic vector currents in beta and kaon decay by the experimental value $G_F = 1.166\,37 \times 10^{-5}\ \text{GeV}^{-2}$ obtained from the muon decay rate. Since only the a term in (33) is relevant to muon decay, we must identify

$$G_F = \hat{G}_F a \ . \tag{68}$$

Combining (66) and (68) we have the relation

$$\tilde{U}^L_{ij} = U^L_{ij} \left| 1 + \frac{cU^R_{ij}}{aU^L_{ij}} \right| \tag{69}$$

between the apparent and true values of U^L_{ij}. Combining the experimental values in (67) with the limit $|\tilde{U}^L_{ub}| \leq 0.012$ (Ref. 52) and three-family unitarity ($\sum_i |U^L_{ui}|^2 = 1$), one obtains

$$\begin{aligned} \sum_i |\tilde{U}^L_{ui}|^2 &= 1 + 2\zeta_g \sum_i U^L_{ui} \text{Re}(e^{i\omega} U^R_{ui}) \\ &= 0.9979 \pm 0.0021 \ , \end{aligned} \tag{70}$$

where we have expanded to lowest order in ζ_g and β_g. Clearly, there is little room for mixing-induced nonunitarity in $\tilde{U}^L$. Neglecting U^L_{ub} this implies

$$\begin{aligned} \zeta_g(\tilde{U}^L_{ud}|U^R_{ud}|\cos\delta_d + \tilde{U}^L_{us}|U^R_{us}|\cos\delta_s) \\ = -0.001\,05 \pm 0.001\,05 \ , \end{aligned} \tag{71}$$

where

$$\cos\delta_i \equiv \frac{\text{Re}(e^{i\omega}U^R_{ui})}{|U^R_{ui}|} \ . \tag{72}$$

Equation (71) is a generalization and updating of the constraint originally derived by Wolfenstein[33] for the case of PMLRS.

Equation (71) is very stringent for small *CP*-violating phases $\cos(\delta_{d,s} \sim 1)$. For maximal phases ($\cos\delta_{d,s} \sim 0$) only the second-order $|c|^2/|a|^2$ term in $|\tilde{U}|^2$ survives, implying $|\zeta_g|^2 = -0.0021 \pm 0.0021$, or

$$|\zeta_g| < 0.049 \quad (\cos\delta = 0, \text{ heavy } \nu_R) \tag{73}$$

at 90% C.L., independent of U^R.

Equations (71) and (73) were derived for heavy neutrinos. However, (71) actually holds for very light or massless neutrinos as well. In this case (68) and (69) become

$$\begin{aligned} G_F &= \hat{G}_F \lambda \ , \\ \tilde{U}^L_{ij} &= (|aU^L_{ij} + cU^R_{ij}|^2 + |bU^L_{ij} + dU^R_{ij}|^2)^{1/2}/\lambda \ , \end{aligned} \tag{74}$$

where $\lambda \equiv (|a|^2 + |b|^2 + |c|^2 + |d|^2)^{1/2}$. The extra terms in (74) are due to the emission of right-handed neutrinos. Expanding to lowest order in β_g and ζ_g one recovers (71). There is no second-order constraint analogous to (73) for massless neutrinos, because the $|a|^2 + |b|^2 + |c|^2 + |d|^2$ terms in the numerator and denominator of $\sum_i |\tilde{U}^L_{ui}|^2$ cancel.

The linear constraint (71) also holds for intermediate mass or nondegenerate neutrinos. The appropriate quadratic constraint analogous to (73) can easily be derived for each case.

3. *Nonleptonic kaon decays*

Donoghue and Holstein[34] have argued that the standard-model PCAC relations between the nonleptonic amplitudes for $K \to 3\pi$ and $K \to 2\pi$ are extremely sensitive to small admixtures of right-handed currents, because the latter lead to new operators that are enhanced by the $\Delta I = \frac{1}{2}$ rule. They further argue that the $K_{\pi 3}$ predictions are successful to 10% in amplitude. Generalizing their results and forbidding fine-tuned cancellations, this implies[50]

$$\begin{aligned} |U^L_{ud}||U^R_{us}||b|\cos\delta_s &< 8 \times 10^{-4} \ , \\ |U^L_{us}||U^R_{ud}||b|\cos\delta_d &< 5 \times 10^{-4} \ , \end{aligned} \tag{75}$$

where $|b| \simeq |\zeta_g|$ is defined in (33). For $\cos\delta_{d,s}=0$ one can still derive a nontrivial second-order constraint[50]

$$\begin{aligned} &|U^L_{ud}|^2|U^R_{us}|^2|b|^2<1\times10^{-5}\,, \\ &|U^L_{us}|^2|U^R_{ud}|^2|b|^2<5\times10^{-6}\,. \end{aligned} \tag{76}$$

Donoghue and Holstein also obtain a constraint on $|d/a|$ which implies $\beta_g|U^R_{us}||U^R_{ud}|<0.01$. This is comparable to the constraint from Δm_K and is not useful for our purposes. These limits are independent of the neutrino properties.

4. *Shift in M_1*

W_L-W_R mixing shifts the mass M_1 of W_1 below the standard-model prediction[7]

$$M_L=\frac{A_0}{\sin\theta_W(1-\Delta r)^{1/2}}\,, \tag{77}$$

where $A_0=(\pi\alpha/\sqrt{2}G_F)^{1/2}=37.281$ GeV and Δr is a radiative correction predicted to be 0.0713 ± 0.0013 for $m_t=45$ GeV, $M_H=100$ GeV. One has[35]

$$\tan^2\zeta=\frac{M_L^2-M_1^2}{M_2^2-M_L^2}\,, \tag{78}$$

so that a large ζ would shift M_1 significantly below M_L. One complication is that the value of $\sin^2\theta_W$ extracted from neutral-current data (needed to compute M_L) could be modified by the presence of the second Z' boson in the $SU(2)_L\times SU(2)_R\times U(1)$ model. We have therefore performed a fit to the neutral-current and boson mass data[56] in which ζ, $\sin^2\theta_W$, $M_{Z'}$, and θ (the Z-Z' mixing angle) were all free parameters, for various fixed values of M_2. The limits on ζ were almost uncorrelated with the other parameters, and are presented in Table VI along with limits from the $|\zeta_g|\le\beta_g$ constraint. The present limits are less stringent than those from universality and $K_{\pi3}$ decay, even for $\cos\delta=0$. However, expected high-precision measurements of M_Z, M_W, various e^+e^- asymmetries, etc., will allow very stringent limits on ζ from (78) in the next few years.

C. Mixed constraints for light and intermediate mass neutrinos

1. *Muon decay*

From precise measurements of the positron asymmetry at the end point the TRIUMF group has obtained[20]

$$\xi P_\mu\delta/\rho=0.997\,90\pm0.000\,88\,, \tag{79}$$

which is 2.4σ away from the standard-model prediction of unity. Herczeg[21] has analyzed this result within general $SU(2)_L\times SU(2)_R\times U(1)$ models. Assuming that the right-handed neutrinos are light enough to be produced without kinematic suppression, he obtains[23]

$$\begin{aligned} R &\equiv 1-\xi P_\mu\delta/\rho \\ &=2\beta_g^2\left[1+\frac{|U^R_{ud}|^2}{|U^L_{ud}|^2}\right]+2\zeta_g^2-4\beta_g\zeta_g\frac{|U^R_{ud}|}{|U^L_{ud}|}\cos\delta_d \\ &=0.0021\pm0.000\,88\,, \end{aligned} \tag{80}$$

where $\cos\delta_d$ is defined in (72), and the U^R dependence enters through the π^+ decay which produced the polarized muons.

The regions in β_g and ζ_g that are allowed at 90% C.L. by (80) are shown in Fig. 1. The limits on β_g alone are given in Table VII, and the limits obtained by combining muon decay with Δm_K and $B_d\bar{B}_d$ mixing are given in Table II.

In Fig. 1 it is apparent that the standard model ($\beta_g=0$, $\zeta_g=0$) is not within the 90%-C.L. allowed region. This is of course a reflection of the fact that R is 2.4σ away from zero. We are taking the conservative attitude of using (80) to give an upper limit on new physics rather than taking it as positive evidence for deviations. However, if one does want to take the apparent derivation seriously the best fit values for the various cases are given in Table VIII. For forms $U^R_{(\mathrm{III})}$ and $U^R_{(\mathrm{LR})}$ the best fit values for β_g are excluded by Δm_K, while the value for case IV conflicts with $B_d\bar{B}_d$ mixing. In the other cases the best fit values are allowed by other data, at least for ζ_g near zero.

TABLE VI. 90%-C.L. upper limits on $|\zeta|$ from the shift in M_1 compared to the standard-model prediction, Eq. (78), for various fixed values of M_2 and β. The results are for $m_t=45$ GeV. Larger m_t would yield more stringent limits. Also shown are the 90%-C.L. limits on $|\zeta|$ from the 1σ constraint $|\zeta|<\beta$ (for $g_R=g_L$). Both limits are independent of $|U^R|$, $\cos\delta_{d,s}$, and the neutrino properties.

M_2 (GeV)	β	$\lvert\zeta\rvert$ (M_L-M_1)	$\lvert\zeta\rvert$ ($\lvert\zeta\rvert<\beta$)
300	0.073	<0.059	<0.12
500	0.026	<0.035	<0.043
700	0.013	<0.025	<0.022
1000	0.0065	<0.017	<0.011
1500	0.0029	<0.011	<0.0048
2000	0.0016	<0.0087	<0.0026
2500	0.0010	<0.0069	<0.0016

TABLE VII. 90%-C.L. upper limits on β_g and lower limits on M_{2g} (in GeV) obtained from muon decay (80) for $\cos\delta_d=1$ and 0, with ζ_g fixed at 0 and ζ_g free. Case $U^R_{(LR)}$ is almost identical to cases $U^R_{(\mathrm{I})}$ and $U^R_{(\mathrm{II})}$.

Case	$\zeta_g=0$ β_g	M_{2g}	ζ_g free β_g	M_{2g}
	$\cos\delta_d=1$			
$U^R_{(\mathrm{I})}, U^R_{(\mathrm{II})}$	0.027	490	0.036	430
$U^R_{(\mathrm{III})}, U^R_{(\mathrm{IV})}$	0.039	410	0.036	430
	$\cos\delta_d=0$			
$U^R_{(\mathrm{I})}, U^R_{(\mathrm{II})}$	0.027	490	0.025	510
$U^R_{(\mathrm{III})}, U^R_{(\mathrm{IV})}$	0.039	410	0.036	430

TABLE VIII. Best-fit values for β_g and ζ_g from the muon decay constraint (80). Case $U^R_{(LR)}$ is almost identical to cases $U^R_{(\mathrm{I})}$ and $U^R_{(\mathrm{II})}$. In each case β_g^2 is $\sim 2.4\sigma$ away from zero.

Case	$\zeta_g=0$ β_g	M_{2g}	ζ_g free β_g	M_{2g}	ζ_g
$U^R_{(\mathrm{I})}, U^R_{(\mathrm{II})}$	0.023	540	0.023	530	0.0014 ± 0.008
$U^R_{(\mathrm{III})}, U^R_{(\mathrm{IV})}$	0.032	450	0.028	480	0.016 ± 0.014

2. Astrophysical and cosmological constraints for light ν_R

If the ν_R are lighter than around 1 MeV and were produced in sufficient numbers in the early Universe they would have contributed significantly to the expansion rate, thus affecting the n/p ratio and leading to the production of too much helium. Steigman *et al.* have calculated[18] that if all three ν_R are light then the observed ^{4}He abundance requires that the ν_R decoupled before $T \sim 0.2$ GeV. The dominant production mechanism is $e^+e^- \to \nu_R \overline{\nu_R}$ via W_R and Z' exchange. The limit depends on both M_{2g} and the Z' mass in a complicated model-dependent way, but typically yields a lower bound of around 1 TeV on M_{2g}, independent of U^R.

Two groups[19] have recently obtained stringent constraints from the energetics of Supernova 1987A. If the ν_{eR} is lighter than about 10 MeV then it could be produced by the charged-current processes $e_R^- p \to \nu_{eR} n$ and $e^+e^- \to \nu_{eR} \overline{\nu}_{eR}$. If the production rate is too large, the ν_{eR} would drain too much energy out of the supernova.[57] Generalizing these results, the $e_R^- p \to \nu_{eR} n$ reaction implies

$$|U^R_{11}|\beta_g < 2.5\times10^{-5}, \quad |\zeta_g| < 3\times10^{-5} . \qquad (81)$$

Hence, for forms $U^R_{(\mathrm{I})}$, $U^R_{(\mathrm{II})}$, and $U^R_{(\mathrm{LR})}$ ($U^R_{11}\sim 1$) one has the very strong limit $M_{2g} > 16.2$ TeV. The ζ_g limit is independent of U^R_{11}. For forms $U^R_{(\mathrm{III})}$ and $U^R_{(\mathrm{IV})}$ one has the weaker limit

$$\beta_g < 0.013 \; (M_{2g} > 720 \text{ GeV}) \qquad (82)$$

from $e^+e^- \to \nu_{eR}\overline{\nu_{eR}}$ (Ref. 58). [The limit $\beta_g < 2.5\times10^{-5}/\epsilon$ would result from (81) if we apply the same kind of fine-tuning restrictions as for Δm_K.]

IV. RESULTS

The limits on $\beta_g \equiv g_R^2 M_1^2/g_L^2 M_2^2$, $M_{2g} \equiv g_L M_2/g_R$, and $\zeta_g \equiv g_R \zeta/g_L$ from global fits to all data are summarized in Tables II and III for various right-handed neutrino properties, for PMLRS and for the special forms $U^R_{(\mathrm{I})} - U^R_{(\mathrm{IV})}$. The latter are the extreme cases of the two regions of parameters $U^R_{(A,B)}$ in (3) which give the weakest limits (barring extreme fine-tuning). We have checked that allowing U^R to be completely arbitrary except for three-family unitarity constraints does not yield weaker or otherwise interesting solutions. The limits on β_g are mainly determined by Δm_K, $B_d\overline{B}_d$ mixing, b decay, $\beta\beta_{0\nu}$, and μ decay, and are largely uncorrelated with the mixing limits from universality, kaon decay, the M_1 mass shift, and the theoretical constraint $|\zeta_g| \le \beta_g$.

The 90%-C.L. allowed contours in β_g and ζ_g for $\cos\delta_{d,s}=1$ (small CP-violating phases) are shown in Figs. 5–7. In each case the contours allowed by combining individual constraints on ζ_g with the relevant mass constraints (Δm_K, $B_d\overline{B}_d$, b decay, $\beta\beta_{0\nu}$) are shown, as well as the result of the overall fit to all appropriate data.

For heavy Majorana neutrinos the combination of Δm_K, $B_d\overline{B}_d$ mixing, b decay, and $\beta\beta_{0\nu}$ requires $M_{2g} > 670$ GeV, with more stringent limits for most U^R. Also, $\beta\beta_{0\nu}$ implies that the ν_R should be quite heavy (comparable to M_2) unless either M_2 is unobservably large or ν_R is lighter than ~ 1 MeV. The correlated limits on β_g and ζ_g are shown in Fig. 5 for $\cos\delta_d=1$ (cases I, II, LR) or for $\cos\delta_s=1$ (cases III, IV). It is seen that the combination of universality and $K_{\pi3}$ always gives limits on ζ_g comparable to the case of PMLRS, even though the β_g limits are much weaker. The case $\cos\delta_i=0$ (Ref. 59) is discussed below.

For heavy Dirac neutrinos (i.e., for ν_R heavier than around $m_\mu/2$) one loses the $\beta\beta_{0\nu}$ constraint. Δm_K, $B_d\overline{B}_d$ mixing, and b decay allow the weaker limit $m_{2g} > 300$ GeV for $U^R_{(\mathrm{I})}$. The limits on ζ_g are similar to the Majorana case, as can be seen in Fig. 6 for $\cos\delta_i=1$.

For intermediate-mass Dirac neutrinos (between ~ 10 MeV and $m_\mu/2$) the mass limits are given by Δm_K, $B_d\overline{B}_d$ mixing, and muon decay, and one has $M_{2g} > 500$ GeV. Mixing is still limited by universality and $K_{\pi3}$. Figure 7 is for $\cos\delta_i=1$. For $\cos\delta_i=0$ the muon constraints are modified slightly, as can be seen in Fig. 1. However, the mass limits in Table II are unchanged to two significant digits.

The limits for the intermediate-mass neutrinos continue to hold for light or massless right-handed neutrinos. However, for neutrinos lighter than ~ 10 MeV the additional more stringent constraints from cosmology and SN 1987A apply as well.

For $\cos\delta_{d,s}=0$ (maximal CP-violating phases) (Ref. 59) the linear universality constraints (71) on ζ_g disappear. The first-order limit from nonleptonic kaon decay also disappears, but the second-order constraints in (76) continue to be important. These lead to the 90%-C.L. upper limits on ζ_g shown in Table III. The largest allowed $|\zeta_g|$ is seen to be ~ 0.013 (cases I and II). Since the $K_{\pi3}$ constraints are perhaps less solid theoretically than some of the others, it should be remarked that slightly weaker limits are obtained from other sources. In particular, $|\zeta_g| < 0.049$ from universality for heavy neutrinos, with a comparable limit from muon decay for light and intermediate mass neutrinos. Limits of a few percent (depending on M_2) follow from the M_1 mass shift and $|\zeta_g| \le \beta_g$ (Table VI).

It should be commented that a very light W_R could be tolerated if one abandons the prohibitions on extreme fine-tuning. For example, there are small variations on $U^R \sim U^R_{(\mathrm{I})}$ or $U^R_{(\mathrm{III})}$ for which the Δm_K constraint is satisfied by fine-tuned cancellations between the different diagrams. For a heavy Dirac neutrino and small ζ_g none of the other constraints considered here would be relevant. Even the direct production limits based on leptonic decay modes[36] would be ineffective if the ν_R were comparable to M_2. The UA2 Collaboration[60] has reported the observation of a broad enhancement in the invariant mass distribution for $\bar{p}p \to 2$ jets which is consistent with the nonleptonic decays of the ordinary W and Z. There is no sign in their data for a second bump (from a light W_2) up to 150 GeV. Since the expected production cross section falls more slowly with mass than the background distribution in Ref. 60, it is likely that there is no W_R in that range. However, caution is advised since the UA2 analysis was directed towards observing the ordinary W and Z and not towards looking for new physics.

V. CONCLUSIONS

The simplest extension of the standard electroweak model involving additional charged gauge bosons is the $SU(2)_L \times SU(2)_R \times U(1)$ group, which could either occur in the context of an SO(10) grand unified theory or independently. It is conceivable that the additional gauge bosons could be light enough to be observable at present or future colliders. However, most previous studies of existing limits have involved further assumptions concerning the Higgs structure of the theory, additional discrete left-right symmetries, and/or the masses and nature of the right-handed neutrinos. In this paper we have studied the existing experimental and theoretical constraints on M_R, the mass of the right-handed charged gauge boson W_R, and on ζ, the mixing angle between W_L and W_R, for a very general case of $SU(2)_L \times SU(2)_R \times U(1)$ models. In particular, we have allowed the right-handed quark mixing matrix U^R to be completely arbitrary except for constraints from three-family unitarity (the results would apply equal well if there are more families, provided their mixings with the first three families are small). We also allow the gauge couplings g_L and g_R of the $SU(2)_L$ and $SU(2)_R$ subgroups to be different, though not by orders of magnitude. Finally, we have allowed the right-handed neutrinos to be light or heavy and either Majorana or Dirac. We have generalized constraints that were derived previously for manifest or pseudomanifest left-right symmetry ($|U_{ij}^R| = |U_{ij}^L|$) and specific neutrino properties—or in some cases derived new constraints—from the K_L-K_S mass difference, $B_d\bar{B}_d$ oscillations, b decay properties, neutrinoless double-beta decay, muon decay, universality, nonleptonic K decays, the relation between gauge-boson masses and mixings, and astrophysical and cosmological constraints for light neutrinos.

With so much freedom in U^R and the neutrinos most of the constraints can be evaded in some cases. However, the combination is sufficient to require

$$\beta_g < 0.075 \quad (M_{2g} > 300 \text{ GeV}) \tag{83}$$

at 90% C.L. The limits on β_g are dominated by the K_L-K_S mass difference Δm_K and by $B_d\bar{B}_d$ mixing. The Δm_K limit is well known to be extremely stringent for PMLRS ($M_{2g} > 1.4$ TeV), but it is weaker for other values of U^R, especially in the vicinity of the special forms $U_{(A)}^R$ and $U_{(B)}^R$ in (3). Each of these involves an angle α which smoothly interpolates between the special forms $U_{(\text{I})}^R - U_{(\text{II})}^R$ and $U_{(\text{III})}^R - U_{(\text{IV})}^R$ listed in Table I. Given the observed hierarchy of quark masses, the most likely possibility is that the off-diagonal elements of U^R are small. Of the forms in Table I only $U_{(\text{I})}^R \sim I$ satisfies this criterion, but we consider all four cases for completeness.

In regions $U_{(A)}^R$ and $U_{(B)}^R$ much smaller values of M_{2g} are allowed than for PMLRS, but we emphasize that this is only the case for relatively small, though not excessively fine-tuned, regions of parameter space. The limits may be strengthened somewhat by b decay, $\beta\beta_{0\nu}$, and μ decay, depending on the neutrino properties, and the result for $U_{(\text{IV})}^R$ is greatly strengthened by $B_d\bar{B}_d$ mixing.

The lower limits on M_{2g} for the various cases are listed in Table II. The lightest possible W_R occurs for a heavy Dirac neutrino. Somewhat stronger limits of 500 and 670 GeV are obtained for the (theoretically more popular) cases of neutrinos light enough to be produced in muon decay or for heavy Majorana neutrinos, respectively. (In fact the muon decay constraints deviate from the standard model predictions by 2.4σ, suggesting the possible existence of a W_R with a mass around 500 GeV. However, we choose to be conservative and interpret the results as upper limits on new physics.) Comparison with the expected production cross sections in Fig. 4 indicates that there is an excellent chance of discovering a W_R at the SSC, which should be sensitive up to 8–9 TeV, and even a window at the Tevatron (sensitive to 500–700 GeV). Fortunately, the cross section for $pp \to W_R^+$, which occurs via $u\bar{d}$ or $u\bar{s} \to W_R^+$, is not very sensitive to the form of U^R as long as U_{ub}^R is small.[62]

In contrast to M_{2g}, the limits on the mixing angle ζ_g, which is dominated by universality and $K_{\pi 3}$ decay, are almost as stringent as for the case of PMLRS as long as $\cos\delta_{d,s} \sim 1$ (small CP-violating phases). This can be seen in Table III and in Figs. 5–7. The weakest limit is

$$|\zeta_g| < 0.0025 \quad (\cos\delta \sim 1) \tag{84}$$

for cases I and II.

However, it is possible that the CP-violating phases in $e^{i\omega}U^R$ are large, and in fact large phases are one of the mechanisms for evading the predictions of PMLRS if there is an underlying L-R symmetry. We have therefore allowed for the possibility of maximal phases ($\cos\delta_{d,s} \sim 0$). In that case, one obtains the weaker limits on $|\zeta_g|$ in Table III. These are dominated by second-order constraints from nonleptonic kaon decay, and yield

$$|\zeta_g| < 0.013 \quad (\cos\delta \sim 0) \tag{85}$$

for $U_{(\text{I,II})}^R$. Other constraints from universality, muon decay, $|\zeta_g| \le \beta_g$, and the M_1 mass shift place limits of a few percent on $|\zeta_g|$. The mass shift limit should be significantly improved in the future. Constraints from CP violation in the kaon system may also be significant but have not been investigated here. Limits for values of $\cos\delta_i$ between 1 and 0 (Ref. 59) interpolate between the extremes in Table III.

The limits we have quoted are not rigorous. There are a number of theoretical uncertainties, and we have incorporated reasonable but not rigorous prohibitions on extreme fine-tuning (in fact we discuss a finely tuned solution which could possibly allow $M_R \simeq M_L$). However, we believe they represent reasonable guides to the most likely places to look for new physics.

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