Asymmetry of Positrons from the Decay $\pi^+ \to \mu^+ \to e^+$ in the Longitudinal 140 000 Oe Magnetic Field

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We measured in photoemulsion the energy-integrated angular distribution of positrons from the $\pi^+ \to \mu^+ \to e^+$ decay in a longitudinal (along direction of the μ-meson spin) 140 000 Oe magnetic field. The total number of the measured events is 190 000. The experimental angular distribution of the positrons has the form $1 - a\cos\theta$ with $a = 0.325 \pm 0.005$. This result agrees with the theory of the universal $V - A$ interaction.

1. INTRODUCTION

The $V - A$ theory of weak interaction leads to the following form of the energy-integrated angular distribution of the $\mu \to e$ decay electrons:

$$\frac{dW}{d\theta} = \frac{1 - a\cos\theta}{2}, \qquad (1)$$

where the asymmetry coefficient is $a = 1/3$; θ is the angle between the electron momentum and the muon spin. The experimental determination of the coefficient a has been the subject of many investigations.[1)] The difficulty in determining the true angular distribution of the $\mu \to e$ decay electrons lies in the fact that the medium in which the μ meson is decelerated and stopped depolarizes the muon. Therefore the experimentally determined value a_{meas} is always smaller than the true value: $a_{meas} = Pa$, where P is the residual polarization of the muon at the instant of $\mu \to e$ decay. The degree of depolarization of the μ^+ meson is different in different substances. The minimum depolarizing action is possessed by metals and certain organic compounds such as bromoform. For μ^+ mesons stopping in bromoform[2] $a = 0.325 \pm 0.015$. The value obtained in hydrogen[3] is $a = 0.31 \pm 0.02$. These are close to the value $a = 1/3$ expected from the $V - A$ theory. However, such measurements yield only the lower limit of the value of a, since the residual polarization P of the muon in the target material remains unknown as before.

There exists, however, a method of "actively" suppressing the depolarizing action of the medium by applying a longitudinal magnetic field H directed along the muon spin direction. It was found experimentally[4–11] that a_{meas} increases with increasing H. To explain the experimentally observed $a(H)$ dependence, let us consider briefly the presently accepted picture of μ^+ meson polarization in matter. We divide the μ^+-meson depolarization process into three stages.

The first stage consists of depolarization of the μ^+ meson during the course of ionization deceleration to velocities exceeding those of the atomic electrons. Experiment[11] has shown, in agreement with the theory, that practically no μ^+ meson depolarization is observed during this stage.

The depolarization takes place essentially during the second stage, when the μ^+ meson slows down to the velocity of the atomic electrons. Then the μ^+ meson captures an electron from the medium and forms a bound state—the hydrogenlike muonium atom. The spin J of the produced muonium can be equal to 0 or 1. Both these states are produced with equal probability, since the μ^+ meson was initially polarized. Production of muonium with $J = 1$ does not lead to depolarization of the μ^+-meson spin, by virtue of the angular-momentum conservation. State with $J = 0$ is not an eigenstate of the μ^+-meson spin, and therefore leads to complete depolarization of the μ^+ meson within a time $T = \hbar/\Delta E = 3.6 \times 10^{-11}$ sec (here ΔE is the energy of the hyperfine splitting of the muonium). As a result, in half the cases (muonium state with $J = 1$) the initial direction of the μ-meson spin is conserved, and in the other half of the cases (muonium state with $J = 0$) the muon is fully depolarized, i.e., the residual polarization of the μ^+ meson is $P = 1/2$. The fact that a polarization $P < 1/2$ has been observed in experiments for a number of substances is a consequence of the possible depolarization of the muonium electron as a result of interaction with the medium. The depolarization of the muonium electron leads to a transition of the muonium from the state with $J = 1$ into the state with $J = 0$, causing addition depolarization of the μ^+ meson. The time τ that the μ^+ meson stays in the muonium state is small for the overwhelming majority of substances ($\tau < 10^{-7}$ sec). This follows from the experimental fact that the residual polarization of the μ^+ meson in a substance does not change in time when $t > 10^{-7}$ sec. The only exception is boron carbide.

The third stage begins when the muonium enters into a chemical bond with a molecule of the medium, replacing a hydrogenlike atom in the latter. After the μ^+ meson has entered into a chemical bond or after it occupies its place in the crystal lattice, its depolarization as a rule ceases. It is the μ^+-meson residual polarization prevailing at the instant when it enters into the chemical bond which determines the values of $a_{meas} = Pa$ for different substances, which are usually measured at $t > 10^{-7}$ sec.

Let us now attempt to explain, on the basis of this model, the experimentally observed dependence of $a_{meas} = Pa$ on the longitudinal magnetic field H. This phenomenon was originally attributed by Orear et al.[7] to the influence of the field H on the muonium atom produced in the medium. The Paschen-Back effect on muonium was considered in [7] and the following dependence of the polarization P of a μ^+ meson in the muonium bound state on the field intensity H was obtained:

$$P = P_0 \frac{1 + 2x^2}{2(1 + x^2)}. \qquad (2)$$

Here P_0 is the primary polarization of the muon (at the instant of the $\pi \to \mu$ decay); $x = H/H_0$, $H_0 = 1580$ G is the "characteristic" field determined from the relation $\mu H_0 = \Delta E$, where μH_0 is the energy of the muonium electron in the field H_0 and ΔE is the muonium hyperfine-splitting energy. From relation (2) it is seen that when H increases the residual polarization tends to its initial value P_0. Formula (2) describes the experimental $P(H)$ dependence only qualitatively. In the experiment, the fields H required to produce the corresponding change in P are several times larger than would follow from formula (2). Relation (2) likewise does not explain the experimental fact that for a number of substances $P(x = 0) \sim 0$. In the model considered above, agreement can be obtained between theory and experiment by taking into account the interaction of the muonium electron with the medium. As was already indicated, this leads to additional depolarization of the muon and therefore large magnetic fields are required to suppress the depolarizing action of the medium. The $P(H)$ dependence with allowance for the depolarization of the muonium electron was obtained by Nosov and Yakovleva,[12] who consistently analyzed the entry of the muonium into a chemical reaction and obtained the following formula:

$$P = P_0 \frac{1 + 2x^2}{2(1 + x^2 + \nu\tau)}, \qquad (3)$$

where τ is the lifetime of the free muonium before entering into the chemical bond; $1/\nu$ is the average time of depolarization of the muonium electron by interaction with the medium. The parameter $\nu\tau$ thus shows how many times the electron bound in the muonium has been depolarized during the muonium lifetime. The introduction of the parameter $\nu\tau$ makes it possible to explain satisfactorily[2)] the experimental data on the $P_{exp}(H)$ dependence. The quantity $\nu\tau$ cannot be calculated and must be determined only by comparison with experiment. In other words, in order to obtain the quantitative $P(H)$ dependence it is necessary to obtain this dependence experimentally in a certain interval of variation of H.

We have measured the angular distribution of the positrons from the $\pi^+ \to \mu^+ \to e^+$ decay in emulsion in a longitudinal pulsed magnetic field of 140 000 G. The preliminary results of this experiment were published in [13]. The use of photographic emulsion to determine the true value of the coefficient a has a number of advantages.

1. In emulsion it is possible to measure accurately the emission angles of the electron and the muon.

2. There is practically no energy threshold for the registration of the $\mu \to e$ decay electrons, so that one actually measures the true energy-integrated angular distribution of the electrons.

3. In emulsion measurements, there is no kinematic depolarization[1] of the muons at all, since the π mesons decay at rest.

4. The $P(H)$ has been sufficiently well measured for emulsions in the interval $\Delta H = 0$–$35\,000$ G[4–10] making it possible to determine the "material parameter" $\nu\tau$. This makes it possible to estimate the field intensity H which is sufficient for practically complete elimination of the depolarizing action of the medium.

Formula (3) becomes somewhat more complicated in the case of emulsion, owing to the fact that the emulsion is a heterogeneous mixture of two components, gelating and AgBr crystals. The expression for P therefore contains two terms, one of which describes the depolarizing action of the gelatin and the other the action of the AgBr crystal:[12]

$$P = P_0\left[f\frac{1 + 2x^2}{2(1 + x^2)} + (1 - f)\frac{1 + 2x^2}{2(1 + x^2 + \nu\tau)}\right]. \qquad (3')$$

Here f is the fraction of the muons stopped in the gelatin; $(1 - f)$ is the fraction of the muons stopped in AgBr. The quantity $\nu\tau$ for gelatin is close to zero, since $P_{gel}(x = 0) \approx 1/2$. The parameters f and $\nu\tau$ were determined from a comparison of (3′) with the experimental $P_{exp}(x)$ dependence for emulsion at $x = 0$–22. The best agreement between formula (3′) and $P_{exp}(x)$ is obtained for $f = 0.63$ and $\nu\tau = 80$.[12] Figure 1 shows a plot of (3′) at these values of f and $\nu\tau$ and the experimental values of P for $x = 0$–22. It is seen from Fig. 1 that in the case of emulsion, magnetic fields with $x = 22$ ($H = 35\,000$ G) are insufficient for complete removal

[1)] A detailed bibliography can be found in [1].

[2)] Formula (3) fails to explain certain details of the experimentally observed $P_{exp}(H)$ dependence, for example the strong $P(H)$ dependence of a number of substances in weak fields ($H < 500$ G).[11]

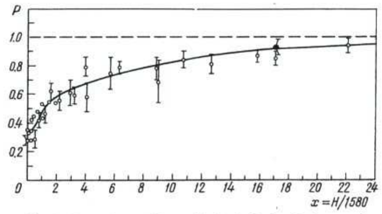

Fig. 1. Dependence of the residual polarization P of a muon in photographic emulsion on the applied magnetic field, measured in relative units $x = H/1580$, where H is in Gauss. Solid curve – relation (3′), light circles – data obtained in [4–10] in a constant magnetic field, full circle – data obtained in the present investigation in a pulsed magnetic field $H = 27\,000$ G.

of the depolarization. It follows from (3′) that for the field H = 140 000 G used in the present work ($x = H/H_0 = 89$) the residual polarization P differs from P_0 by less than 1%.

2. EXPERIMENT

The experimental setup is shown in Fig. 2. A μ^+ meson beam of 80 MeV energy extracted from the accelerating chamber of the synchrocyclotron of the Laboratory of Nuclear Problems of JINR passed through a deflecting magnet, a collimator, and entered a pellicle stack placed in the coil of the pulsed magnetic field. The coil was made of beryllium bronze and constituted a flat spiral 130 mm long with inside and outside diameters 80 and 180 mm respectively, each turn being 3 mm thick. The coil was fed from a capacitor bank rated 0.1 F, charged to 2000 V. The setup for producing the pulsed magnetic field is described in detail in [14]. The magnetic-field pulses were synchronized with the pulses of the π^+-meson beam extracted from the accelerator. An oscillogram of a magnetic-field pulse is shown in Fig. 3, from which it is seen that the waveform of the field pulse is well approximated by a sinusoidal half-wave with period 20 msec. The extraction of the π^+ mesons lasted approximately 0.6 msec. Therefore the magnetic field did not vary by more than 1% during the π^+-meson pulse. The operation of the synchronization circuit was automatically monitored with the aid counters placed in the π^+-meson beam (see Fig. 2). During the entire time of operation, 258 extractions of π^+ mesons without a magnetic field were recorded, amounting to $\kappa = 0.44\%$ of the number of extractions with the field (H = 140 000 G). The π^+ mesons were decelerated prior to their incidence on the pellicle stack. The thickness of the moderator was chosen to be such as to make the π^+ mesons stop in the scanned region of the stack. The $\pi \to \mu$ decay cases selected during the scanning of the emulsion were those in which the muon traveled along the direction of the magnetic field. Since the muons of the $\pi \to \mu$ decay were polarized in the direction of their motion, this meant that the selected muons were polarized along the direction of the magnetic field. In measuring the angular distribution of the positrons of the $\mu^+ \to e^+$ decay in the emulsion, extraction of the π^+ mesons from the accelerator without an accompanying magnetic-field pulse leads to a decrease in the measured value of the coefficient:

$$a_{\text{meas}} = (1 - \varkappa)\, a_H + \varkappa a_{ph} = {}^1\!/_3 - 0.23\varkappa. \tag{4}$$

Here $a_H = {}^1\!/_3$ is the limiting value of the asymmetry coefficient a in a strong longitudinal magnetic field; $a_{ph} = 0.10$ is the coefficient a in photoemulsion at $H = 0$; κ is the fraction of the π^+ mesons extracted without a magnetic field. For $\kappa = 0.44\%$ we have $a_{meas} = {}^1\!/_3 - \Delta a$, where $\Delta a = 0.23\kappa = 0.001$.

We used in the investigation NIKFI type BR-2 emulsion stacks with increased grain density – 40 grains per 100 μ of track of minimally ionizing particle. The use of high-sensitivity emulsions made it possible to register the positrons of the $\mu^+ \to e^+$ decay with high efficiency. Out of the 190 000 $\pi^+ \to \mu^+ \to e^+$ decays, we observed only 130 cases in which the positron of the $\mu^+ \to e^+$ decay was not seen. Each stack consisted of 70 pellicles measuring 100 × 40 mm. In the unscanned region of the pellicles, x-ray lines of 30 μ thickness were marked, fixing the direction of the coil axis with accuracy of 0.5°. Alltogether we irradiated nine stacks in the 140 000 G pulsed field and two stacks in a 27 000 G pulsed field. The total number of π^+ mesons stopped in the scanned region of these stacks was approximately 2×10^6. This required approximately 60 000 out of the 140 000 G magnetic field pulses.

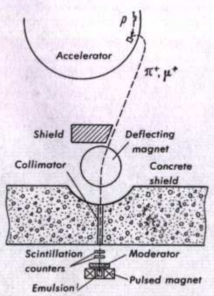

Fig. 2. Experimental setup

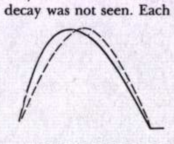

Fig. 3. Oscillogram of magnetic-field pulse. The time scale is marked on the horizontal axis: One marker corresponds to 1 msec. For comparison, the dashed curve is half a sinusoid.

3. SCANNING OF THE EMULSIONS AND SELECTION OF THE $\pi^+ \to \mu^+ \to e^+$ DECAY CASES

The scanned part of the pellicle stack was located symmetrically with respect to the central plane of the coil and was chosen in such a way that the direction of the magnetic field was in essence the same as the direction of the coil axis within the limits of the scanned region. This part of the stack is shown schematically in Fig. 4. Figure 5 shows the distribution of the angles ϵ between the direction of the magnetic field and the direction of the coil axis. It is seen from Figs. 4 and 5 that in the scanned region of the stack, the angles ϵ did not exceed 3°. Such a choice of the scanned region made it possible to disregard differences between the directions of the magnetic field in different parts of the emulsion stack. In the measurement of the angular distribution of the positrons of the $\mu^+ \to e^+$ decay, we assumed the direction of the magnetic field to be the direction of the coil axis. The associated decrease in the measured value of the asymmetry coefficient is negligibly small:

$$a_{\text{meas}} = a \langle \cos \varepsilon \rangle \approx a (1 - 0.0006). \tag{5}$$

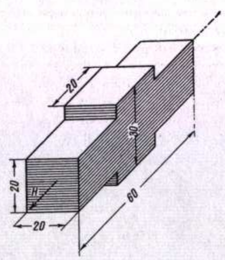

Fig. 4. Scanned part of the emulsion stack. The emulsion layers are arranged horizontally. The indicated direction of the magnetic field H coincides with the axis of the coil. The dimensions are indicated in mm.

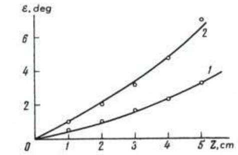

Fig. 5. Change of angles ϵ between the directions of the magnetic field and the coil axis z along the axis for two distances from the latter, r = 1 cm (curve 1) and r = 2 cm (curve 2).

For the measurement we chose cases of $\pi^+ \to \mu^+ \to e^+$ decays in which the muon was contained entirely within a single emulsion layer and traveled along the direction of the magnetic field. In practice the selection was effected in such a way, that the projection angle γ (on the emulsion plane) between the direction of muon emission and the direction of the magnetic field lay in the range γ = 0–30° and γ = 150–180°. Altogether we chose 189 547 such cases in the field H = 140 000 G and 12 964 cases in the field H = 27 000 G.

The distribution of the selected cases of $\pi \to \mu$ decay with respect to $\cos \varphi$, denoted $F(\cos \varphi)$ (φ is the three-dimensional angle between the direction of emission of the muon and the direction of the magnetic field), is shown in Fig. 6. This distribution was obtained by measuring the angles φ for approximately 12% of all the registered cases of $\pi \to \mu \to e$ decay uniformly during the entire scanning time. The angles φ were determined from the relation $\cos \varphi = \cos \gamma \cos \beta$, where β is the angle between the direction of emission of the muon and the emulsion plane. The distribution $F(\cos \varphi)$ shown in Fig. 6 is the summary distribution for the cases with γ = 0–30° and γ = 150–180°, i.e., it is actually a distribution of the value of $|\cos \varphi|$. Figure

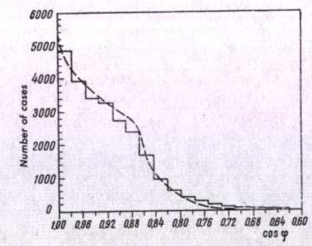

Fig. 6. The distribution with respect to $\cos \varphi$ of 25,056 selected cases of $\pi \to \mu$ decay. Dashed curve – calculated distribution $F_{calc} = f(\cos \varphi)$ under the assumption that a muon emitted in any direction is registered by the scanner with equal probability. The average value of $\langle \cos \varphi \rangle$ is 0.9190 ± 0.0017.

6 shows also the calculated relation $F_{calc} = f(\cos\varphi)$, obtained under the assumption that the emission of the muon at any angle is registered by the scanners with equal probability. It is seen from the figure that the calculated an experimental distributions $F(\cos\varphi)$ obtained in this manner differ insignificantly. However, in the subsequent calculations we shall use only the experimental $F(\cos\varphi)$ dependence obtained without any additional assumptions.

The foregoing selection of the $\pi \to \mu$ decay cases was carried out by scanning the emulsions without immersion, with magnification $15 \times 10 \times 1.5$. At this magnification one could clearly see the tracks of only the π and μ mesons, while the positron from the $\mu \to e$ decay was not registered. Measurement of the emission angle of the positron of the $\mu \to e$ decay was made with magnification $15 \times 60 \times 1.5$, once the given $\pi \to \mu$ decay case was selected for the measurements. Such a procedure excluded the possibility of systematically selecting cases in which the positron of the $\mu \to e$ decay was emitted predominantly at some definite angle. In scanning 190 000 cases of the $\pi^+ \to \mu^+ \to e^+$ decay, we observed no anomalous decays at all. We registered a case of $\mu \to 3e + \nu + \bar{\nu}$, when the bremsstrahlung γ quantum produced a Dalitz pair.

4. ANGULAR DISTRIBUTION OF THE POSITRONS OF THE $\pi^+ \to \mu^+ \to e^+$ DECAY

When a muon decays in a magnetic field, the muon spin is not a conserved quantity, all that is conserved is the projection of the spin on the direction of the magnetic field. Therefore the angular distribution $f(\theta)$ of the $\mu \to e$ decay electrons must be measured in such an experiment with respect to the direction of the magnetic field. The "non-parallelism" of the muon spin and the direction of the magnetic field H deforms the angular distribution (1) of the $\mu \to e$ decay electrons. This deformation is the result of the precession of the spin (and consequently of the initial angular distribution of the $\mu \to e$ decay electrons) about the direction of the magnetic field H. A simple quantum-mechanical calculation shows that the deformation of the angular distribution (1) reduces to a decrease in the asymmetry coefficient a:

$$a_H = a|\cos\varphi|,$$

where φ is the angle between the direction of the muon spin and the direction of the magnetic field. In the $\pi \to \mu$ decay, the direction of the muon spin is collinear with the direction of its momentum,[3)] and therefore the angle φ is determined by the direction of the muon emission. Consequently, for the selected cases of the $\pi \to \mu$ decay, the magnitude of the asymmetry coefficient a_H measured in the magnetic field is

Table I
Mean values of $\langle|\cos\varphi|\rangle$
(errors – purely statistical)

Scanner	$\langle\lvert\cos\varphi\rvert\rangle$
I	0.9132±0.0007
II	0.9212±0.0008
III	0.9184±0.0007
IV	0.9219±0.0007
V	0.9194±0.0010
Average	0.9190±0.0003

$$a_H = \langle|\cos\varphi|\rangle a. \qquad (6)$$

Here $\langle|\cos\varphi|\rangle$ is the mean absolute value of $\cos\varphi$ for the selected cases of $\pi \to \mu$ decay. The value of $\langle|\cos\varphi|\rangle$ was determined from the experimentally measured $F(\cos\varphi)$ distribution shown in Fig. 6. The value of $\langle|\cos\varphi|\rangle$ obtained in this manner is

$$\langle|\cos\varphi|\rangle = 0.9190 \pm 0.0017. \qquad (7)$$

The error indicated in (7) was determined experimentally from the scatter of the values of $\langle|\cos\varphi|\rangle$ measured in individual emulsion stacks. This error is connected with the inaccuracy of measuring the angles φ, shrinkage of the emulsion during development, and other measurement errors. It greatly exceeds the purely statistical error, which in this case is $\delta\langle\cos\varphi\rangle = 0.0003$. Table I lists the values of $\langle\cos\varphi\rangle$ for the cases of $\pi \to \mu$ decay selected by five different scanners. It is seen from the table, that the obtained values of $\langle\cos\varphi\rangle$ are close to one another, i.e., the selection of the $\pi \to \mu$ decay cases by different scanners was approximately uniform.

In the present work, the angle θ between the emission direction of the positron of the $\mu \to e$ decay and the direction of the magnetic field was not measured, all that was measured was the projection α of this angle on the emulsion plane. Figure 7 shows the $\pi^+ \to \mu^+ \to e^+$ decay scheme projected on the emulsion plane and the angles α and γ. The angles α were measured with a goniometer at a microscope magnification $15 \times 60 \times 1.5$. The theoretically expected $\Phi(\alpha)$ distribution follows from the distribution (1) for the angle θ. After simple transformations we obtain

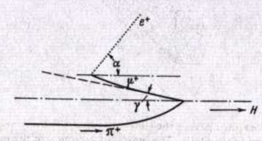

Fig. 7. Schematic representation of the projection of the $\pi^+ \to \mu^+ \to e^+$ decay on the emulsion plane.

[3)] We have in mind the momentum of the muon at the instant of the $\pi \to \mu$ decay. As the muon moves further in the emulsion, multiple Coulomb scattering leads to a deviation of the momentum from its initial direction, but the spin direction remains unchanged.

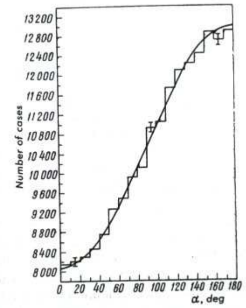

Fig. 8. Distribution $dN/d\alpha$ of the $\pi^+ \to \mu^+ \to e^+$ decay positrons with respect to the angle α in a longitudinal magnetic field of 140 000 G. The smooth curve represents the theoretical distribution (9) for $a = 0.323 \pm 0.005$ and $(\pi/4)\, a\langle\cos\varphi_\mu\rangle = 0.233$.

$$\frac{d\Phi}{d\alpha} = \frac{1}{2\pi}\left(1 - \frac{\pi}{4} a \cos\alpha\right). \qquad (8)$$

In a longitudinal magnetic field, with allowance for (6), the distribution (8) takes the form

$$\left.\frac{d\Phi}{d\alpha}\right|_H = \frac{1}{2\pi}\left(1 - \frac{\pi}{4} a\langle|\cos\varphi|\rangle \cos\alpha\right). \qquad (9)$$

The corresponding experimental angular distribution $dN/d\alpha$ obtained in the present work for 189 547 positrons of the $\pi^+ \to \mu^+ \to e^+$ decay is shown in Fig. 8. In Fig. 9*A*, the distribution $dN/d\alpha$ is plotted as a function of $\cos\alpha$. It is seen from this figure that the experimental distribution $dN/d\alpha$ can be well interpolated by means of a straight line, and consequently confirms the correctness of the theoretical distribution (1). The degree of agreement between the experimental distribution $dN/d\alpha$ and the theoretical distribution (9) is determined by the quantity

$$M = \sum_{i=1}^{18}\left[\left(\frac{dN}{d\alpha}(\text{theor}) - \frac{dN}{d\alpha}(\text{exp})\right)\Big/\, \delta\frac{dN}{d\alpha}(\text{exp})\right]_i^2,$$

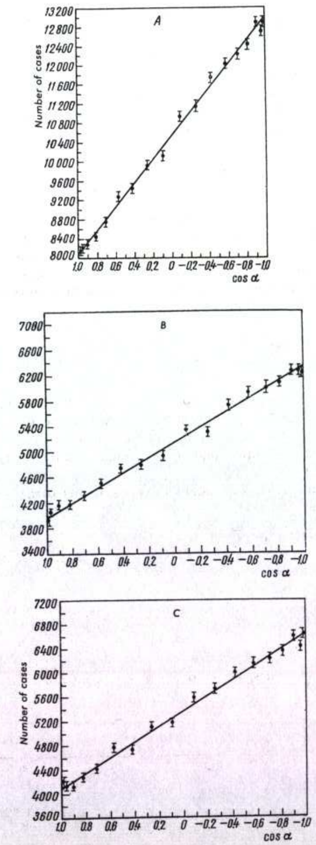

Fig. 9. Distribution $dN/d\alpha$ of the $\pi^+ \to \mu^+ \to e^+$ decay positrons with respect to $\cos\alpha$. The straight lines correspond to the distribution (9) at the corresponding value of the asymmetry coefficient a: *A*–distribution for 189 547 cases with the muons emitted both along the magnetic field and opposite to it, $a = 0.323$, $M = 21.9$; *B*–distribution for 92 198 cases with the muons emitted in the direction of the field, $a = 0.317$, $M = 17.2$; *C*–distribution for 97 349 cases with the muons emitted opposite to the field, $a = 0.329$, $M = 23.2$

where the summation over the number of experimental points. For the case of Fig. 9*A* with $a = 0.323$ we get $M = 21.9$ for 18 degrees of freedom.

The asymmetry coefficient a, the value of which is in accord with the $V - A$ theory is $a = {}^1\!/_3$, was determined from the experimentally measured angular distribution $dN/d\alpha$ by two methods.

1. From the distribution (9) we get

$$a_H = a\,\langle\cos\varphi\rangle = \frac{2(N_b - N_f)}{N_b + N_f}, \qquad (10)$$

where N_b and N_f are the number of the cases of $\mu \to e$ decay corresponding to positron emission "backward" ($\alpha > 90°$) and "forward" ($\alpha < 90°$) relative to the projection of the muon momentum on the direction of the magnetic field. The statistical error in the determination of a is in this case

$$\delta a = \frac{1}{\langle\cos\varphi\rangle}\frac{2}{\sqrt{N_0}}, \text{ where } N_0 = N_b + N_f. \quad (11)$$

For $N_0 = 189\,547$ and $\langle\cos\varphi\rangle = 0.9190$ we obtain $\delta a = 0.005$. The error in the coefficient a, connected with the inaccuracy of determining the quantity $\langle\cos\varphi\rangle$ (7), is negligibly small and is equal to $\delta a(\langle\cos\varphi\rangle) = 0.0006$.

The value of a obtained in this manner turned out to be

$$a_{\text{meas}} = \frac{2(N_b - N_f)}{N_0} = \frac{56648}{189547} = 0.2989 \pm 0.0046,$$

$$a = \frac{a_H}{\langle\cos\varphi\rangle} = 0.3252 \pm 0.0050.$$

We have assumed here for a_H and $\langle\cos\varphi\rangle$ the averages of the values obtained by all the scanners. We can calculate the coefficient a in a somewhat different form:

$$a = \sum_i P_i a_i,$$

where $a_i = (a_H/\langle\cos\varphi\rangle)_i$ is calculated from the data of individual scanners (see Tables I and II), and P_i is the statistical weight of the cases obtained by the corresponding scanner. The value of a obtained in this manner agrees with that obtained above to within the fourth significant figure.

2. The second method of determining a consists in comparing the obtained angular distribution $dN/d\alpha$ (Figs. 8 and 9*A*) with the theoretical distribution (9). The best agreement between the experimental and theoretical distributions is obtained when $a = 0.321$. The minimum value of the Pearson parameter corresponding to this value of a is $\chi^2 = 21.5$ for 18 degrees of freedom ($a = 0.325$ corresponds to $\chi^2 = 23.0$).

The coefficients a in a longitudinal field of 140 000 G, obtained from formula (10) and from the angular distribution $dN/d\alpha$, are statistically compatible. Since there are no grounds for giving preference to any one of these two methods of determining a, we assume as the final experimental result the mean value of these two quantities

$$a = 0.323 \pm 0.005.$$

The largest errors (within 1%) of this value of a will be considered in Sec. 5. Table II lists the asymmetry coefficients a as measured by six scanners.

It is seen from Table II that the results obtained by individual scanners are in statistical agreement with one another. The values of the coefficients a determined from formula (10) and from the angular distributions by the least-squares method are likewise in statistical agreement. It also follows from Table II that the asymmetry coefficients a (along H) $= 0.319 \pm 0.007$ and a (opposite to H) $= 0.332 \pm 0.007$ for muons emitted in a direction along and opposite the magnetic field H coincide within the limits of errors. The experimental angular distributions $dN/d\alpha$ corresponding to these two cases are shown in Figs. 9*B* and *C*. The theoretical distributions (9) shown in these figures were obtained for values a(along H) $= 0.317$ and a(opposite H) $= 0.329$, which are mean values obtained by determining the coefficient a by the two methods described above. The fact that a(along H) and a(opposite H) are equal within the limits of errors indicates that there are no noticeable systematic errors connected with the data reduction for these two groups of cases.

Table II

The coefficients a

Scanner	Section A: Direction of muon momentum p — Along the field H	Opposite to the field H	Arbitrary	Section B: Direction of p arbitrary — a	χ^2_{min}
I	0.333±0.017	0.359±0.017	0.346±0.012	0.338	30.2
II	0.299±0.019	0.334±0.018	0.317±0.013	0.319	28.0
III	0.322±0.016	0.316±0.016	0.319±0.011	0.319	12.8
IV	0.304±0.016	0.314±0.016	0.309±0.011	0.301	11.6
V	0.314±0.017	0.325±0.017	0.320±0.012	0.311	12.6
VI	0.348±0.021	0.356±0.020	0.352±0.014	0.349	10.4
All scanners	0.319±0.007	0.332±0.007	0.325±0.005	0.321	21.5

Note. In Sec. A we give the coefficients a determined from formula (10); in Sec. B – the coefficients a were determined from the angular distributions by least squares (minimum χ^2) for 18 degrees of freedom. The errors are statistical.

In order to compare the experimental values of the asymmetry coefficient a obtained with emulsions irradiated in pulsed and constant (see [4–10]) magnetic fields, we irradiated two stacks in a pulsed magnetic field of 27 000 G. All the remaining experimental conditions, the scanning, and the reduction of the obtained results were fully analogous to those described above for the field $H = 140\,000$ G. The asymmetry coefficient $a_{pulse}(H = 27\,000)$ for a pulsed field $H = 27\,000$ G, obtained in the measurement of $N_0 = 12\,964$ cases of $\pi^+ \to \mu^+ \to e^+$ decay turned out to be

$$a_{\text{pulse}}(H = 27000) = \frac{1}{\langle\cos\varphi\rangle}\frac{2(N_b - N_f)}{N_0}$$

$$= 0.311 \pm 0.019.$$

This value of a_{pulse} coincides, within the limits of errors, with the value obtained in a constant magnetic field [4]

$$a_{\text{const}}\,(H = 27000) = 0.284 \pm 0.016.$$

The agreement between a_{pulse} and a_{const} at $H = 27\,000$ G (see Fig. 1) confirms that there are no effects leading to additional depolarization of the muons when pulsed magnetic fields are used.

5. ERRORS IN THE OBTAINED VALUE OF THE ASYMMETRY COEFFICIENT

In this section we consider small errors, within 1%, of the experimentally obtained value $a = 0.323 \pm 0.005$.

1. The main error in the value of a given in Sec. 4 results from the fact that the residual polarization P of the muon tends only asymptotically to its initial value P_0 with decreasing H. The difference between P_0 and P in a longitudinal magnetic field $H = 140\,000$ G can be estimated by using formula (3′). From a comparison of the theoretical dependence (3′) with the experimental values[4] to the line of $P_{exp}(H)$ it follows that the best agreement is obtained with the parameter values $f = 0.63$ and $\nu\tau = 80$.[12] Substituting these values into (3′) we find that $P = 0.995P_0$ in a field $H = 140\,000$ G. The corresponding error in the coefficient a is $\Delta a = +0.0015$.

2. Error connected with the fact that in $\kappa = 0.44\%$ of the cases we registered extraction of π^+ mesons from the accelerator without a magnetic field in the coil. The corresponding error in the coefficient a is calculated by means of formula (4) and is equal $\Delta a = 0.23\kappa = +0.001$.

3. Differences between the assumed and true directions of the magnetic field in the scanned region of the emulsion stack leads to an error (5) $\Delta a = ({}^1\!/_2)\epsilon^2 a = +0.0002$.

4. Radiative corrections to the coefficient a in the $\mu \to e$ decay were calculated in [15, 16]. It was found that radiative effects increase the coefficient a by 0.3%, i.e., $\Delta a = -0.0011$.

5. The influence of the radiative corrections on the degree of polarization of the muons in the $\pi \to \mu$ decay was considered in [17]. It was found that the probability of muon spin flip due to the radiative effects is 4×10^{-6}. Therefore the corresponding error in the asymmetry coefficient can be neglected.

Thus, the total error in the coefficient a, due to all the effects considered above, is $\Delta_\Sigma a = +0.0016$. The experimental value of the coefficient a corrected for this quantity, in a longitudinal field $H = 140\,000$ G, is equal to $a = 0.325 \pm 0.005$.

In concluding this section it should be noted that the final value of the coefficient a differs by only 0.5% from the value $a = 0.323$ which was directly measured in the experiment. The small value of the computational errors indicates that the obtained coefficient a is subject to practically no systematic errors which usually arise in those cases when the errors are large.

6. DISCUSSION

1. The main result of the investigation consists in determining, with high degree of accuracy, the asymmetry coefficient of the angular distribution of the positrons produced in the decay of polarized μ^+ mesons. The experimentally obtained angular distribution of the positrons for 190 000 cases of $\pi^+ \to \mu^+ e^+$ decay agrees well with the distribution (1)

$$\frac{dW}{dO} = \frac{1}{2}(1 - \lambda\sigma_\mu \mathbf{P}_e) = \frac{1}{2}(1 - a\cos\theta)$$

with $a = 0.325 \pm 0.005$; the obtained value of a is statistically compatible with the value $a = {}^1\!/_3$ that follows from the theory of the universal weak $V - A$ interaction, and is thus an experimental confirmation of this theory.

2. An interaction more general than $V-A$ is $V-kA$, which requires only a two-component neutrino. Knowledge of the coefficient a makes it possible to estimate experimentally the relative contribution of the V and A interactions in the $\mu \to e$ decay, i.e., the coefficient k:

$$a=\frac{1}{3}\frac{2\,\mathrm{Re}\,k}{1+|k|^2}=\frac{1}{3}\frac{2|k|\cos\psi}{1+|k|^2}. \tag{12}$$

Assuming the experimental value $3a_{min} = 0.975 - 0.015 = 0.96$, we obtain for $|k|$ the estimate $0.75 \leqslant |k| \leqslant 1.34$. The maximum possible deviation of $|k|$ from unity at a given value of a is obtained when k is real, i.e., for the T-invariant theory. We can obtain from (12) also the estimate $\cos\psi \geqslant 3a$, from which we get $\psi \leqslant 16°$. Unfortunately, relations (12) is very insensitive to $|k|$ and ψ, in spite of the high accuracy with which a is determined.

3. The accuracy attained in the present work in determining the coefficient a allows us to present an experimental estimate of the parameter $\Lambda = G_1/G_0$ in the weak-interaction theory constructed in accordance with the principle of baryon and lepton symmetry.[18] According to this theory

$$a=\frac{1}{3}\frac{1-3\Lambda^2}{1+\Lambda^2}.$$

when $\Lambda = 0$ the coefficient a becomes equal to $1/3$, as should be the case for $V-A$ interaction. The theory of [18] makes no definite predictions with respect to the quantity Λ. We can only estimate Λ quantitatively from $\Lambda^2 = (G_1/G_0)2 \approx 1/20$, starting from the fact that the probabilities of lepton decays with and without change of strangeness are related approximately like 1:20. The obtained value $a = 0.325 \pm 0.005$ leads to an experimental estimate $\Lambda = (8 \pm 3) \times 10^{-2}$, i.e., $\Lambda < 0.1$.

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Measurement of Angular *eν* Correlation in Free-neutron Decay

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The angular correlation between the electron and neutrino in the β decay of a free neutron is determined from the form of the energy spectrum of the recoil protons. The angular correlation constant is found to be $\lambda = -0.091 \pm 0.039$. The ratio of the constants $|G_A|$ and χχ $|G_V|$ calculated from these data is 1.22 ± 0.08.

Measurement of the angular correlation between the electron and the neutrino in the β decay of a free neutron, together with the measurement of the lifetime of the neutron and experiments with a polarized neutron, yields information on the structure of the weak interaction of nucleons. According to the theory of the universal interaction, the amplitude of the neutron β decay is

$$A=\frac{1}{\sqrt{2}}\,\bar{u}_p(G_V\gamma_\alpha - G_M\sigma_{\alpha\beta}q_\beta + G_A\gamma_\alpha\gamma_5 - G_Pq_\alpha\gamma_5) \times u_n\bar{u}_e\gamma_\alpha(1+\gamma_5)u_\nu. \tag{1}$$

Here G_V, G_A, G_P, and G_M are weakly varying functions of the momentum q transferred to the leptons; u are the wave functions of the fermions participating in the β decay process; γ_α, γ_5, and $\sigma_{\alpha\beta} = (1/2)(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$ are Dirac matrices. In the β decay of the neutron, the momentum transfer is small compared with the mass of the nucleons, and the quantities G_V, G_A, G_P, and G_M are in fact constants. To check on the theoretically predicted form of the amplitude and to determine the values of the constants it is necessary to compare the results of all the neutron experiments. The accuracy with which the angular correlation was measured was heretofore greatly inferior to the accuracy of experiments with polarized neutrons and to the accuracy with which the lifetime was determined. This is seen from Table I, which lists the ratio of the constants of the axial and vector variants of the interaction, obtained in different experiments. The purpose of the present paper was to greatly increase the accuracy of angular-correlation measurement.

The standard experimental scheme for measuring the angular $e\nu$ correlation consists of measuring the energy spectrum of the recoil nuclei; the form of this spectrum depends on the correlation constant λ.[5] However, this method is difficult to use for a free neutron, since the nuclear reactor and the open beam of thermal neutrons serve as powerful background sources. It is therefore necessary to resort to the coincidence-counting technique.

In this experiment we measured the energy spectrum of the recoil protons at a fixed decay-electron momentum. The spectrum of the recoil protons has in this case the following form:

$$dN=C\left(1+\lambda\frac{e}{E_e}\cos\theta_{e\nu}\right)dT_p, \tag{2}$$

where e is the electron momentum, E_e is its total energy, T_p the kinetic energy of the proton, $\theta_{e\nu}$ the angle between the emission directions of the electron and neutrino, C is a constant, and λ is the $e\nu$-correlation coefficient. We disregard here the contribution of the "weak magnetism" and of the effective pseudoscalar[6], and the proton energy is assumed to be small compared with the lepton energies. The quantity $\cos\theta_{e\nu}$ is connected linearly with the proton energy T_p:

$$\cos\theta_{e\nu}=\frac{2T_p-(T_{p\,max}+T_{p\,min})}{T_{p\,max}-T_{p\,min}}. \tag{3}$$

Thus, the spectrum of the protons on an energy scale turns out to be linear, and the slope of the spectrum depends on λ. The maximum and minimum values of $T_p = {}_p^2/2M_p$ (M_p – proton mass, p – its momentum) are determined from the relations

$$p_{max}=e+\nu=E_0-E_e+e\approx E_0-\frac{1}{2e},$$
$$p_{min}=|e-\nu|=|-E_0+E_e+e|. \tag{4}$$

Table I
Ratio $|G_A/G_V|$ obtained in different experiments

Measurement method	$\left\lvert\frac{G_A}{G_V}\right\rvert$	Data from
Lifetime of neutron	1.19±0.04	[1]
Electron-spin correlation in polarized-neutron decay	1.25±0.04	[2]
eν correlation in polarized-neutron decay	1.14±0.40	[3]
	0.89±0.25	[4]
	1.22±0.08	Present work

*Deceased