Optimization of the M13 Surface Muon Beam for Experiment TWIST, and its Injection into the Solenoid

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Abstract

This paper describes a new tune for beamline M13, which is used for optimum injection into a 22.5 kG solenoid. The depolarization is between 3.0 10^{-4} and 6.0 10^{-4} for intensities of a few thousand muons per second. The uncertainty in the depolarization is of the order of 1.0 10^{-4}, if the beam parameters before the solenoid are measured sufficiently close to the final focus of M13. This paper should be read in conjunction with reference [1], because it relies partly on the formulas and methods, developed there.

1 Introduction

An essential requirement for the beam is that in the solenoid the angle α of the muons with respect to the solenoid axis is very small. A recent report on the injection of muons into a solenoid [1] shows that it is possible to express α accurately as function of the beam parameters before the solenoid. The expression is

$$\alpha = \sqrt{\frac{1000}{2\rho Q}\left[\theta^2 + (\phi - Qx)^2\right]} \tag{1}$$

Here, ρ is the radius of curvature of a particle with momentum P that moves in a plane perpendicular to the direction of the constant homogeneous field B. It is given by

$$\rho(\mathrm{mm}) = 33.35641\frac{P(\mathrm{MeV/c})}{B(\mathrm{kG})} \tag{2}$$

θ and ϕ are the horizontal and vertical angles in the beam. Together with the coordinate x, they have to be evaluated at a specific longitudinal position z_{aim} in the solenoid, after a coordinate rotation that makes the vertical coordinate y equal to zero. Q is the aspect ratio of the optimally shaped phase space ellipse of the beam with emittance ϵ. Q and z_{aim} are determined by the shape of the fringe field. For an other fringe field those parameters will different. Q_{new} is the aspect ratio of the new optimally shaped phase ellipse. For the same emittance ϵ, every point of the new phase space corresponds with an equivalent point in the old phase phase space, in such a way that

$$x_{\text{new}} = \sqrt{\frac{Q}{Q_{\text{new}}}}x \tag{3}$$

$$\theta_{\text{new}} = \sqrt{\frac{Q_{\text{new}}}{Q}}\theta \tag{4}$$

$$\phi_{\text{new}} = \sqrt{\frac{Q_{\text{new}}}{Q}}\phi \tag{5}$$

Substitution in equation 1 with Q_{new} shows that α is the same for the corresponding points. Therefore, the full distribution of α is exactly the same for the two fringe fields. It turns out that a workable emittance for a beam entering the solenoid is about 80π mm.mr. For the magnetic field of the report [1]

$$Q = \frac{\theta_0}{x_0} = \frac{16.0 \text{ mr}}{5.0 \text{ mm}} = 3.2 \tag{6}$$

An other fringe field, for a solenoid without yoke, gave the optimal values $\theta_0 = 20.0$ mr and $x_0 = 4.0$ mm, giving $Q = 5.0$ for the same 80π mm.mr emittance. Most reasonable fringe fields give comparable values. All those fringe fields give the same distribution for α. The question becomes now, which emittance structure is easier to realize with M13.

2 New tune for M13

The normal M13 tune has focus positions F1, F2 and F3. At F1, after bending magnet B1, are horizontal and vertical foci, which are imaged by a quadrupole

triplet at F2, before B2. The final quadrupoles Q6 and Q7 give a double focus at F3, typically 1.0 m from the exit of Q7. There are momentum slits in the horizontal plane at F1 and F2. Formerly, there were also vertical slits at F1 and F2, which were removed a long time ago. As will become clear below, it is essential that vertical slits are again installed at F2. The function of the vertical slits is to restrict the vertical source size that is seen by the channel. The angle acceptance can be varied by the horizontal and vertical jaws before B1.

The coordinates at any longitudinal position in the beam can be expressed in first order as function of the coordinates at the production target, namely the positions x_0 and y_0, and the horizontal and vertical angles θ_0 and ϕ_0.

$$x = R_{11}x_0 + R_{12}\theta_0 + R_{16}\delta \tag{7}$$

$$\theta = R_{21}x_0 + R_{22}\theta_0 + R_{26}\delta \tag{8}$$

$$y = R_{33}y_0 + R_{34}\phi_0 \tag{9}$$

$$\phi = R_{43}y_0 + R_{44}\phi_0 \tag{10}$$

Here, R_{16} and R_{26} are the position and angle momentum dispersion. $R_{16} = R_{26} = 0$ before B1, and after B2. The beam is doubly achromatic, a condition that is necessary to keep the position and angle small at the final focus F3.

It is very important for TWIST that the beam size does not exceed ± 5 mm and that the angles are less than about ± 15 mr. Normally, M13 accepts vertical angles of ± 165 mr, and horizontal angles of ± 65 mr. The requirements for TWIST imply that the jaws have to be closed to a small aperture, in order to restrict the vertical angle acceptance to about ± 30 mr, and the horizontal acceptance to about ± 25 mr. Equation 9 shows that the vertical size at the jaws is determined by ϕ_0 and y_0. Under normal circumstances, the contribution of ϕ_0 dominates. However, for TWIST, due to the small ϕ_0, both contributions are about equal. Although a similar argument can be made for the horizontal plane, the situation is much worse in the vertical plane, because quadrupole Q1 is focussing in the vertical plane to make possible a vertical focus at F1. This results in a fairly small size at the jaws, even for the full angle acceptance. For TWIST, it becomes then very difficult to select the accepted vertical angle. In the horizontal plane, the angle acceptance is already much lower than the vertical angle acceptance. Furthermore, Q1 defocusses horizontally, increasing

Table 1
The difference in strength of the quadrupoles for the new tune with respect to the normal tune

quadrupole	difference in percent
Q1	-8.0
Q2	-1.7
Q3	-0.5
Q4	-12.4
Q5	-0.5
Q6	-1.5
Q7	-2.4

the horizontal size. Therefore, even though Q2 focusses at F1, the horizontal beam size is still sufficiently large for TWIST.

A new tune was made, in which there is not longer a vertical focus at F1, but only a horizontal focus. The symmetric triplet after F1 has two independent parameters, which are used to give a horizontal focus at F2, which is the mirror image of the focus at F1, and also make the beam doubly achromatic after B2. Quadrupole Q1 is now tuned to give a vertical focus at F2. Therefore, the vertical beam is now larger at the jaws, and it becomes easier to accurately select the vertical angle acceptance. Furthermore, a vertical slit at F2 restricts the accepted vertical source size, which helps further to reduce the contribution due to y_0 in equation 9 compared to the contribution due to ϕ_0. The new tune has one more special feature. Equation 10 shows that the final angle ϕ is due to $R_{43}y_0$ and $R_{44}\phi_0$. The first contribution is large, but that does not matter so much under normal operating conditions, when the final angle ϕ is large anyway. For TWIST, however, this is a serious drawback, that can only be partly overcome by restricting the beam intensity even more. Fortunately, in the case of the new tune, it turned out to be possible to make $R_{43} = 0$. Therefore, the influence of the vertical source size on the final angle ϕ, should be small. Table 1 list the differences in strength of the quadrupoles for the new tune, compared to the normal tune. The main difference is for the vertically focussing quadrupoles Q1 and Q4. The reason for the lower Q1 is that there is now no vertical focus at F1. The lower strength of Q4 makes the matrix element R_{43} in equation 10 small.

Table 2
Full slit apertures for 6,000 and 2,000 muons per second

intensity	6,000 per second	2,000 per second
slit	full slit width	full slit width
	(mm)	(mm)
horizontal jaws	82	62.4
vertical jaws	18	13.6
momentum slit at F1	12	12
vertical slit at F2	28	21.3
effective target length	72	54.8

3 Function and use of jaws and slits

Due to the cylindrical symmetry of the solenoid, the optimal beam entering the solenoid has to be rotationally symmetric. The horizontal and vertical beam profiles and angle distributions at the final focus of M13 have to be the same. The vertical angle and the vertical beam size can be controlled independently of each other, by the vertical jaws, and by the vertical slit at F2. The horizontal angle is determined by the horizontal jaws. The horizontal slit af F1 selects the momentum bite. However, there is no device to influence the horizontal beam size, which therefore can only be varied by varying the length of the production target.

Table 2 shows the slit settings for 6,000 and 2,000 muons per second. Note that the momentum slit has been kept the same, because the momentum bite has been kept the same. It may, however, be advantageous from the point of aberrations, to reduce the momentum bite. This is discussed in the next section.

The intensity can in principle be varied by changing just one of the slits. However, it is important to keep the horizontal and vertical beam profiles the same. Simultaneously, the shape of the phase ellipses has to be kept constant. That means, changing the vertical jaws and the vertical slit at F2 by the same fraction. The horizontal jaws are changed by the same fraction. Since it is only possible to change the horizontal size, by changing the length of the production target, the target length has also to change in the same way. In other words, the slits have to be changed proportional to $(I/I_0)^{0.25}$, where I_0 is the initial intensity and I the required intensity. This is done in table 2. The normal intensity of M13 is about 200,000 muons per second in a 1.0 percent momentum bite. Experiment TWIST needs only a few thousand muons per second. Due to the closing of the slits, the widths of the final spot and the

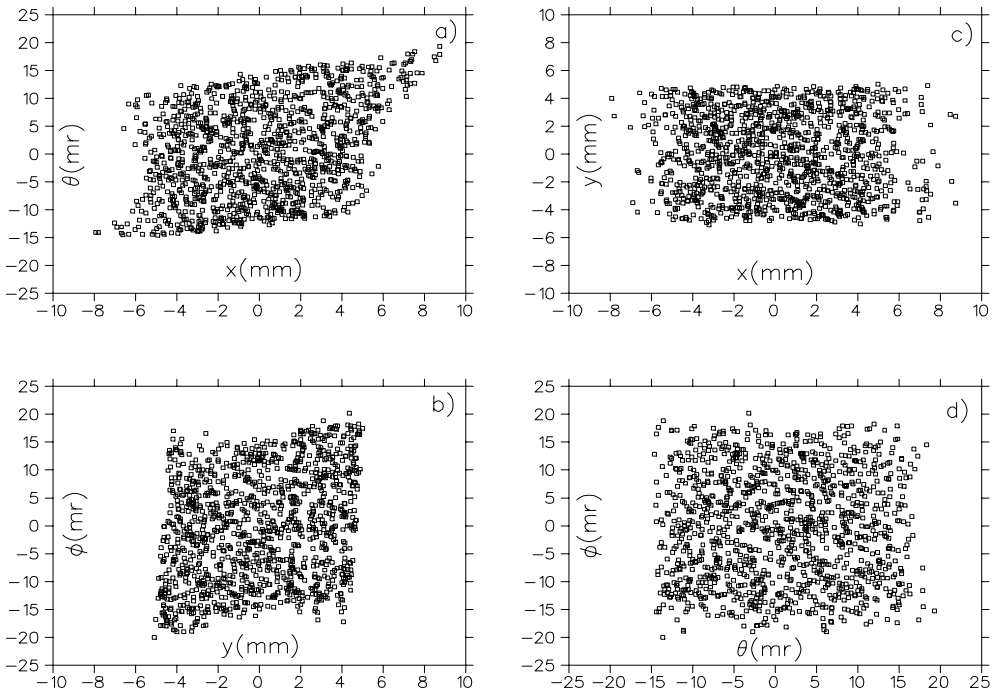

Fig. 1. The horizontal $x - \theta$ and $y - \phi$ phase space for the case of 6K muons per second

angle distribution are smaller than for the full intensity. Therefore, the slits make possible a fairly large range of spot sizes and divergences. The present tune has a focus 1.0 m downstream of Q7. A tune that focussed the beam 1.2 to 1.5 m downstream would also be acceptable. For the same slit apertures, the spot sizes would be larger, and the divergences would be smaller. However, by choosing different apertures, the same final beam characteristics can be obtained as for the focus at 1.0 m. On the basis of the same reasoning, it is not necessary to optimize the fringe field of the solenoid, because the requirements for the beam can always be fulfilled by a judicious choice of the slit apertures and the target length.

4 Tracing of M13 beam into solenoid

The slits, jaws and target length were chosen as in table 2 for the case of 6K and 2K muons per second. The resulting phase space at the final M13 focus can be seen in figure 1 for the 6K case, and in figure 2 for the 2K case.

The $x-\theta$ and $y-\phi$ phase spaces are very similar, namely ± 5 mm by ± 16 mr for the 6K case and ± 4 mm by ± 12 mr for the 2K case. The beam is aimed in such a way that the focus occurs at 153 cm upstream of the center of the solenoid when the field is turned off, as is indicated in figure 2 of reference [1].

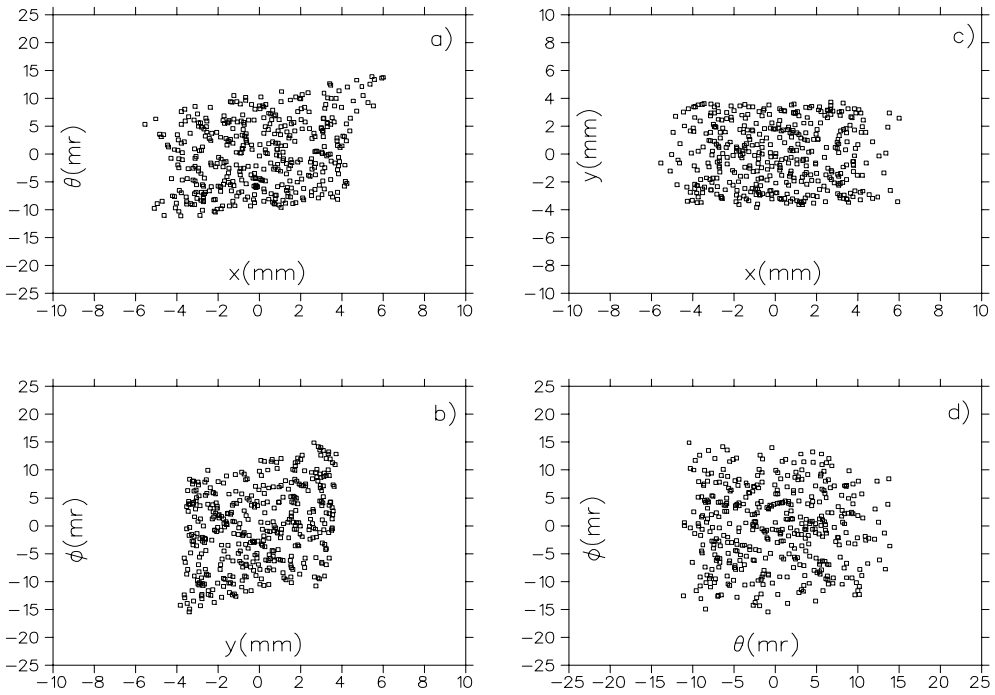

Fig. 2. The horizontal $x - \theta$ and $y - \phi$ phase space for the case of 2K muons per second

Figures 3 and 4 show α and the depolarization, $1 - \cos\alpha$, as indicated. The angle α was obtained from a direct raytracing to the center of the solenoid. α was also calculated using equation 1, following the procedure outlined in section 7 of reference [1]. The difference dα is quite small, and is due mainly to the momentum dependence, as can be seen in figures 3c and 4c, where dα is plotted against dp/p. The average depolarization is 6.52 10^{-4} for the 6K case and 3.72 10^{-4} for the 2K case.

5 The depolarization

The depolarization was calculated as function of the beam intensity. The intensity was varied according to the procedure, outlined in section 3, varying the slit widths and the target length as $(I/I_0)^{0.25}$, where I is the desired intensity and I_0 is a reference intensity. Figure 5a shows the depolarization as function of the beam intensity. Figure 5b shows the sensitivity to a change of 1.0 percent in the strength of each quadrupole. The points at zero on the horizontal axis show the results for the optimal quadrupole settings.

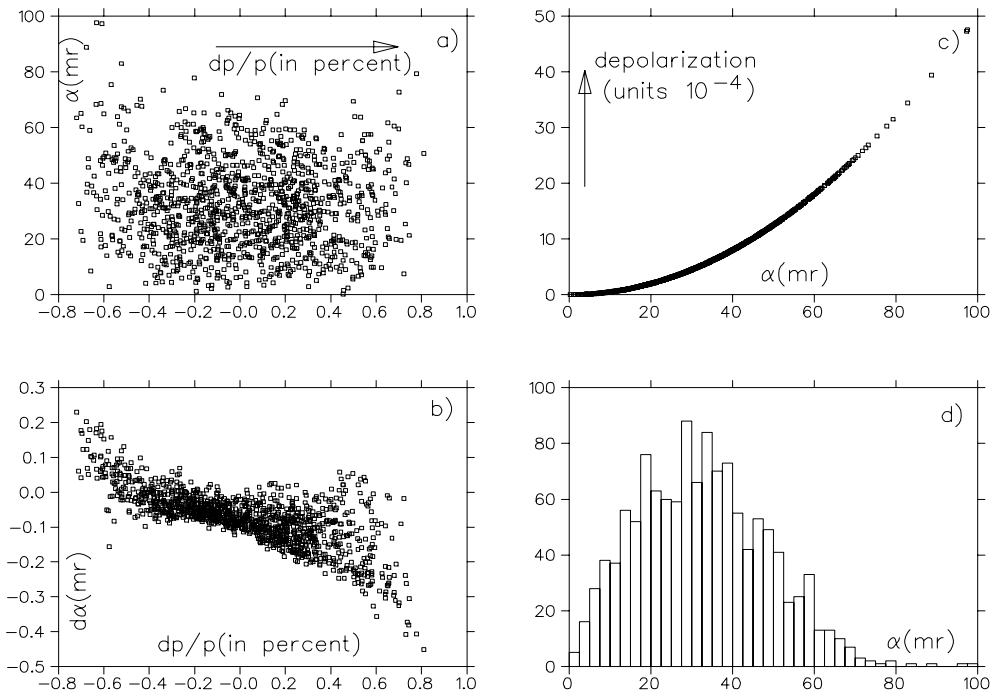

Fig. 3. The case of 6K muons per second. Figure a shows α as function of dp/p. Figure b shows the difference in α as traced directly with the calculation of α from equation 1. Figure c shows the depolarization as function of α. Figure d shows the distribution of α.

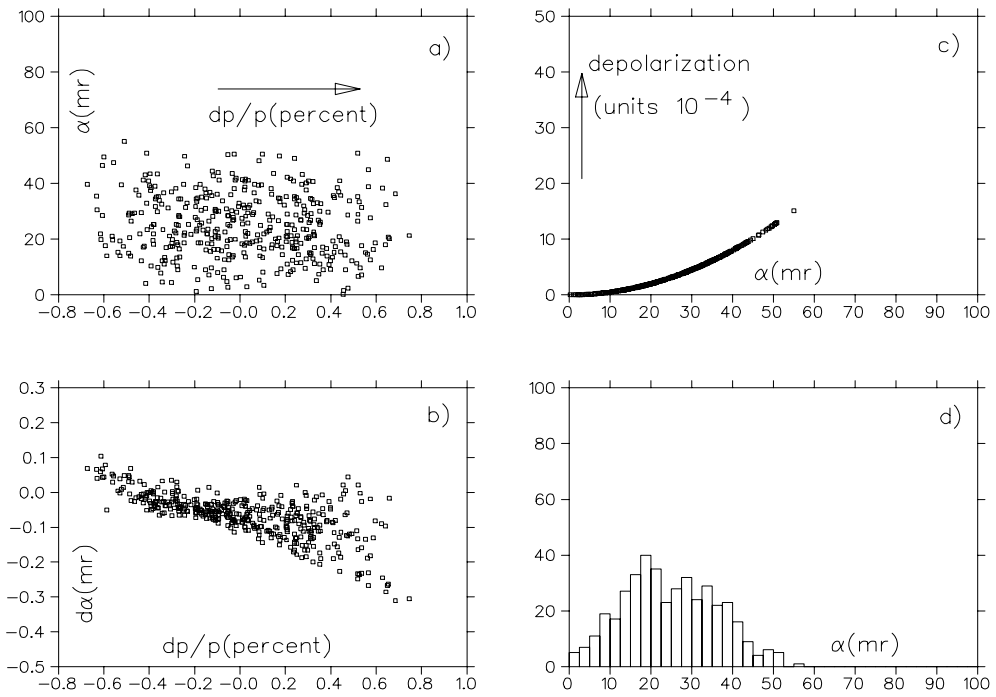

Fig. 4. The case of 2K muons per second. See further the caption of figure 3.

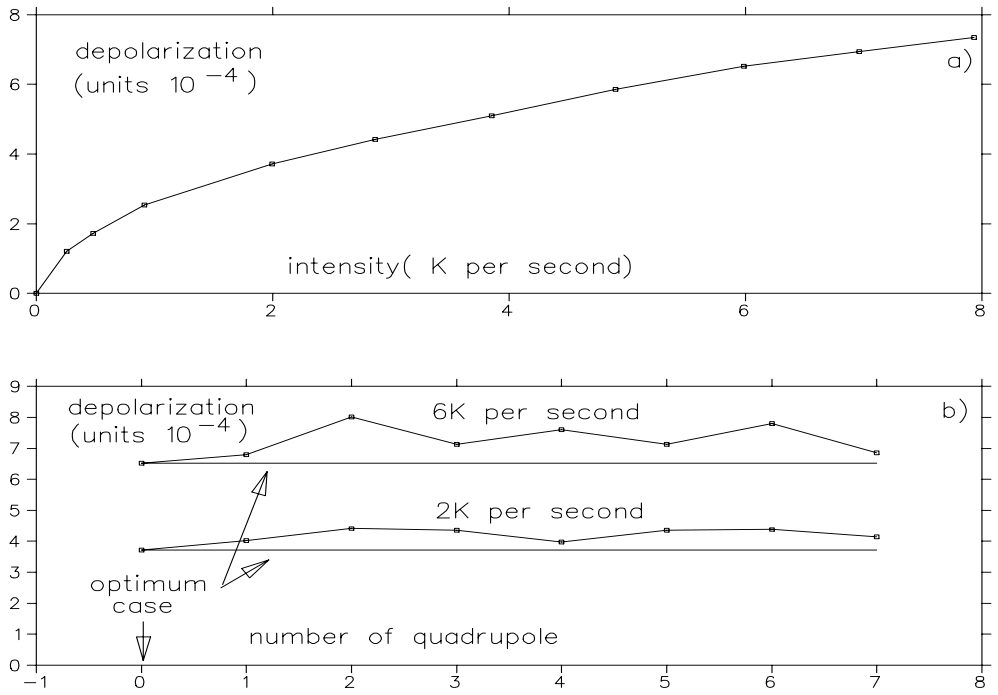

Fig. 5. Figure a shows the depolarization as function of the beam intensity. Figure b shows the sensitivity, namely the change in depolarization if the strength of each quadrupole in turn is changed by 1 percent.

6 The uncertainty in the depolarization as function of the uncertainty in the beam parameters before the solenoid

Equation 1 can be used to quickly determine the effect that uncertainties in the measurement of the beam parameters before the solenoid have on the uncertainty in the value of the depolarization inside the solenoid. Tracing the M13 beam directly into the solenoid gives the distribution for α, and therefore the distribution in the depolarization as well as the average depolarization. To estimate the effect of the uncertainties, the following procedure was used. The M13 beam was traced to the final focus at position $z_{\rm aim}$ in the solenoid. Then the beam was traced back 0.5 m or 1.0 m, the θ and ϕ coordinates were changed randomly with an amount chosen from a uniform distribution between -5 mr and +5 mr. Similarly, x and y were changed by a random amount between -0.5 mm and +0.5 mm. Then equation 1 was used in the way, given in section 7 of reference [1], to determine a new α distribution in the solenoid, from which the average depolarization could be determined. This was done for 6K and 2K muons per second. The uncertainty of $\pm$5 mr is large compared to the divergence of $\pm$10 to $\pm$15 mr in the beam. If the measurements are assumed to be done 1.0 m upstream, the resulting uncertainty in the x and y coordinates at the focus are large, namely up to $\pm$5 mm, which is equal to the beam size. The result is that the correlation between the original α and the folded

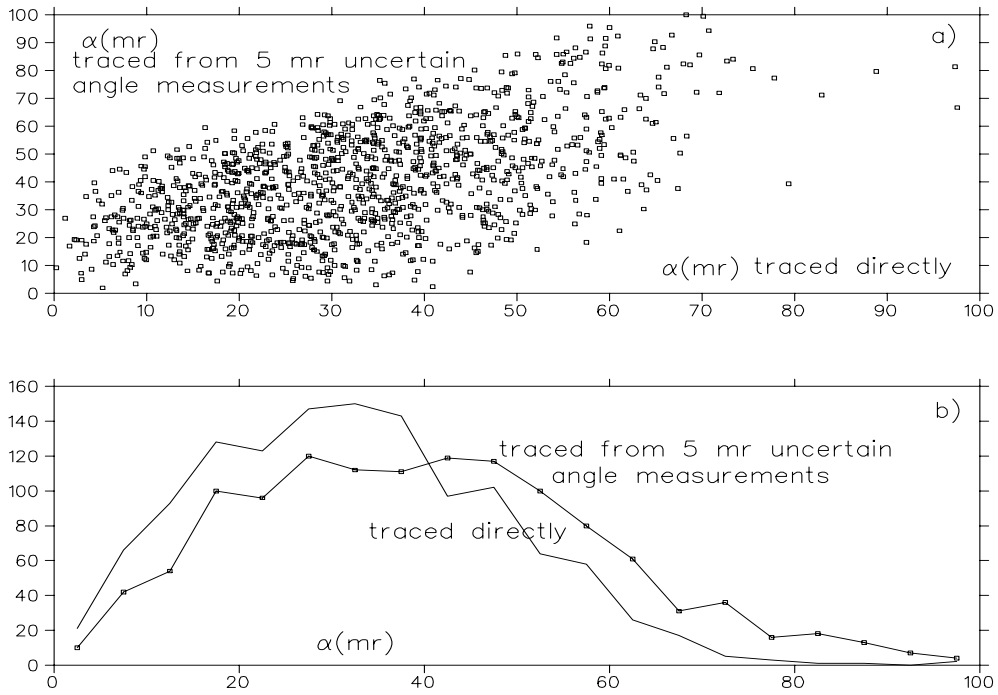

Fig. 6. The 6K case. The beam parameters are assumed to be measured 1.0 m upstream of the focus. Figure a shows the correlation between the directly traced α and the α from the folding with an uncertainty in the measured beam parameters. Figure b shows the α distributions for the two cases

Table 3
Average depolarization for different measurement situations. See text

intensity per second	upstream drift (m)	depol direct 10^{-4}	depol folded 10^{-4}	depol increase 10^{-4}
6K	1.0	6.52	9.75	3.23
6k	0.5	6.52	7.71	1.19
2k	1.0	3.72	7.39	3.67
2k	0.5	3.72	4.84	1.12

α is quite poor. This is illustrated in figure 6a for the 6K case. The folded α distribution in figure 6b is much wider than the original α distribution. The average depolarization is also much larger. Table 3 gives the average depolarization for the folded and original α distributions for the 6K and 2K case, for measurements either 1.0 m or 0.5 m upstream of the M13 focus.

Figures 7, 8 and 9 show the results for the other three cases.

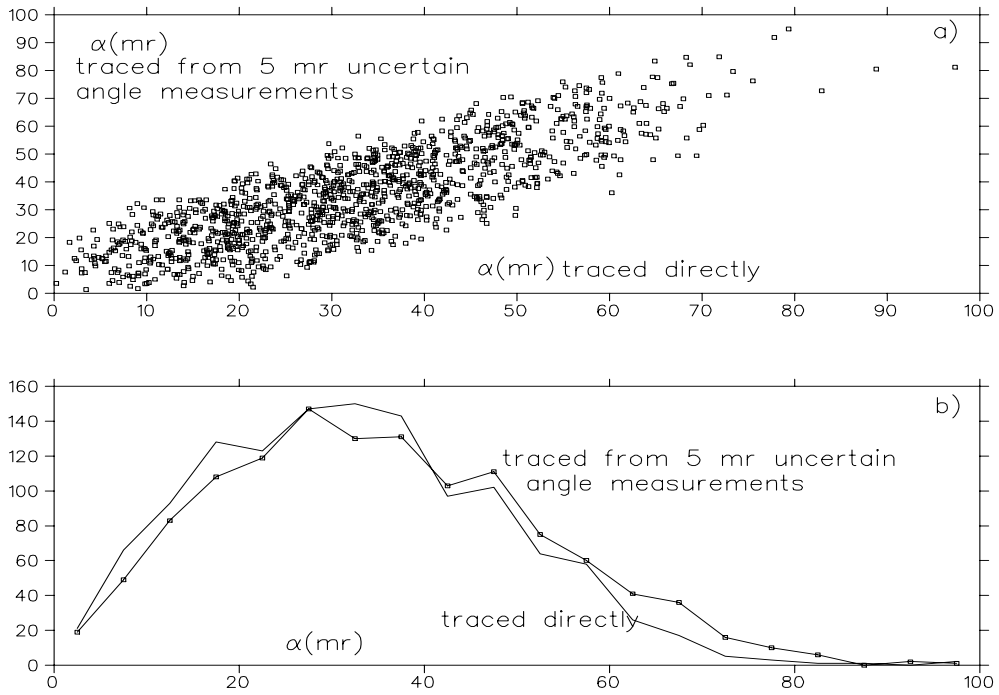

Fig. 7. The 6K case. The beam parameters are assumed to be measured 0.5 m upstream of the focus. See also the caption of figure 6.

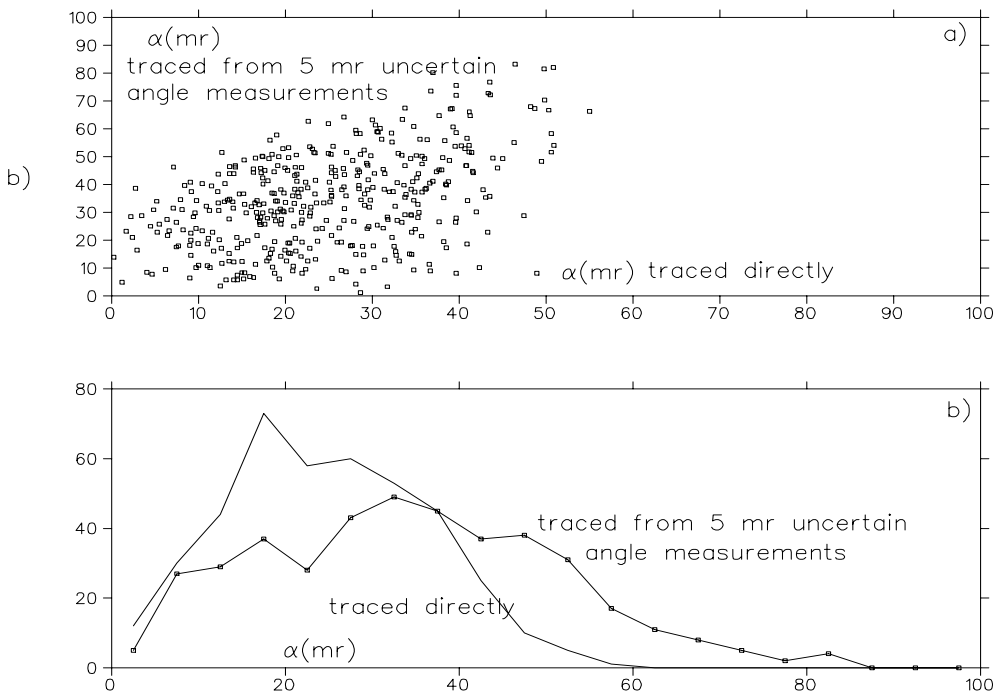

Fig. 8. The 2K case. The beam parameters are assumed to be measured 1.0 m upstream of the focus. See also the caption of figure 6.

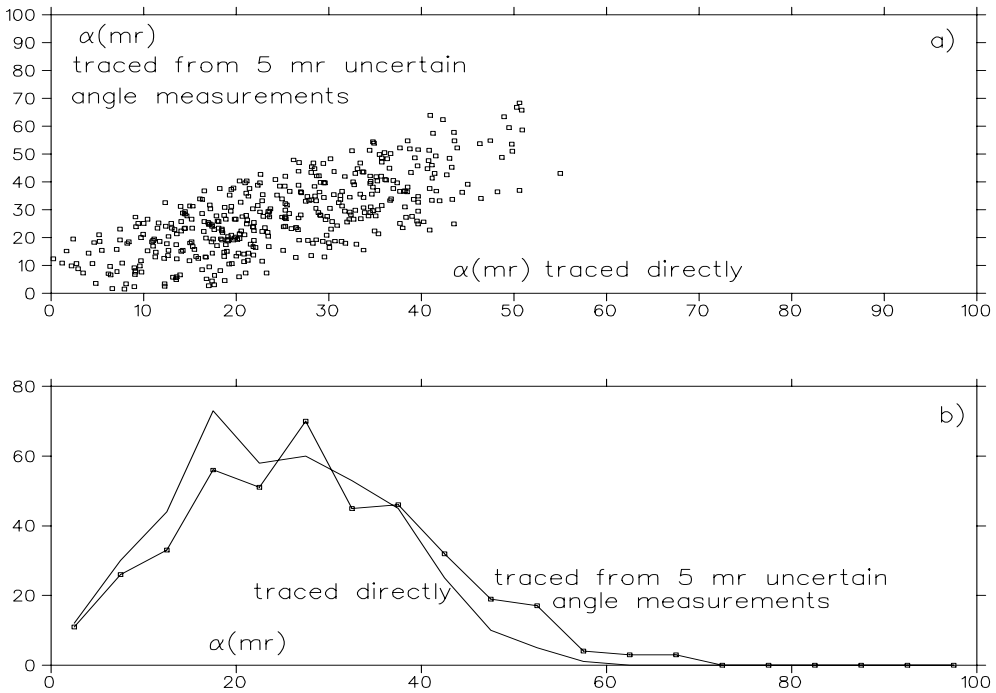

Fig. 9. The 2K case. The beam parameters are assumed to be measured 0.5 m upstream of the focus. See also the caption of figure 6.

7 Summary and conclusion

A new tune was made for beamline M13, which maximizes the useability of slits and jaws, and minimizes disturbing effects contributing to the size of the beam spot at the final focus. Settings were found for slits and jaws, and for the production target length, which give equal horizontal and vertical phase spaces, and optimize the shape of the phase space ellipses to make the ellipse aspect ratio equal to the optimum aspect ratio, required by the specific fringe field of the 22.5 kG solenoid. The depolarization was calculated as function of the beam intensity and is shown in figure 5a for intensities between zero and 8K muons per second in a 1.0 percent FWHM momentum bite.

The formulas of reference [1] for determining the beam parameters in the solenoid from measured parameters before the solenoid are very useful. In particular, the formula for α, reproduced in equation 1, was used in this paper to estimate the uncertainty in the depolarization as function of the uncertainty in the beam parameters, measured at distances of 1.0 m and 0.5 m before the final focus of M13. If the uncertainty in the measurements of the angles is ± 5 mr, this is big compared to the beam divergence of ± 15 mr. Therefore, the correlation between the real α in the solenoid, and the α, inferred from the measurements, is weak. If the measurements are made 1.0 m upstream of the focus, this will result in a very uncertain depolarization. If the measurements

are made 0.5 m upstream, the uncertainty in the depolarization will be of the order of 1.0 10^{-4}.

The conclusion is that a tune has been developed, which gives, with appropriate settings of slits and jaws, and the length of the production target, useable beam intensities of a few thousand muons per second with a depolarization between 3.0 $10^{-4}$and 6.0 10^{-4}. The uncertainty in the depolarization can be made of the order of 1.0 10^{-4}, if the beam parameters before the solenoid are measured sufficiently close to the final focus of M13.

References

[1] Optics Considerations for the Injection of a 29.8 MeV/c Surface Muon Beam into a Solenoid, Jaap Doornbos, TRIUMF, July 26, 2000. http://www.triumf.ca/people/doornbos/homepage.html