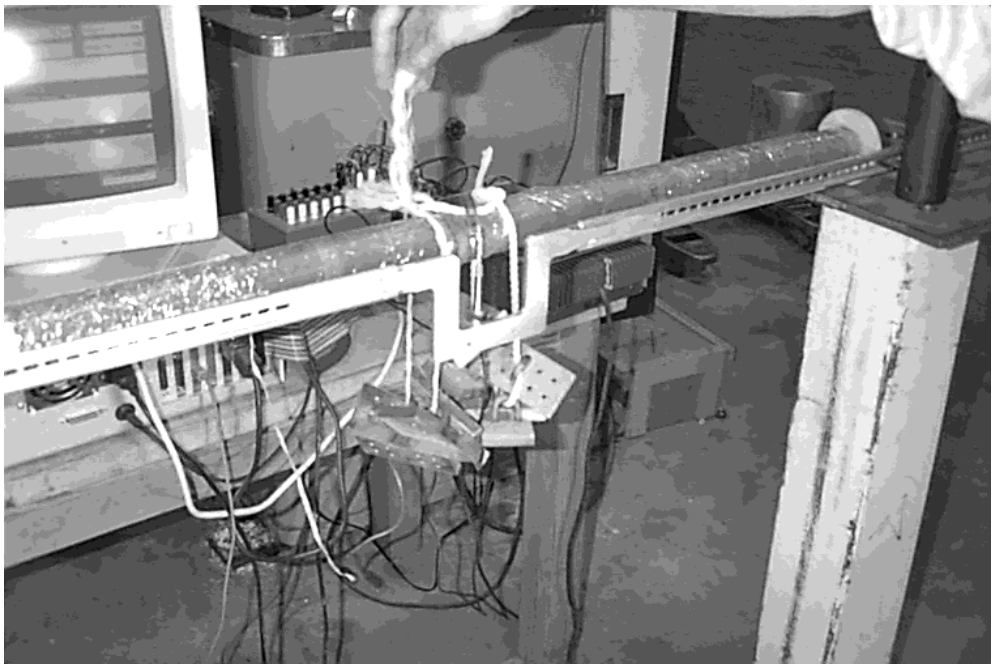
DESIGN PROPERTIES OF CITAL CO-115M FOR USE IN E614 DETECTOR SUPPORT STRUCTURE



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EXECUTIVE SUMMARY

Tests and analyses were carried out at the Civil Engineering Department of the University of British Columbia to predict the behaviour of the proposed E614 detector support structure design. Material properties of CITAL CO-115M were determined and analytical predictions of support structure behaviour were performed.

Experimental work determined the following material properties of CITAL CO-115M:
1) the elastic modulus was found to be 57.0 GPa; and
2) the maximum strength in compression was found to be about 900 MPa.

For a brittle material, such as CITAL CO-115M, the compression strength results are dependent on the load apparatus and specimen dimensions. A certain amount of variation in strength is to be expected depending on the particular application. For the expected range of operating loads, however, the maximum strength is well above the design stress maximums.

Friction between finished surfaces of adjacent segments of CITAL was not determined. Experiments aimed at determining load-slip behaviour of the friction surfaces resulted in fracture of the segments without ever exhibiting slip between adjacent segments.

Computer analysis for a 1.06 m long single beam predicts behaviour for two extreme end-support conditions: *simply-supported* and *fixed-fixed*. Equations based on the analysis are provided for determining the minimum requirements for combinations of axial stress and load per 4 mm CITAL segment. Also reported are prediction equations for maximum mid-span deflection for a variety of load cases.

The expected range of stresses and mid-span deflection for the 1.06 m long four-beam structure carrying 4 kg per load plane are:

	Simply-Supported	Fixed-Fixed
Minimum Required Axial Stress (MPa)	0.55 – 34.0	0.35 – 23.0
Mid-span Deflection (μm)	39 – 2500	0.8 - 500

The large range is due to the unknown amount of rigidity provided by the G10 plates which link the four beams together. If there is no rigidity, then each beam behaves independently and the higher values result. The best case, therefore, is the fixed-fixed condition with very rigid links between the beams. Although the minimum axial stress is quite low for the ‘best case,’ it is recommended that at least 20 MPa be provided.

Design equations are provided and further recommendations are made.

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1. Introduction

A test programme was carried out to determine the elastic modulus and strength properties of CITAL CO-115M ceramic/glass segments. These values are required for the final design of the detector support-structure.

The CITAL CO-115M pieces are hollow cylindrical segments and are available in three (3) lengths: 4, 20 and 40 mm as shown in Figure 1. The segments will be aligned, end-to-end, and compressed to form one long beam, as shown in the sketch in Figure 2.

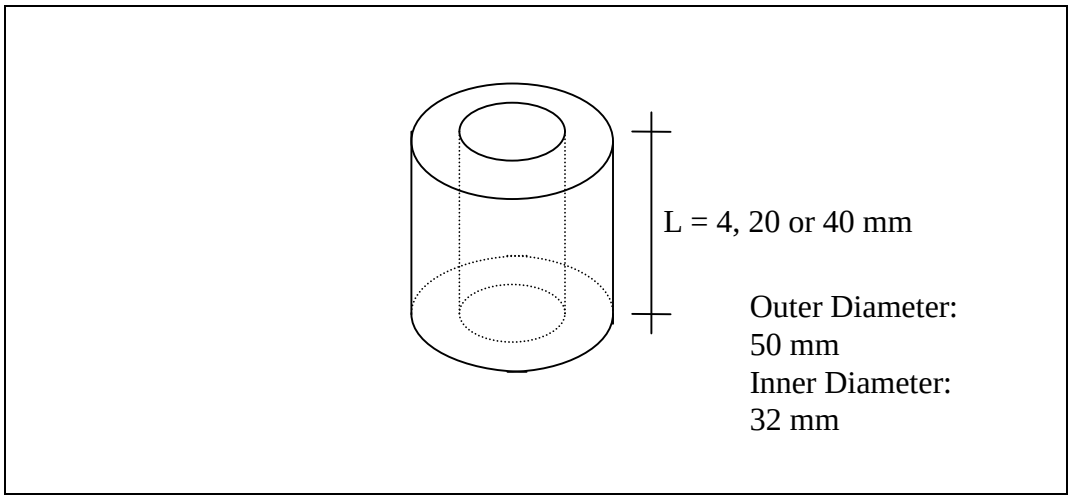


Figure 1. CITAL CO-115M Segment.

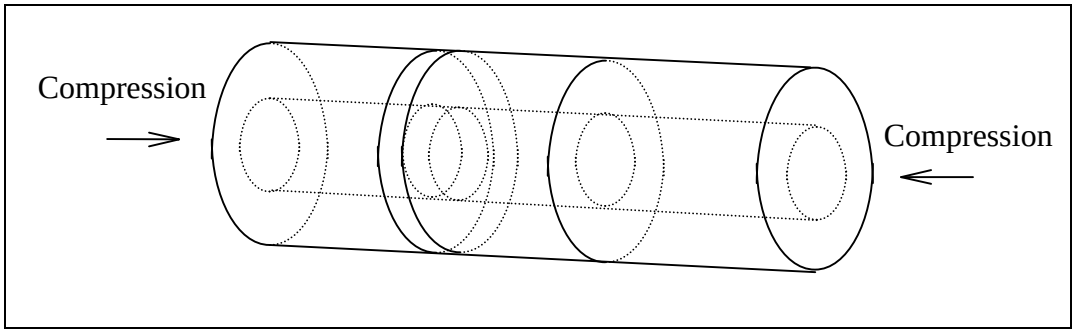


Figure 2. Sample Single Beam.

Four of these long beams will be used to form the structural frame of the E614 Detector at the TRIUMF facility in Vancouver. They will be arranged in the pattern shown in Figure 3. The distance, R, shown in the figure is 0.260 m. For the single beam and the group of four beams, an analysis can be performed

assuming *fixed-fixed* support at the ends or *simple-support* at the ends, as will be discussed in sections 1.2 and 4.

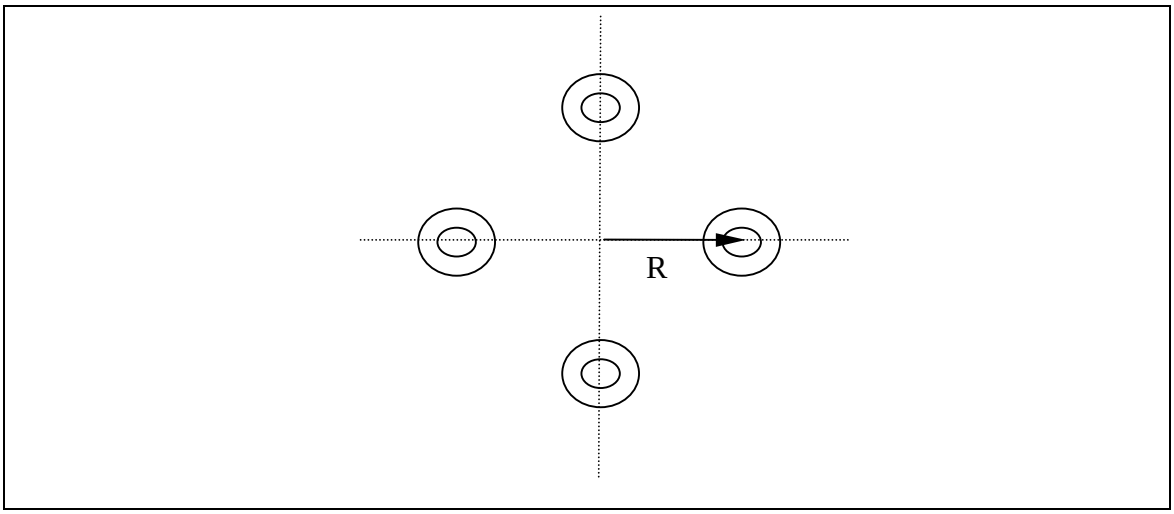


Figure 3. Cross-section of Detector Support-Structure

1.1. Loading

Loads will only be placed on the 4 mm rings. It is estimated that the load per ring will be between 1 kg to 5 kg. There will be seventy-five (75) 4 mm rings distributed along the length of the beam, for a total load of between 75 and 375 kg. Thus, the worst case of shear load between segments occurs near the ends of the beam where approximately half of the entire load is transmitted to the supports. If the load per ring is 5 kg, then the shear load at the end rings is in the range of 200 kg.

The total load per ring is, however, distributed between the four supporting beams. If it is assumed that the load is equally distributed between the beams, then the load per ring per beam only ranges from 0.25 to 1.25 kg, giving a maximum shear load at the end rings of approximately 50 kg.

1.2. Basic Beam Theory

Beams, such as those shown in Figure 4, where support is provided at each end to resist lateral forces, will deflect an amount which is dependent on the type of constraint at the end supports. The *simply-supported* case, shown in the left hand figure, is defined as allowing rotations to occur at each end. This is, as its name implies, a simple case. It is the easiest to build since the beam need only rest on top of supports as the supports resist movement in the vertical direction.

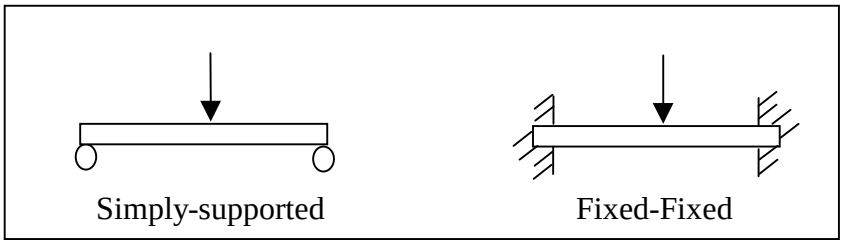


Figure 4. Beam Support Conditions.

A more rigid configuration is the *fixed-fixed* support condition. In this case, each end is 'fixed' against rotations, as well as displacements in the horizontal and vertical directions. Because of the increased rigidity, a fixed-fixed beam will deflect less than one which is simply-supported. The amount of deflection depends on:

1) the magnitude and location of the applied forces;
2) the type of material from which the beam is fabricated; and
3) the geometry of the beam (length and cross-sectional properties).

For the simply-supported beam, with one load (force) at centre span, the maximum mid-span deflection (i.e. the point directly under the load), is:

[1] $$\Delta_{max} = \frac{PL^3}{48EI}$$

where:

Δ_{max} is the maximum vertical displacement (mid-span) in metres,
P is the vertical force in Newtons,
L is the total length of the beam in metres,
E is the modulus of elasticity of the beam material in Pascals, and
I is the moment of inertia (second moment of area) in metre^4, see section 1.2.2.

Equation [1] is modified by a constant for the fixed-fixed case and becomes:

[2] $$\Delta_{max} = \frac{1}{192}\frac{PL^3}{EI}$$

If a load is continuously distributed over the length of the beam, such that load W, expressed in Newtons per metres is applied, then equation [1] becomes equation [3] for a simply-supported beam:

[3] $$\Delta_{max} = \frac{5}{384}\frac{WL^4}{EI}$$

and [2] becomes [4] for a fixed-fixed beam:

[4] $$\Delta_{max} = \frac{1}{384}\frac{WL^4}{EI}$$

Again, the mid-span deflection is dependent on the position and number of lateral loads, P or W. For loading situation where many equal loads are spread over the length of a beam, the maximum deflection tends towards equations [3] and [4]. Note: Only when loads are distributed symmetrically about the mid-span point (L/2) does the maximum deflection occur at L/2. For example, if all forces are on one side of mid-span, the maximum deflection will occur at a point other than the mid-span.

1.2.1. Principle of Superposition

An interesting property is the linear increase in deflection due to the addition of loads along a beam. If one places a single force at a point 1/3 of the beam length from one end of the beam, as shown in Figure 5, a deflection at mid-span will result. If a second load of equal magnitude is placed at 2/3 of the distance from the same end, then the mid-span deflection will be double. This is the *principle of superposition*. The resulting effect of several loads acting on a member simultaneously is the sum of the contributions from each of the loads applied individually.

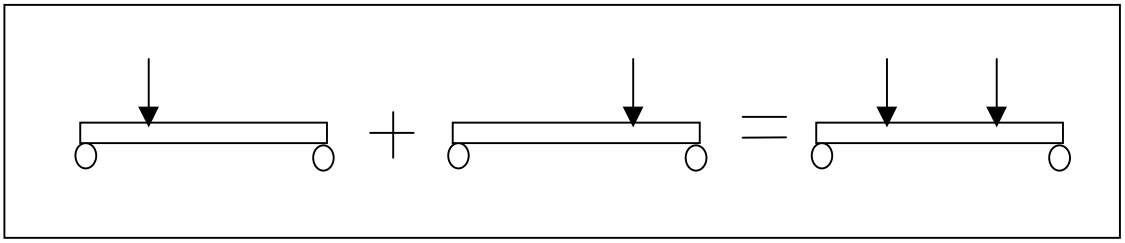


Figure 5. Principle of Superposition: Total Deflection Summation.

This principle can be used to create graphs, known as *influence line diagrams*, which indicate the deflection at mid-span due to a load at any given point along the beam. Figure 6 shows a sample influence diagram for some beam. If the deflection at mid-span is required for the case when a unit load is applied to point A, then the value of deflection at point A can be read from the figure. The same holds for a unit load at point B. If the two loads are in place simultaneously, the total mid-span deflection is the summation of the two individual deflections. The diagram is based on unit loads (1 Newton). If the load is a value which differs from 1, the deflection values from the diagram can be multiplied by the real load level since deflections vary linearly with load. This can be described mathematically, as will be performed in section 3.6.4.

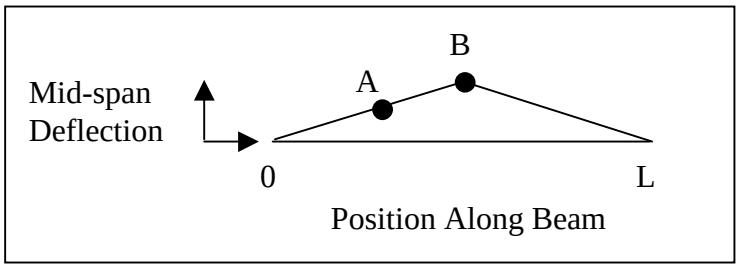


Figure 6. Example Influence Line Diagram.

1.2.2. Moment of Inertia, I

The value of I depends on the geometry of the cross-section. The inner radius, r_i, of one CITAL segment is 0.015 m and the outer radius, r_o, is 0.025 m. The major radius, R, is 0.26 m. These are shown in various configurations in Appendix F. Note that there is not much change in I if the four-beam structure is rotated 45°. The deflections are inversely proportional to I, as indicated in equations [1] through [4].

The four figures in Appendix F each show a different beam geometry. Only F.1 and F.4 will be considered here. Figure F.1 shows the single beam. This is the simplest case. Figure F.4 shows the four-beam configuration. The value of I for Figure F.4 assumes that there is a rigid connection between all of the members so that the radius, R, remains constant throughout loading. If this is not the case, then each of the four beams acts individually, thus having the same moment of inertia as Figure F.1.

The benefit of using the four beam structure is two-fold. First, it reduces the load which must be carried by each beam, and second, it increases the moment of inertia by a factor of more than 600. This increase helps to reduce deflections. Since deflections are inversely proportional to I, deflections will be reduced by a factor of 600. Again, however, this is only possible if the four beams are rigidly linked to one another.

For the detector, the four-beam structure will approach the rigid connection condition, allowing the use of the higher I value, however, the real value will be somewhere between the two cases, F.1 and F.4.

2. Objectives

2.1. Material Properties

The following material properties of CITAL CO-115M are required:

1) Elastic modulus;
2) Ultimate strength;
3) Load-rate sensitivity; and
4) Shear load resistance.

2.2. Analysis

The following parameters must be calculated for the single-beam and four-beam set-ups:

5) Maximum compressive stress;
6) Maximum deflection; and
7) Maximum shear load.

Items 5) and 7) will be compared to items 2) and 4), respectively, to determine if the proposed structure exceeds the material capacities. Item 6) is a serviceability requirement which must be within the client's acceptable limits.

3. Material Properties - Experiments

The following tests were carried out in the Structural Engineering Laboratory at the University of British Columbia.

1) Elastic modulus of CITAL in compression;
2) Time-dependency of CITAL in compression
3) Load-slip behaviour of CITAL;
4) Ultimate strength of CITAL;
5) Load-displacement of G10; and
6) Beam deflection of a single CITAL beam.

The set-up and results of each test will now be discussed.

3.1. Elastic Modulus

3.1.1. Apparatus

One 40 mm long segment was fitted with strain gauges. The gauges were mounted on the outer surface and were oriented to measure strain along the length of the segment. The gauges formed a half-bridge and with a scale factor of 1475.797 με/V (micro-strain per Volt), as shown in Appendix G.

A Baldwin loading frame was used. The maximum capacity of its load cell is 1.8 MN; the full scale was adjusted to suit the scale of measurement required for these tests.

The data acquisition system was PC-based. The acquisition rate was 2 Hz. Two channels were recorded: load and strain.

3.1.2. Specimen

A stack of five (5) segments was tested. The segment with the strain gauges was placed in the middle of the stack. The specimen was pre-assembled and surrounded in plastic wrapping to prevent dust from entering between the segments. The specimen was placed between two pieces of G10 material to prevent scratching of the ends. It is shown in Figure 7.

Figure 7. Compression specimen.

3.1.3. Results

A total of seven (7) tests were performed. Data was recorded during loading and unloading to determine if any permanent deformation occurred. The maximum load reached was 37.1 kN (32.0 MPa) and did not produce any failures. The results are summarised in Table 1.

Table 1. Compression test results.

Summary Information
Compression Specimens

Sample Name	Elastic Modulus (GPa)		Loading Rate (kPa/s)	
	Loading	Unloading	Loading	Unloading
TR1	56.5	55.0	66	59
TR2	56.2	55.4	99	85
TR3	58.3	62.9	290	463
TR4	59.7	64.3	380	303
TR5	58.6	67.1	602	458
TR6	55.5	55.1	189	1062
TR7	54.2	56.1	207	1769
Average	57.0	59.4		
S.D.	1.9	5.2		
C.O.V. (%)	3.4	8.7		

Load displacement curves are available in Appendix A. It was found that no significant difference in loading and unloading behaviour occurred and that there was no apparent permanent deformation of the specimen at this level of loading. Thus, behaviour was elastic and the value of elastic modulus, E, is 57.0 GPa.

Using this information, one can predict the amount of shortening in μm which occurs under axial stress. The shortening over a 1.06 m length (the full length of the CITAL beams in the detector) is:

[5] $$\Delta = \sigma \frac{L}{E}$$

where:

Δ is the amount of shortening in metres,
σ is the axial stress in the beam in Pascals,
L is the length of the beam in metres (1.06 m), and
E is the modulus of elasticity in Pascals (57×10^9 Pa)

For example, if the axial stress is 20 MPa, the shortening of the beam is equal to: $(20 \times 10^6)(1.06)/ 57 \times 10^9) = 372$ μm.

Equation [5] can be re-written in terms of mass, m, in kilograms (applied **axially**, not laterally) if one uses the cross-sectional area of a segment, A=1.963 x 10^{-3} m^2:

$$[6] \qquad \Delta = \frac{mgL}{AE} = \frac{m(9.81\frac{metre}{second^2})1.06}{1.963x10^{-3} \cdot 57x10^{9}} \cdot 10^{6} = 0.0929 \cdot m$$

where: Δ is displacement in μm, and
m is load in kilograms.

3.2. Load Rate Dependency

The results of the compression tests were also used to determine if the rate of loading had any effect on the value of elastic modulus. This test does not conclude if long term displacements under constant load (creep) occur in CITAL.

3.2.1. Results

The results are shown in Table 1 and in graphs in Appendix B. The data from the table (excluding outliers) are plotted in Figure 8 and showed only slight sensitivity to rate of loading in the range of loads and rates observed.

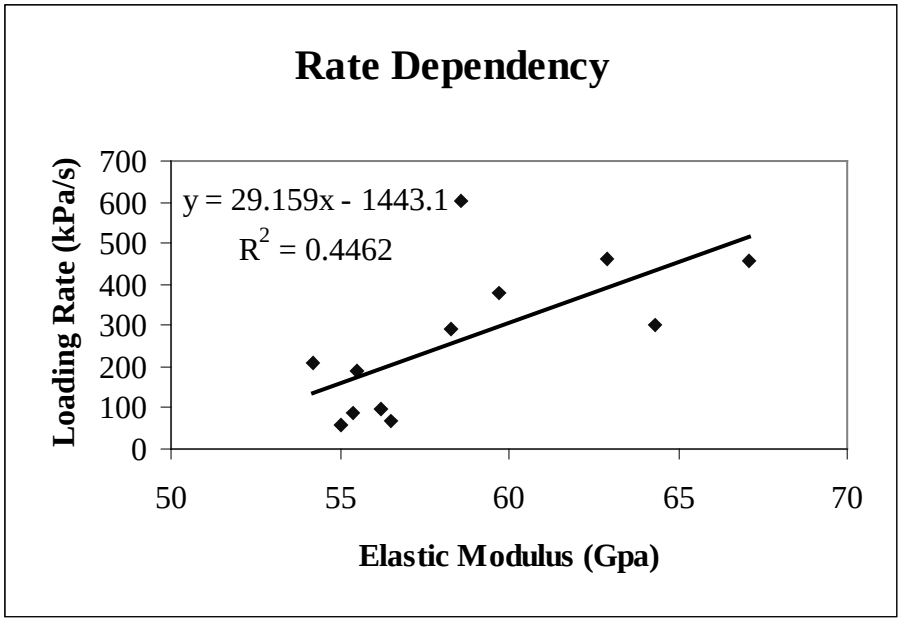


Figure 8. Load-rate dependency.

3.3. Load-slip Behaviour

3.3.1. Apparatus

The same apparatus described in 3.1.1 was used. In addition, a linearly varying displacement transducer (LVDT) was used to determine displacements of one of the CITAL segments. The LVDT was calibrated to 2.1416 mm/V. The data acquisition system was used to record three channels: load, displacement and strain.

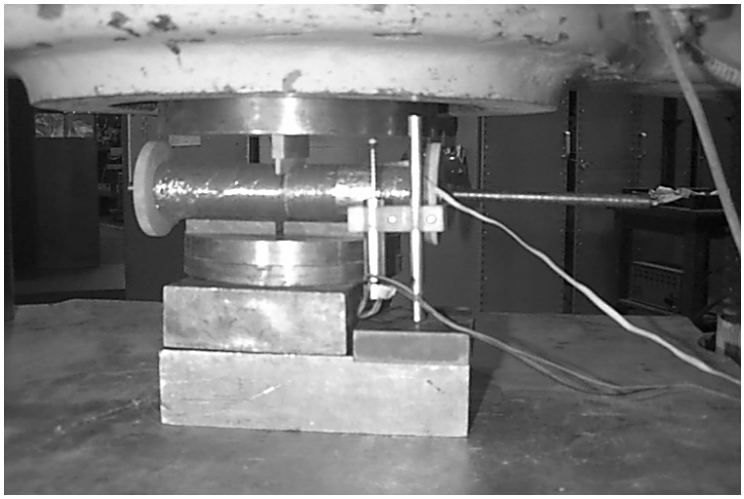
3.3.2. Specimen

A stack of nine (9) segments was made. The stack was formed with three 40 mm long segments at each end and three 4 mm segments in the middle. The stack was pre-assembled and covered in plastic wrapping. A threaded steel rod was placed through the stack and end plates made from G10 were tightened in place with steel nuts. The tension in the steel rod (compression in the CITAL segments) was measured using a torque wrench and using the output from the strain gauges (using the same instrumented segment described in section 3.1.1). It was found that the rod was too weak and needed to be replaced with a higher diameter rod to prevent yielding. A number of levels of axial stress were tested.

The specimen was placed horizontally on supports. A 5 mm gap between the supports allowed for movement of the middle 4 mm segment relative to the other segments. A loading platen made of G10 was placed against the edge of the middle 4 mm segment.

The LVDT measured the relative movement between the Baldwin loading surfaces. This is shown in Figure 9. The support material is included in the displacement measurement but the magnitude of sliding displacement was expected to greatly exceed any deformation in the supports.

Figure 9. Load-slip set-up.



3.3.3. Results

A total of six (6) tests were performed, five (5) of which were conducted on one specimen. Tests C1 to C5 were performed at axial stress levels of between 12.0 and 21.3 MPa. As mentioned previously, it was found that the 12.7 mm (½ inch) diameter steel tensioning rod was yielding and no higher axial compressive stress levels could be achieved. Thus, the results from these first five tests are inconclusive. After dismantling the specimen, it was found that the 4 mm ring had broken. It was difficult to determine the load level at which this breakage occurred.

The final test, C6, was performed using a 19 mm (¾ inch) diameter rod to prevent yielding of the rod at higher stress levels. A compressive stress of 26.2 MPa was achieved. A sudden drop in load level occurred at approximately 400 kg of load, as indicated in the graphs of Appendix C. Upon dismantling the specimen, it was found that all three 4 mm segments had broken. There was no indication of sliding prior to failure.

3.4. Ultimate Strength

3.4.1. Apparatus

The Baldwin machine was used again. The LVDT was placed to measure displacements between the machine platens. The data acquisition system was used to record two channels: load and displacement. Fragments of the 4 mm segments damaged in the load-slip tests were used to determine the maximum strength of CITAL. For the first fragment, two pieces of G10 were placed above and below the specimen during loading. This was found to prevent failure of the CITAL and resulted in crushing the G10 and embedding the CITAL in the G10. The second fragment was placed between steel plates.

3.4.2. Specimens

Two CITAL fragments from the load-slip tests were tested in compression. The cross-sectional area of each fragment was estimated.

3.4.3. Results

A total of three tests were performed. Only the results of test CR3 are valid. CR1 and CR2 were performed on the first fragment and resulted in embedment of the CITAL fragment into the G10 platens. It was decided to place the second fragment directly between the steel platens of the Baldwin machine for this test, CR3. Plots of all three tests are shown in Appendix D.

The maximum stress level achieved was 910 MPa. At this level, the specimen failed suddenly and disintegrated.

3.5. Load-displacement G10

The load-displacement behaviour of G10 in the through-thickness orientation was determined in compression.

3.5.1. Results

The results are indicated in a plot shown in Appendix E. The through-thickness elastic modulus, E, for G10 (11.5 mm thick, as tested), is E = 19.6 GPa. NOTE: This value is approximate and is only for the through-thickness orientation: loading in other directions will produce different results.

3.6. Beam Deflection of Single CITAL Beam

The behaviour of a single 1.06 m beam pre-stressed with a threaded steel rod is required to verify the theoretical predictions of beam deflection.

3.6.1. Apparatus

A single beam was constructed and simply-supported at each end. Loading was provided by up to eight 2.0 kg weights which could be placed at any location along the beam. The mid-span deflection was measured using an LVDT, accurate to ±1 μm. The LVDT was mounted on an aluminium bar at the mid-span location. The aluminium bar was attached only at the beam ends, at mid-depth; this standard beam testing method allows measurement of the mid-span displacements relative to the position of the beam ends.

The level of pre-stressing was measured using the strain-gauge instrumented segment.

3.6.2. Specimen

The specimen comprised 4, 20 and 40 mm long CITAL segments, placed in positions according to the final detector design. One of the 40 mm segments near the left end of the beam was replaced with the instrumented 40 mm segment as in section 3.3.2. The beam was surrounded by plastic wrapping to prevent dust from entering between the segments. The nuts on the threaded rod were tightened to achieve an axial stress level of 22.2 MPa in the beam. The set-up, with loads in place is shown in Figure 10. The self-weight of the beam is disregarded in this test since the LVDT is zeroed in place prior to the addition of the 2.0 kg weights. Following the test, the beam was to be checked for breakage of CITAL segments.

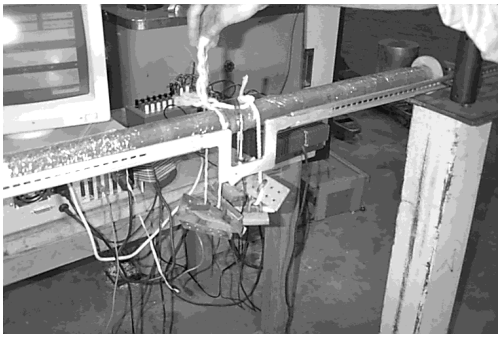


Figure 10. Single beam test set-up.

3.6.3. Results

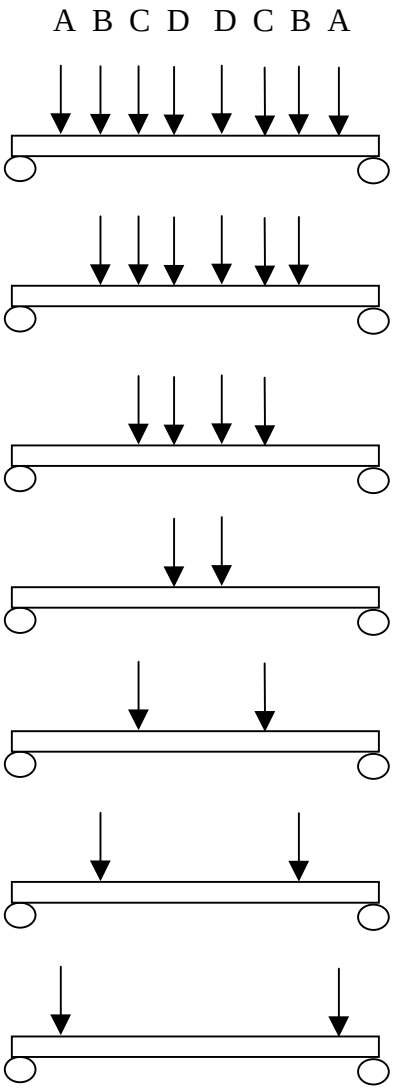
Eight load combinations were tested. The maximum mid-span deflection for each combination is shown in Table 3. For each combination, a computer analysis was performed to predict the mid-span deflection. This analysis is described in section 4 but the results are included here for comparison to the experimental results. Since the experimental beam was simply-supported, the predicted results for this case should match closely.

Each load arrow represents a 2 kg load. Load locations, measured from the beam ends are, in Table 2.

Table 2. Load locations.

Point	Distance From Beam End (mm)
A	122
B	246
C	370
D	494

Table 3. Single beam displacements.



Displacements (μm)		
Test	Predicted	
	Simply-Supported	Fixed-Fixed
181	179	37
164	158	35
120	118	28
67	62	16
50	55	12
36	41	7
22	21	2

From the results indicated in Table 3, the predictions appear accurate. The values for the fixed-fixed case are given here as an indicator of the reduced deflection which occurs when sufficient end constraints are provided.

3.6.4. Mid-span Deflection Prediction Equations

The total deflection at mid-span due to a single point load can be determined from the following. For the case of a simply-supported beam:

[7] $\Delta = -0.155 + 52x - 40.5x^2$

where: Δ is the deflection at mid-span in micrometres (μm), and
x is the distance from the end of the beam to the location of the load in metres as shown in Figure 11, to a maximum of L/2.

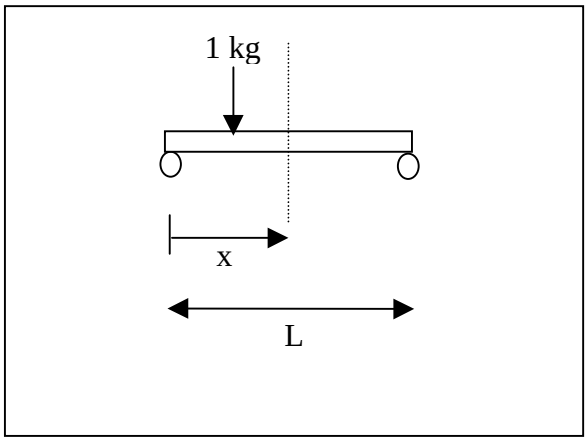


Figure 11. Mid-span deflection parameters.

Equation [7] gives the deflection due to a 1 kg load placed at point x, where $0 < x < L/2$. Since deflections are linearly proportional to the load, the value of Δ may be multiplied by, for example, 2 if the load is 2 kg. For more than one load, the equation is used to determine the deflection at mid-span due to each load, and the sum of these deflections gives the total deflection due to all loads as if they were applied at once. This is the principle of superposition, as described in section 1.2.1. This can be re-written as:

[8] $\Delta_{total} = \sum_{i=1}^{n} \Delta_i$

where: n are the number of loads and Δ_i is the mid-span deflection for each load determined using equation [7].

For the case of the fixed-fixed supports, equation [7] becomes:

[9] $\Delta = -0.002 + 0.015x + 40.5x^2 - 49.3x^3$

with the same variable definitions and restrictions as for equation [7]. Equation [8] also applies.

Example

A single 1.06 m long beam is simply-supported and has two 4 kg loads placed on it, one at 0.122 m from the left, the other at 0.370 m from the right. What is the total deflection at mid-span?

For a simply-supported beam, use equation [7]:

$$\Delta(0.122) = 4\left[-0.155 + 52(0.122) - 40.5(0.122)^2\right] = 22.3\mu m$$

$$\Delta(0.370) = 4\left[-0.155 + 52(0.370) - 40.5(0.370)^2\right] = 54.2\mu m$$

From equation [8] we find Δ_{total} = 22.3 + 54.2 = 76.5 μm.

One can easily verify that these equations satisfy the results given in Table 3.

Note: Only for the case of loads placed symmetrically about the mid-span does the maximum deflection occur at or about the mid-span point.

4. Analysis

A two-dimensional computer model of the single beam was created. The loads were positioned over each 4 mm segment along the beam length according to the proposed detector design. Two support conditions were assumed: simply-supported and fixed-fixed. These assumptions result in different values of maximum stress and maximum displacement along the beam. The actual detector design will have support conditions which fall somewhere between these extreme values.

4.1. Model Geometry and Load

The computer model has the geometry and loads shown in Figure 12.

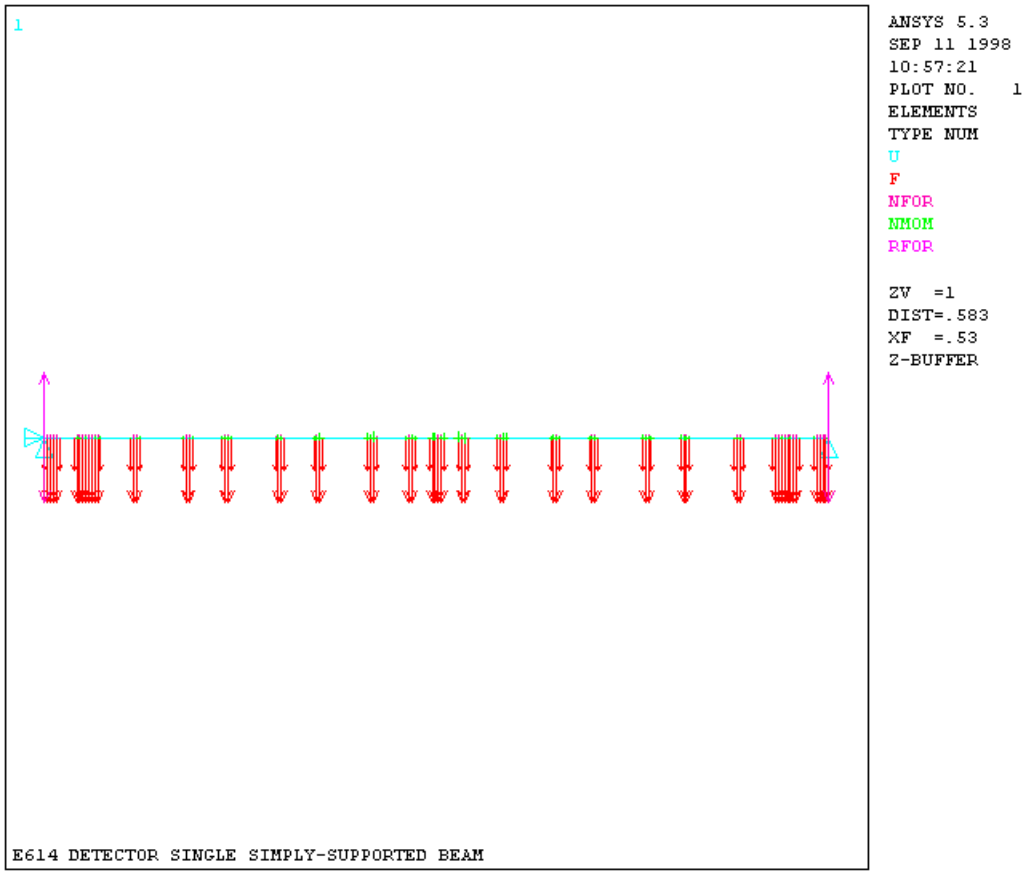


Figure 12. Computer model of single beam.

Each arrow pointing downwards indicated the load hanging from a 4 mm segment. The magnitude of these loads can be varied to determine the relationship be load and maximum mid-span deflection or load and maximum compressive stress.

The compressive load which acts along the length of the beam (axially or longitudinally) is also included but is not shown in Figure 12. This load has no effect on the maximum deflection but does affect the compressive stresses.

The cross-sectional properties of the segments must be included in the model. These include area, A, and moment of inertia, I (also called second moment of area) for a single beam. The values are listed in Table 4.

Table 4. Single Beam Geometric Properties.

Cross-sectional area, A (m^2)	Moment of Inertia, I (m^4)
1.9635×10^{-3}	2.67×10^{-7}

4.2. Results

A range of loads and compressive stresses were analysed. The loads were varied between 1 and 5 kg per loading plane (i.e. per 4 mm segment). The compressive stress ranged from 0 to 40 MPa. It is not physically possible to have zero compressive stress since the axial stress is required to maintain the integrity of the beam.

Deflection

It was found that the maximum deflection could be described as a function of the load per plane as follows.

Table 5. Maximum mid-span deflection of single beam.

Simply-supported beam (mm)	Fixed-fixed beam (mm)
0.624 * Load	0.124 * Load

Note: 1. Load is in kilograms and is per plane (i.e. per 4 mm segment).

Maximum Compressive Stress

The maximum compressive stress (in MegaPascals, MPa) in the beam can be expressed as a function of load per plane and axial stress as follows.

Table 6. Maximum compressive stress of single beam.

Simply-supported beam (MPa)	Fixed-fixed beam (MPa)
7.7 * Load + Axial Stress	5.1 * Load + Axial Stress

Note: 1. Load is in kilograms and is per plane (i.e. per 4 mm segment).

Minimum Required Axial Stress

The equations in Table 6 can be rearranged to determine the minimum axial stress required for a given load, as shown in Table 7.

Table 7. Minimum required axial stress of single beam.

Simply-supported beam (MPa)	Fixed-fixed beam (MPa)
7.7 * Load * S.F.	5.1 * Load *S.F.

Notes: 1. Load is in kilograms and is per plane (i.e. per 4 mm segment).
2. S.F. is a safety factor which can be taken as 1.10.

Example

For example, if the beam has fixed-fixed supports and the load is 1 kg per plane, the minimum axial stress which is required is 5.1 * 1 * 1.10 = 5.6 MPa. Using these values, the beam will have a maximum compressive stress of 5.1 * 1 + 5.6 = 10.7 MPa which is well below the ultimate compressive strength of CITAL (910 MPa as indicated in section 3.4.3). The maximum deflection is expected to be 0.124 * 1 = 0.124 mm or 124 μm.

4.3. Four-beam Structure

If the four beams are assumed to act as one in the finished detector structure, then the entire cross-sectional area and moment of inertia must be used. The values summarised in Table 7 with area calculated as follows: $A_4 = 4 * 1.963 \times 10^{-3} = 7.852 \times 10^{-3}$ m^2.

Table 8. Four-beam geometric properties.

Cross-sectional area, A (m^2)	Moment of Inertia, I (m^4)
7.852×10^{-3}	1.71×10^{-4}

Deflection

The ratio of the moment of inertia of the single beam to the four-beam structure is:
$I_1 / I_4 = 2.67 \times 10^{-7} / 1.71 \times 10^{-4} = 1.56 \times 10^{-3}$. Thus, the deflection equations can be modified by multiplying by this ratio to give the equations in Table 9:

Table 9. Maximum mid-span deflection of four-beam structure.

Simply-supported beam (mm)	Fixed-fixed beam (mm)
9.74×10^{-3} * Load	1.94×10^{-4} * Load

Note: 1. Load is in kilograms and is total load per plane carried by all 4 beams.

Thus, for a 4 kg load with fixed-fixed supports, the deflection should be only 0.8 μm.

Maximum Compressive Stress

The maximum stresses will be found to reduce relative to the moments of inertia and the distance to the outer edge of the beam. The *section modulus*, S, is defined as:

$S_1 = I_1 / r_o = 2.67 \times 10^{-7} / 0.025 = 1.07 \times 10^{-5}$ m^3 for a single beam, and $S_4 = I_4 / R = 1.71 \times 10^{-4} / 0.26 = 6.577 \times 10^{-4}$ m^3 for the four-beam structure. The ratio of $S_1 / S_4 = 0.01627$ is used with the equations in Table 6 to give:

Table 10. Maximum compressive stress in four-beam structure.

Simply-supported beam (MPa)	Fixed-fixed beam (MPa)
0.125 * Load + Axial Stress	0.08 * Load + Axial Stress

Note: 1. Load is in kilograms and is per plane (i.e. per 4 mm segment).

Thus, for the case of a 4 kg load, the necessary axial stress becomes 0.5 MPa, and the maximum compressive stress is only 1.0 MPa.

For a conservative estimate, it is recommended that the equations for the single beam be used to estimate stresses. The actual deflections should fall between the values predicted in Table 5 and Table 9.

5. Conclusions

A) It was determined that the modulus of elasticity, E, for CITAL CO-115M is 57 GPa.

B) The friction between adjacent segments was not determined. The slip tests indicated that the segments would fail prior to any measurable slip between adjacent segments. It should be noted that the fracture did occur under a load of 400 kg (or 200 kg load per shear plane) with 26.3 MPa confining pressure between segments - this level of loading is not expected in the actual structure but could occur if the structure is accidentally overloaded (due to jarring impact or the weight of excess instrumentation).

C) The maximum compressive strength was found to be about 900 MPa. This value was determined from a relatively small specimen which means that the value 900 MPa is an upper bound. For the four-beam structure, it is expected that operating stresses will be well below this level. In brittle materials such as glass, a size effect normally exists which causes specimens of larger volume to fail at lower stresses. This occurs since there is a higher probability of having a flaw (which leads to failure) in the larger volume. Also, the results of such a test are highly dependent on the load apparatus and specimen shape.

D) The maximum tensile strength was not determined experimentally. For brittle materials, such as glass, the tensile strength is at least ten times less than the compressive strength.

E) The single beam experiment showed that the material and geometric properties of the beam could be used to accurately predict the mid-span deflection of the beam. The experimental beam was simply-supported and the computer model prediction for the simply-supported case matched well. If a similar beam is constructed with rotations prevented at each end (fixed-fixed condition) then the computer predictions for this case should also match. Although it is unlikely that a pure fixed-fixed condition can be achieved, it is expected that the results for mid-span deflection will fall between the values of the two cases.

F) The behaviour of the full four-beam structure will depend upon the rigidity of the G10 plates which link the beam at locations along the length of the beam. In order to gain any advantage from using the four-beam structure (‘advantage’ here would be either a reduction in mid-span deflection or a reduction in stresses, or both) the G10 must contribute to the structural stiffness.

As examples, the single beam and four-beam structures will be analysed here:

Example 1 – Single Beam

For a single 1.06 m long beam, simply supported at either end, the expected mid-span deflection with 4 kg per plane is: 0.624 * 4 = 2.5 mm (0.124 * 4 = 0.5 mm if it is fixed-fixed). The axial stress for this beam should be 7.7 * 4 * 1.1 = 34. MPa (5.1 * 4 * 1.1 = 23 MPa if it is fixed-fixed). The ultimate compressive strength of CITAL was found to be 910 MPa. With the 4 kg load and specified axial

stresses, the maximum compressive stress in the beam will be 7.7 * 4 + 34 = 65 MPa (5.1 * 4 + 23 = 43 MPa) which is below the ultimate strength, thus failure in compression is unlikely. The associated shortening due to the axial stress is 34 * 1.06 / 57 000 = 632 μm (23 * 1.06 / 57 000 = 428 μm if it is fixed-fixed). Note that the units have been changed from metres to μm.

Example 2 – Four-Beam Structure

For a 1.06 m long four-beam structure, simply supported at either end, the expected mid-span deflection with 4 kg per plane is: $9.74 \times 10^{-3} * 4 = 39$ μm ($1.94 \times 10^{-4} * 4 = 0.8$ μm if it is fixed-fixed). The axial stress for this beam should be 0.125 * 4 * 1.1 = 0.55 MPa (0.08 * 4 * 1.1 = 0.35 MPa if it is fixed-fixed). The compressive stresses will be much lower in this case. With the 4 kg load and specified axial stresses, the maximum compressive stress in the beam will be 0.125 * 4 + 0.55 = 1.1 MPa (0.08 * 4 + 0.35 = 0.7 MPa) which is well below the ultimate strength, thus failure in compression is unlikely. The associated shortening due to the axial stress is 1.1 * 1.06 / 57 000 = 20.5 μm (0.7 * 1.06 / 57 000 = 13 μm if it is fixed-fixed). Note that the units have been changed from metres to μm.

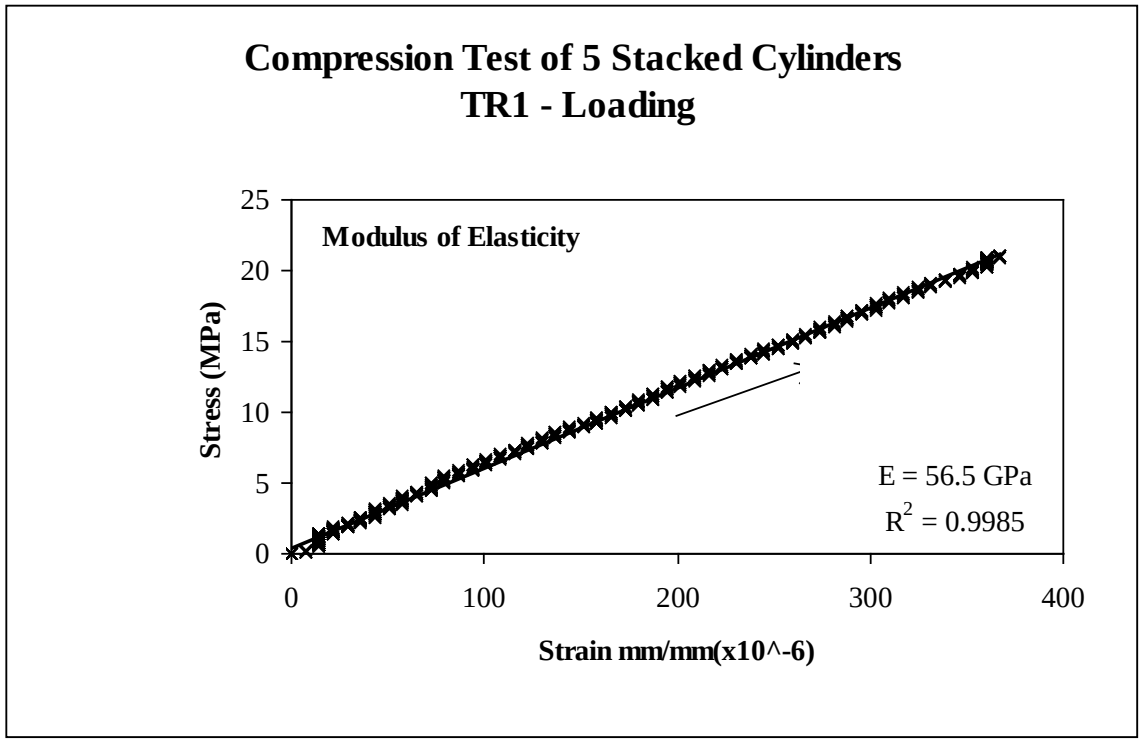
The actual behaviour will fall somewhere between the single beam and four-beam structure design values depending on the rigidity of the G10 plates. These values also depend on the end support conditions.

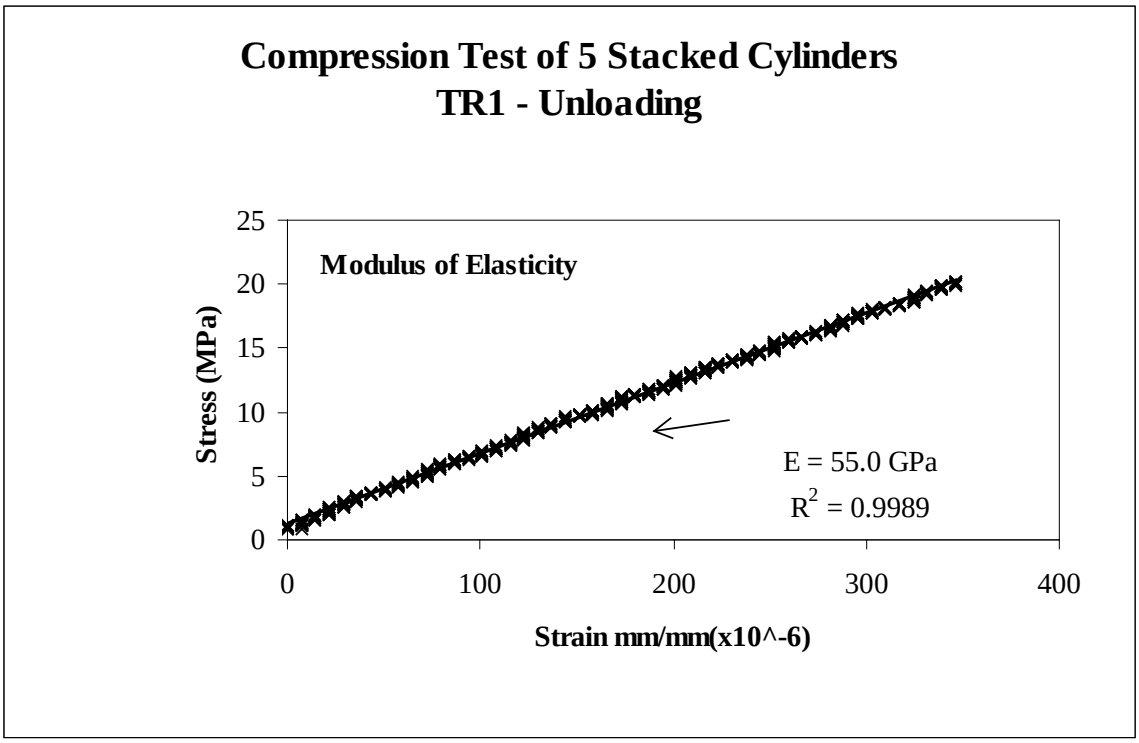
5.1. Recommendations

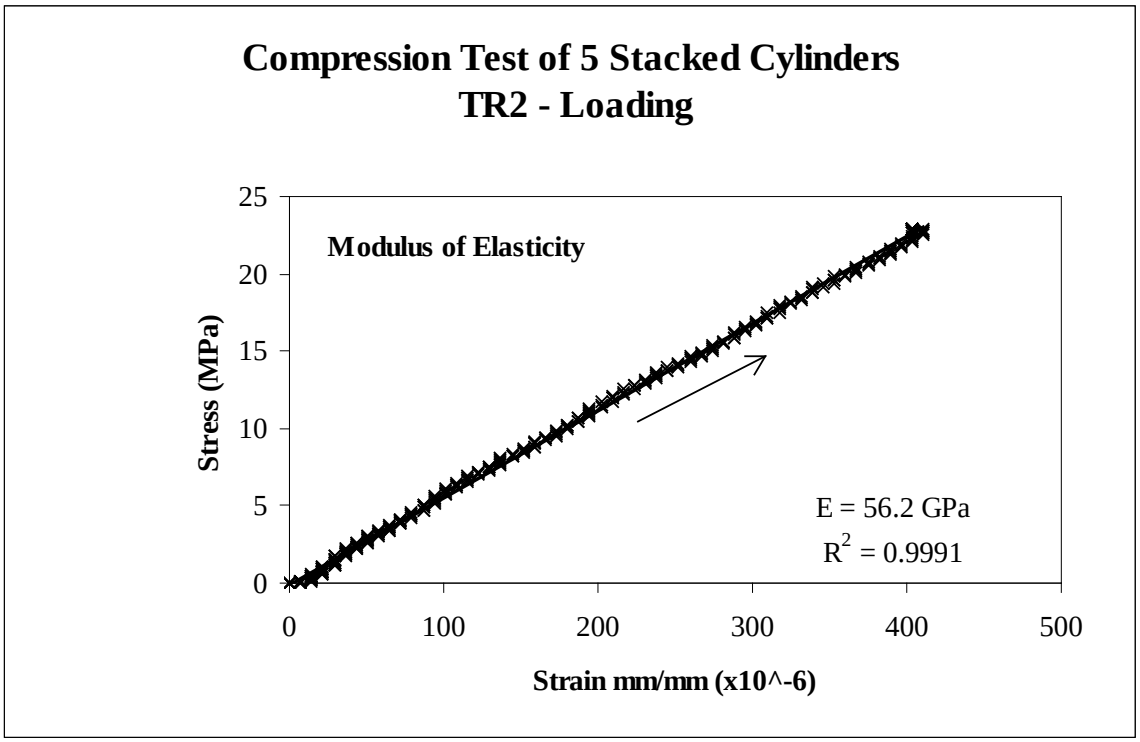
1) The minimum axial pre-stress in each beam should be 20 MPa.
2) One segment with a calibrated strain gauge should be used in each beam to determine the level of pre-stressing. A torque wrench was used during the experiments but was not as accurate as the instrumented segment. This will also allow for long-term monitoring of the axial stress in each beam.
3) Care must be taken to prevent the beams from being accidentally overloaded. This could occur from a jarring impact or from dropping a tool on it. If this does occur, some segments may become fractured.
4) The four-beam structure should have end constraints which prevent end rotations in order to approach the fixed-fixed support condition. In this case, assuming the G10 links are relatively rigid, the maximum mid-span deflection, if the beam is loaded symmetrically with 4 kg loads placed at the locations indicated on the work drawings, is limited to less than 1 μm. The axial stress should be maintained at a level above 20 MPa but should not exceed 50 MPa. This stress will cause an axial shortening of each beam of about 400 μm.

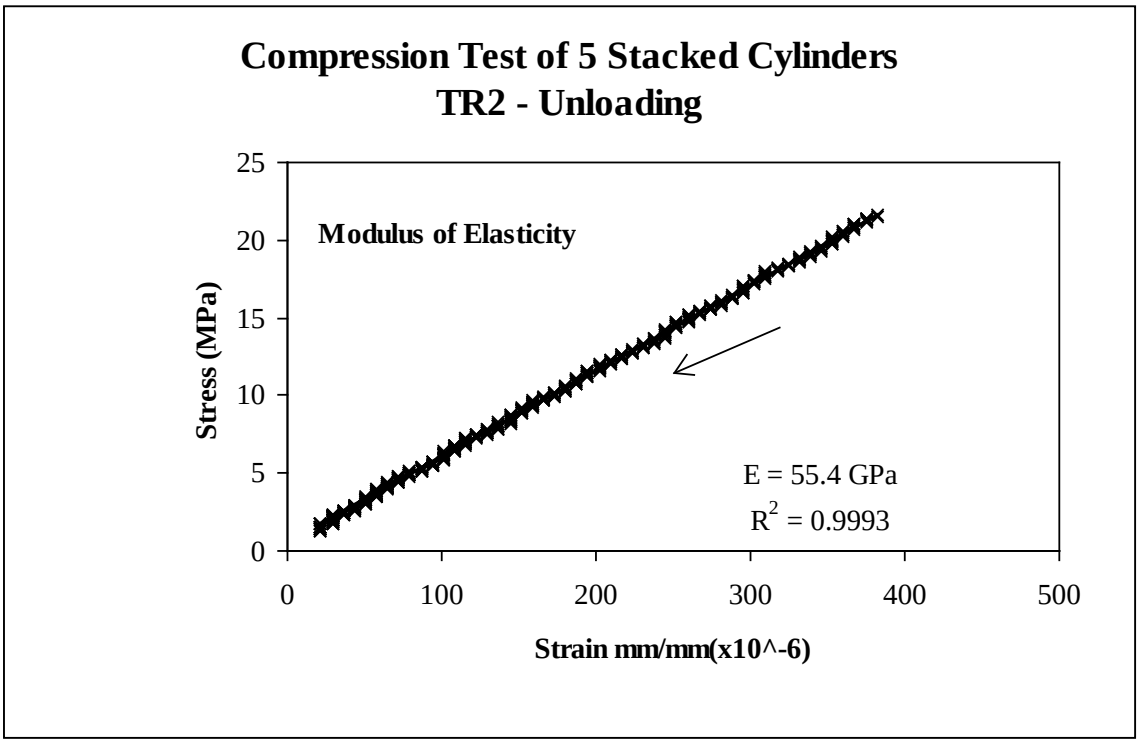
APPENDIX A

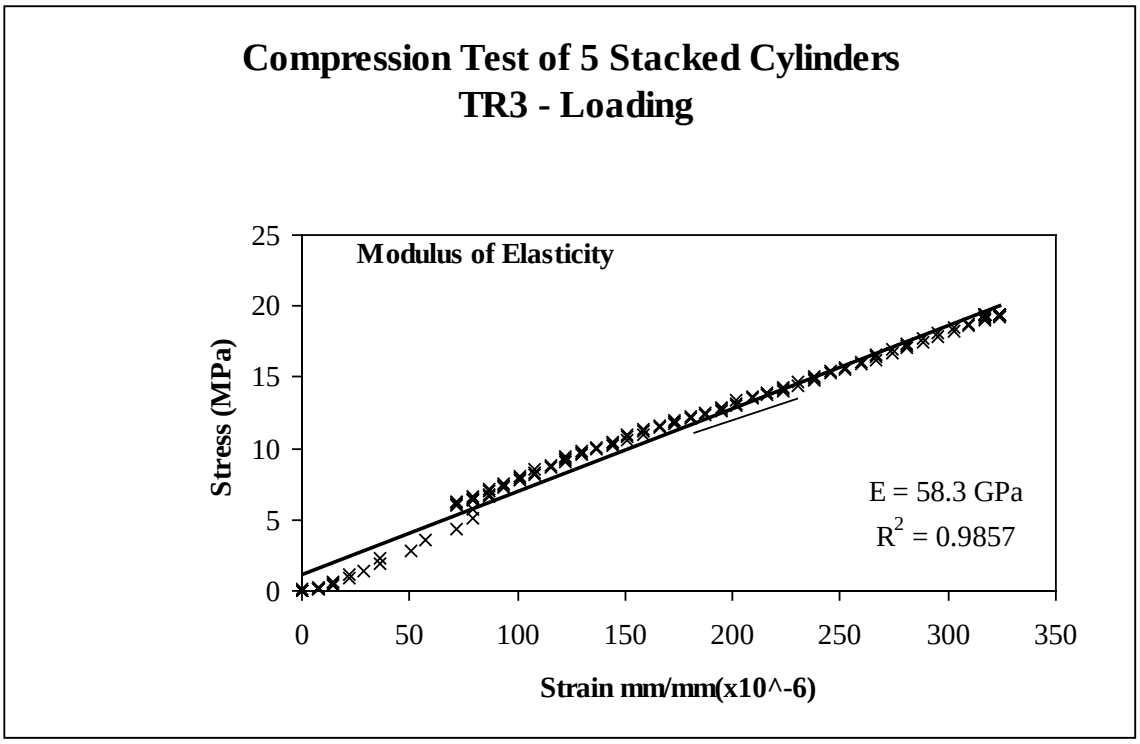
Compression Tests
Stress-strain Curves

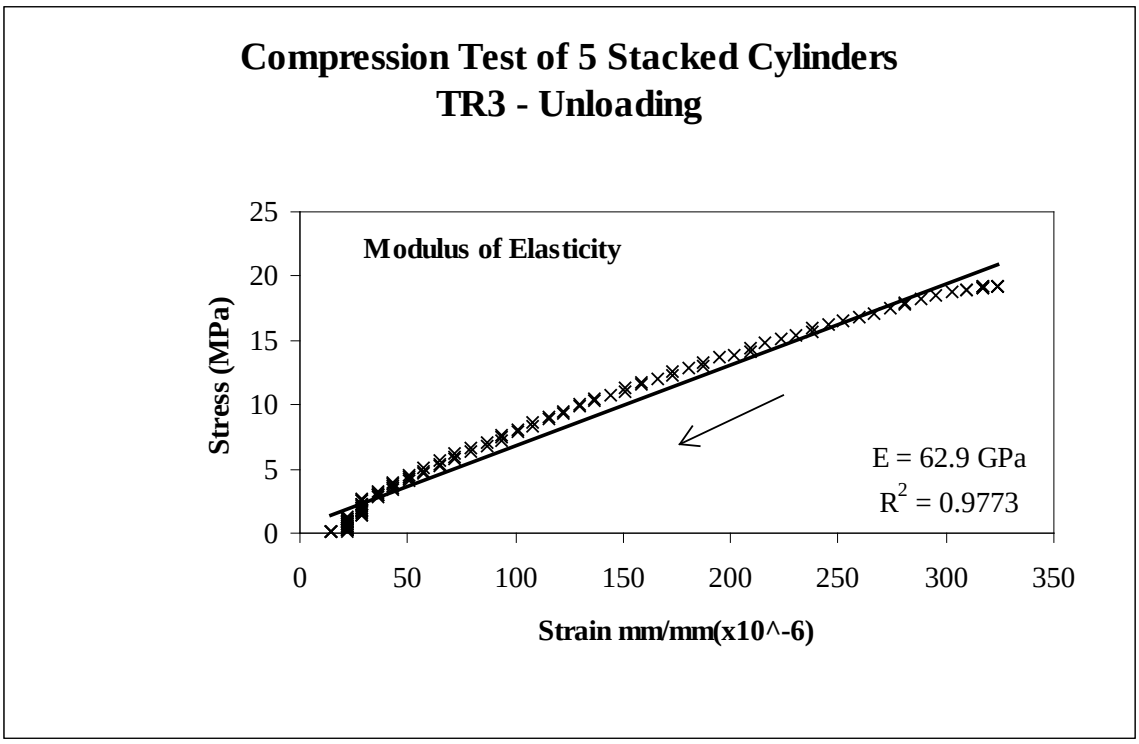

Compression Test of 5 Stacked Cylinders
TR1 - Loading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 56.5 GPa
R2 = 0.9985
0
5
10
15
20
25
0
100
200
300
400

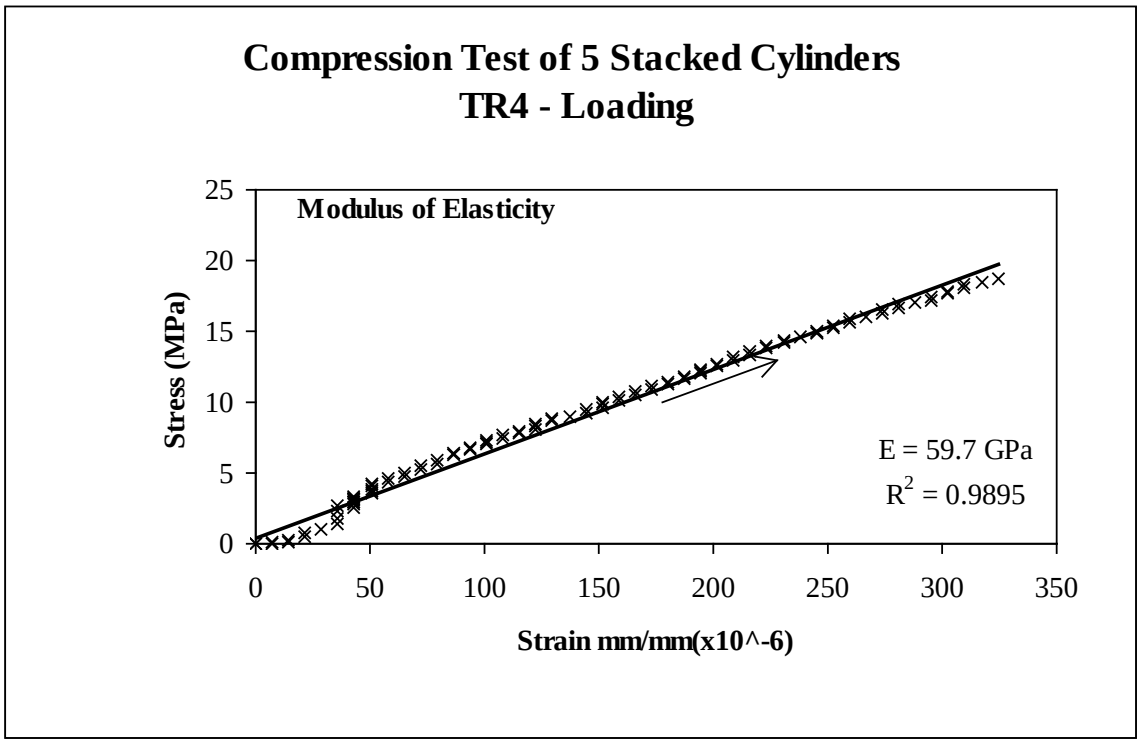

Compression Test of 5 Stacked Cylinders
TR1 - Unloading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 55.0 GPa
R2 = 0.9989
0
5
10
15
20
25
0
100
200
300
400

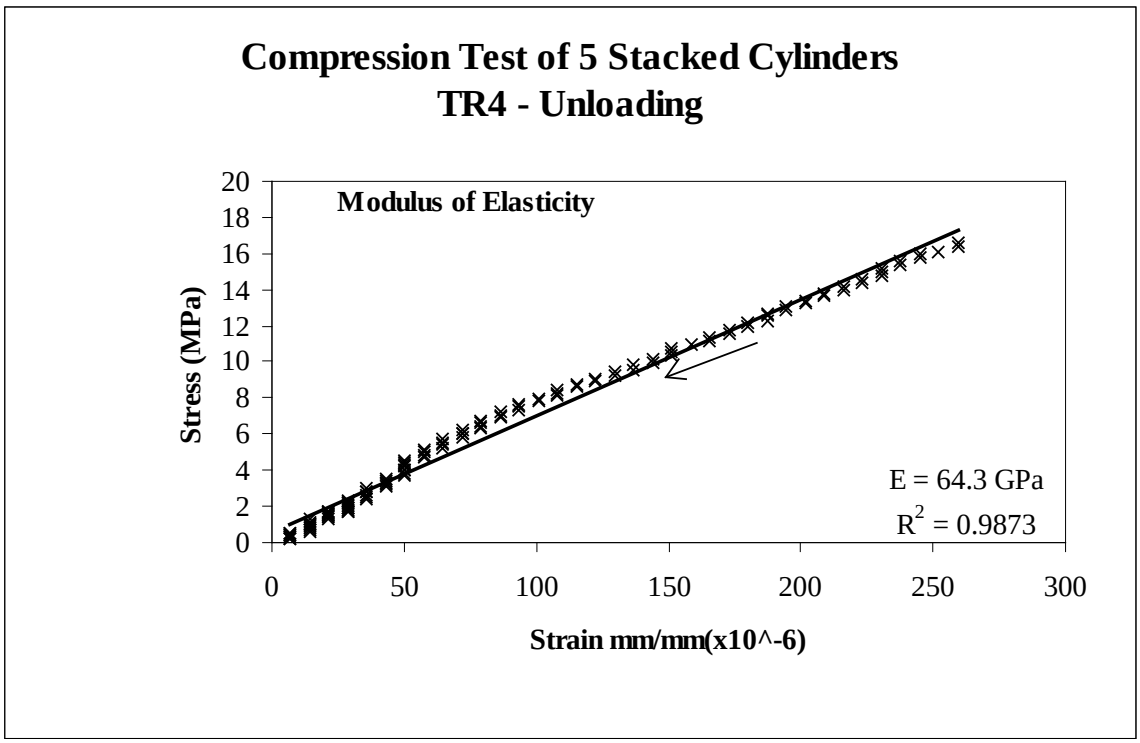

Compression Test of 5 Stacked Cylinders
TR2 - Loading
Modulus of Elasticity
E = 56.2 GPa
R2 = 0.9991
Stress (MPa)
Strain mm/mm (x10^-6)
0
5
10
15
20
25
0
100
200
300
400
500

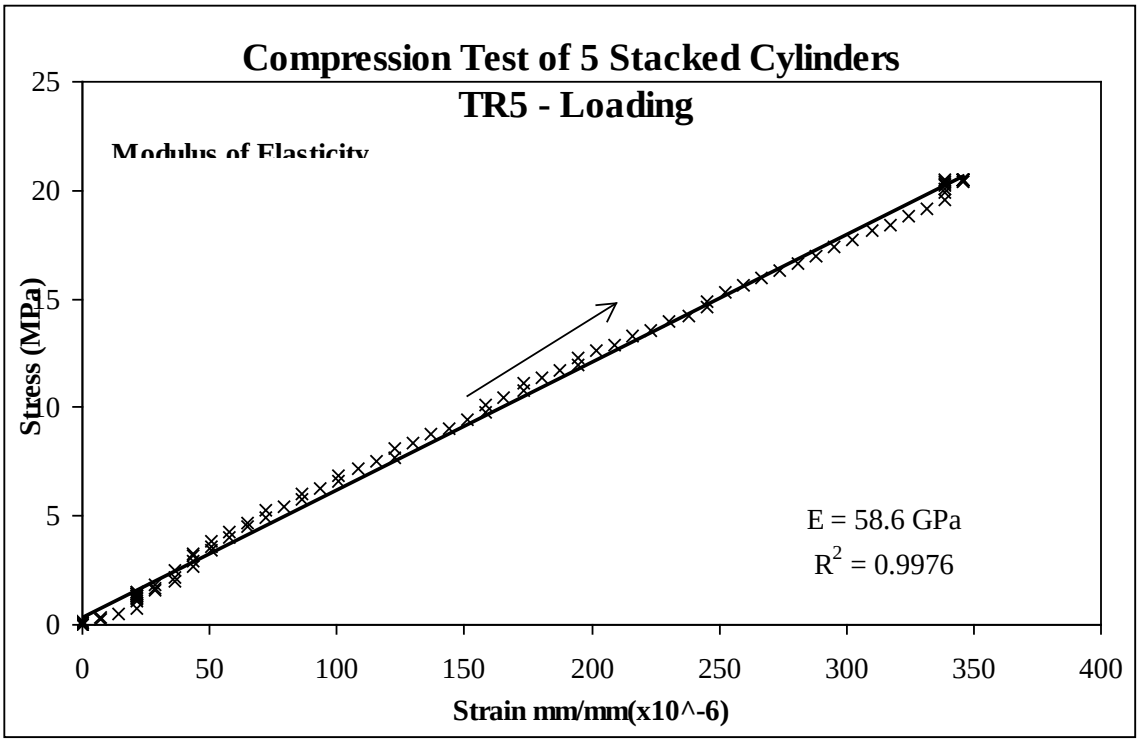

Compression Test of 5 Stacked Cylinders
TR2 - Unloading
Modulus of Elasticity
E = 55.4 GPa
R2 = 0.9993
Stress (MPa)
Strain mm/mm(x10^-6)
0
5
10
15
20
25
0
100
200
300
400
500

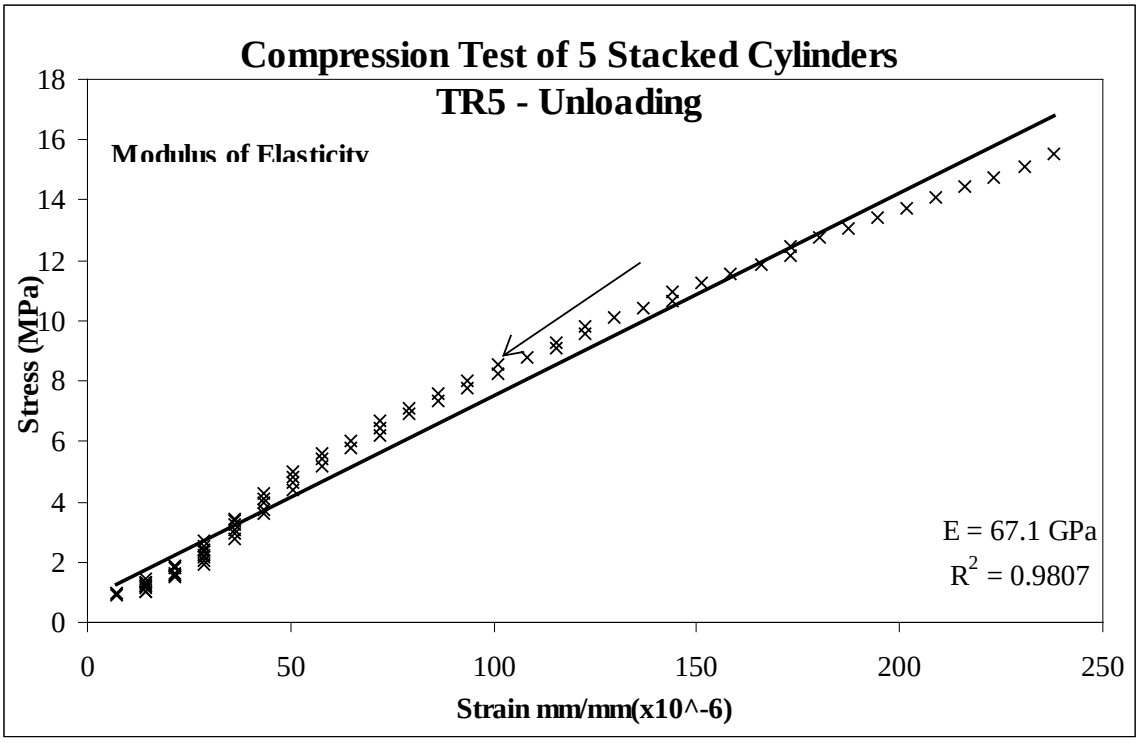

Compression Test of 5 Stacked Cylinders
TR3 - Loading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 58.3 GPa
R2 = 0.9857
0
5
10
15
20
25
0
50
100
150
200
250
300
350

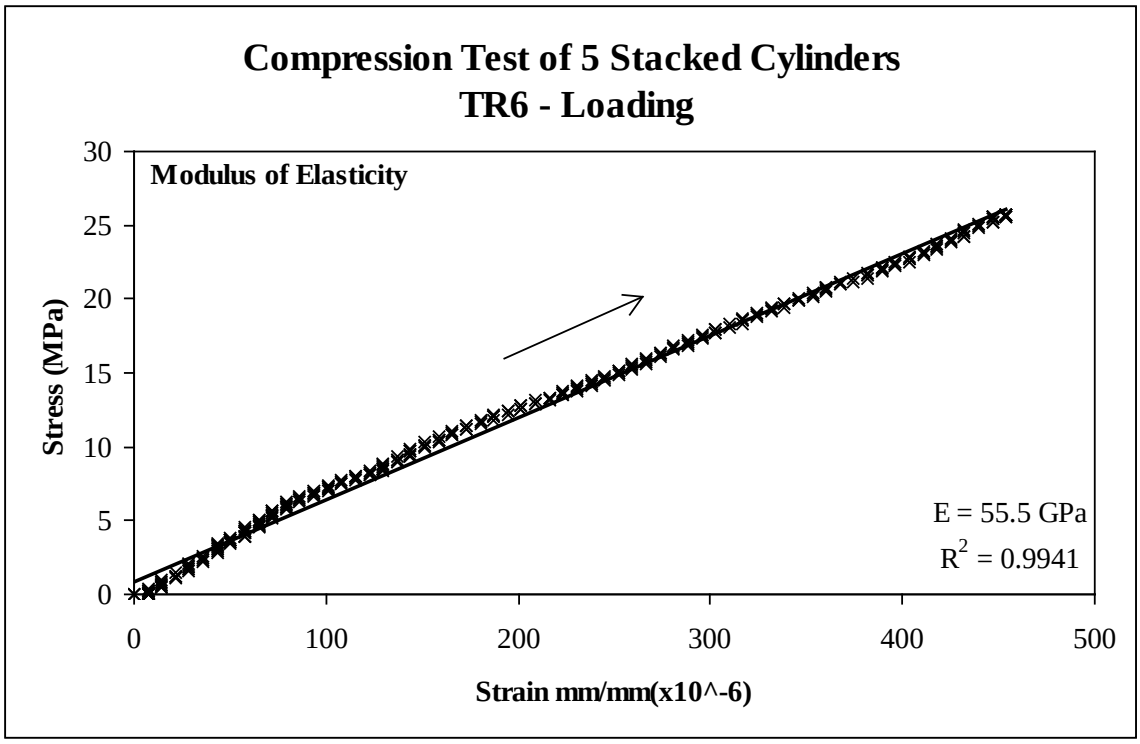

Compression Test of 5 Stacked Cylinders
TR3 - Unloading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 62.9 GPa
R2 = 0.9773
0
5
10
15
20
25
0
50
100
150
200
250
300
350

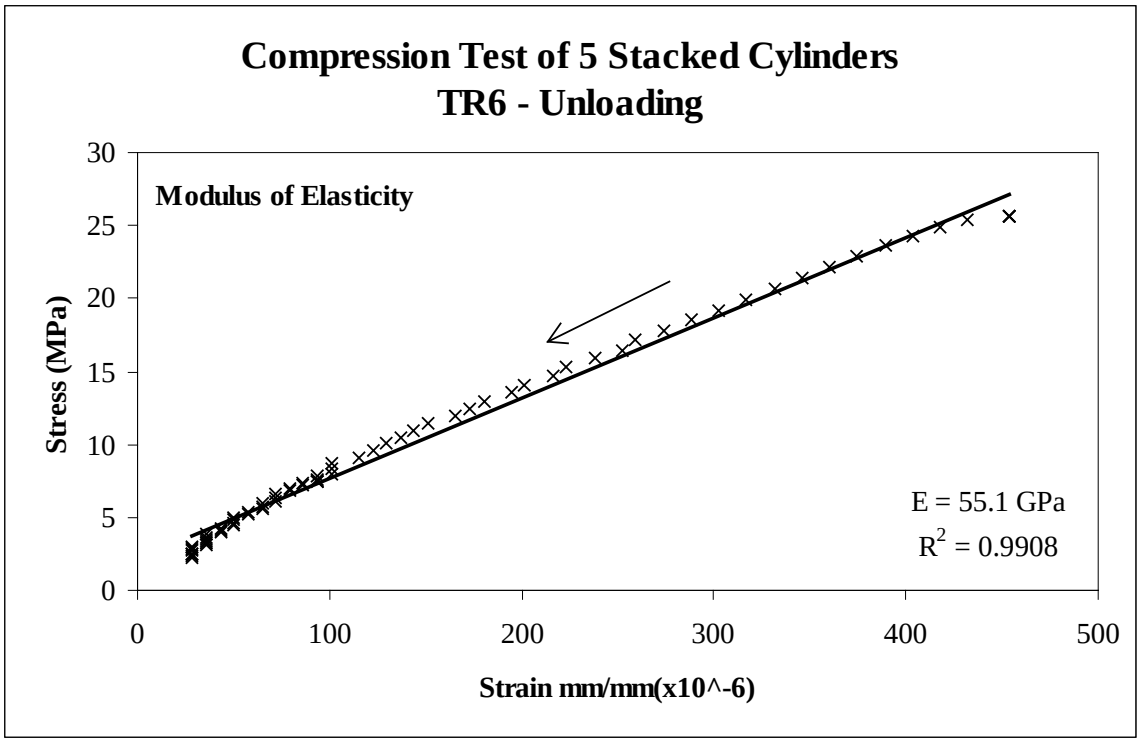

Compression Test of 5 Stacked Cylinders
TR4 - Loading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 59.7 GPa
R2 = 0.9895
0
5
10
15
20
25
0
50
100
150
200
250
300
350

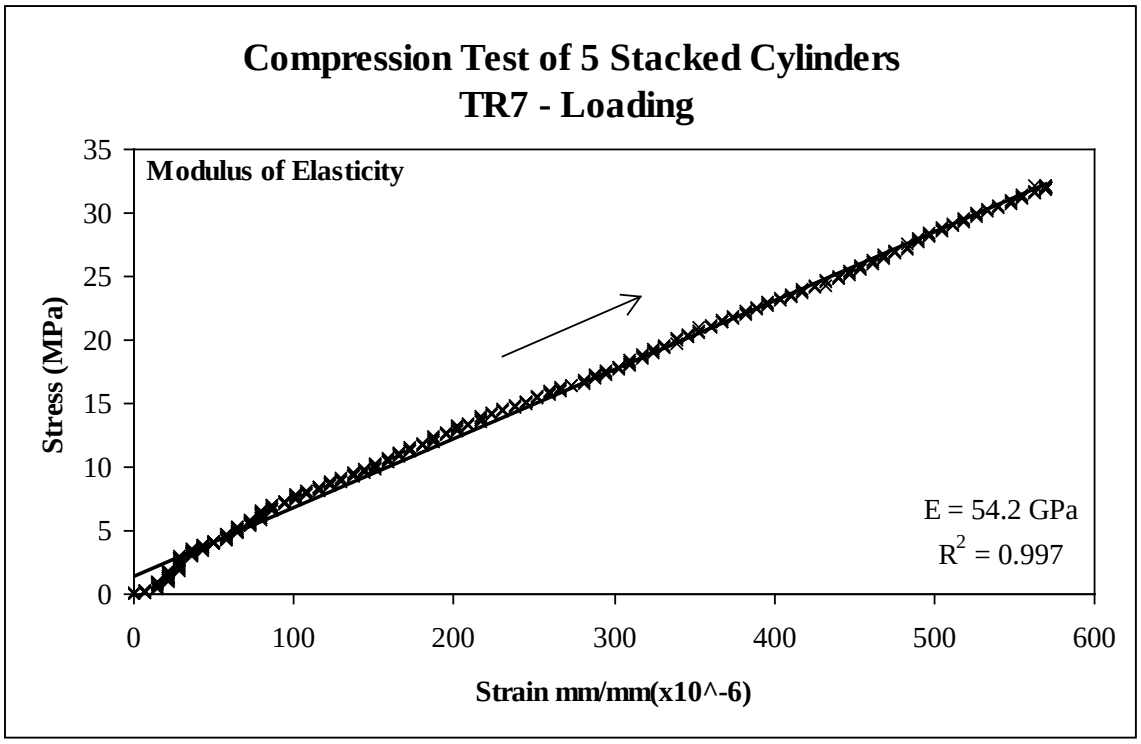

Compression Test of 5 Stacked Cylinders
TR4 - Unloading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 64.3 GPa
R2 = 0.9873
0
2
4
6
8
10
12
14
16
18
20
0
50
100
150
200
250
300

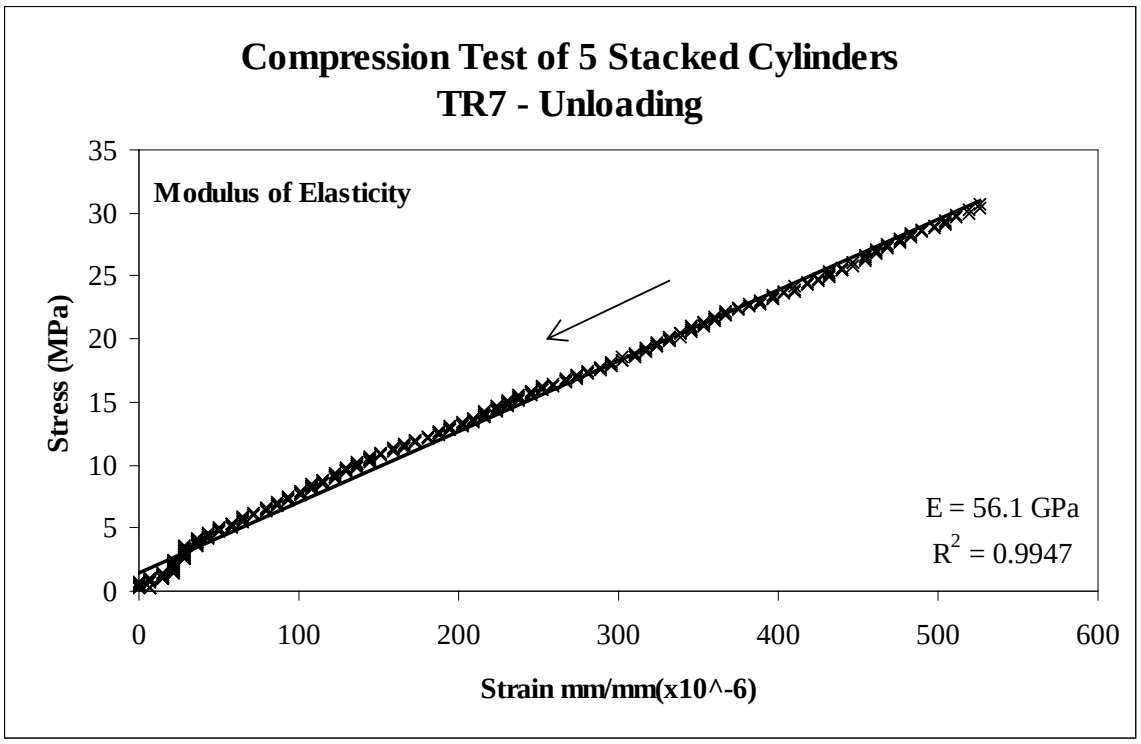

Compression Test of 5 Stacked Cylinders
TR5 - Loading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 58.6 GPa
R2 = 0.9976
0
5
10
15
20
25
0
50
100
150
200
250
300
350
400

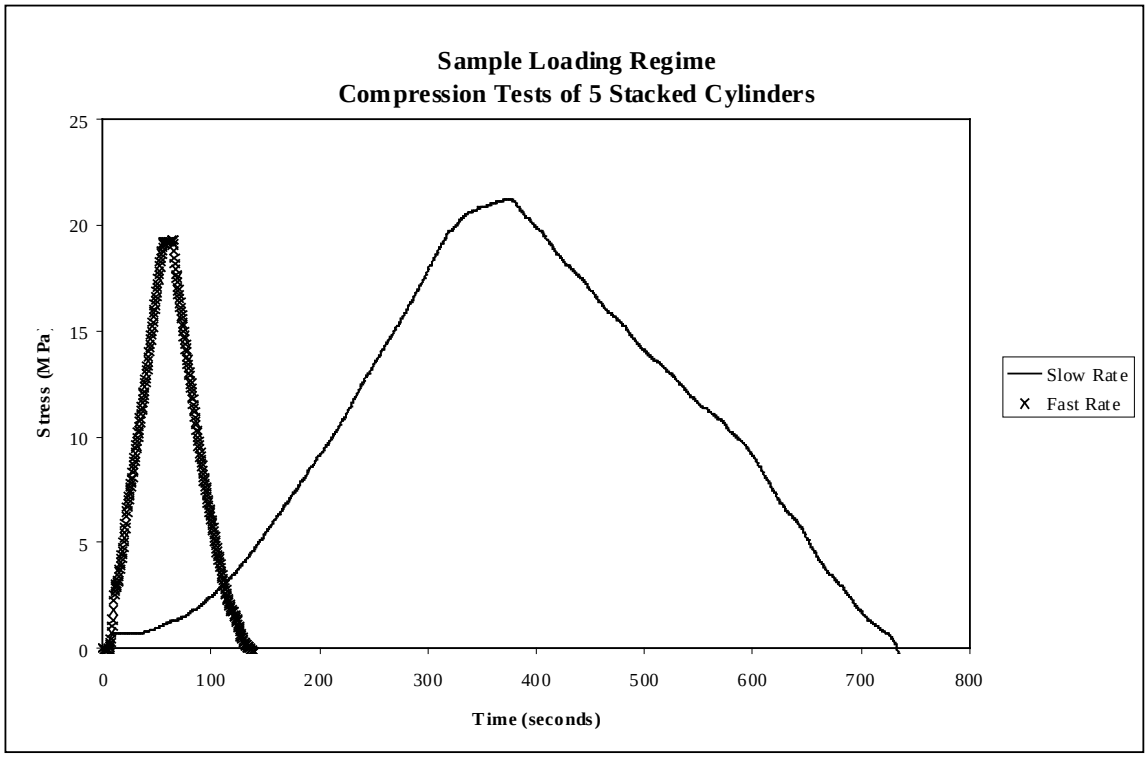

Compression Test of 5 Stacked Cylinders
TR5 - Unloading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 67.1 GPa
R2 = 0.9807
0
2
4
6
8
10
12
14
16
18
0
50
100
150
200
250

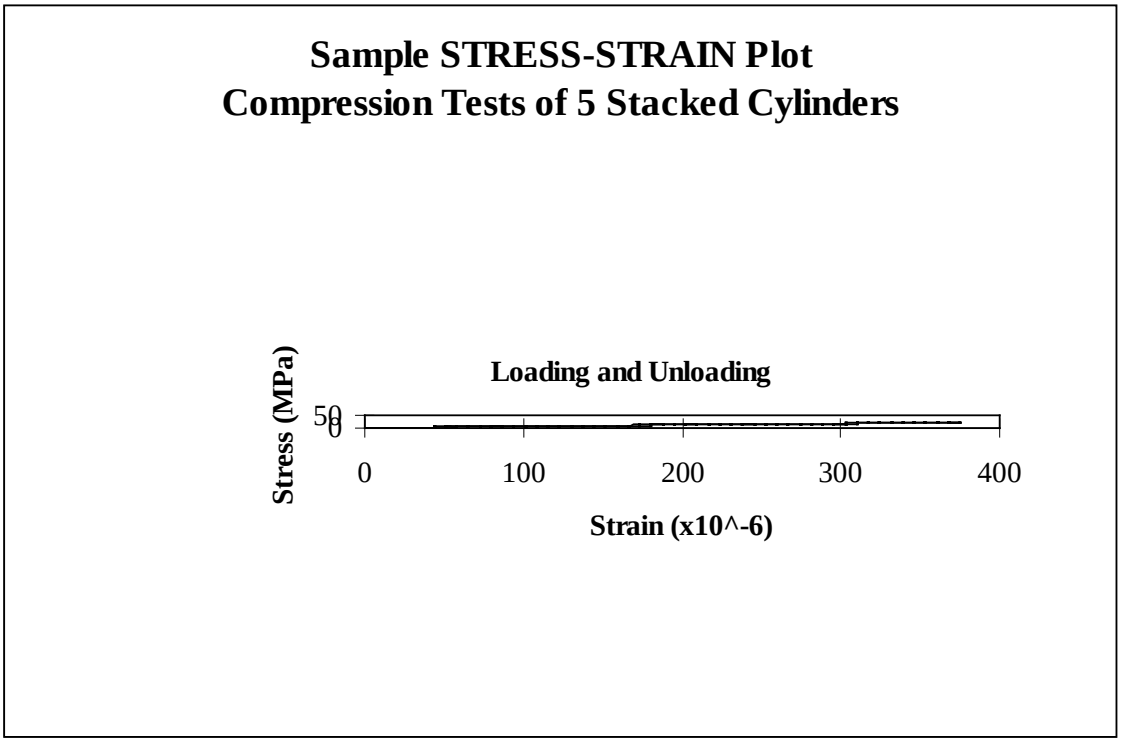

Compression Test of 5 Stacked Cylinders
TR6 - Loading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 55.5 GPa
R2 = 0.9941
0
5
10
15
20
25
30
0
100
200
300
400
500


Compression Test of 5 Stacked Cylinders
TR6 - Unloading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 55.1 GPa
R2 = 0.9908
0
5
10
15
20
25
30
0
100
200
300
400
500


Compression Test of 5 Stacked Cylinders
TR7 - Loading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 54.2 GPa
R2 = 0.997
0
5
10
15
20
25
30
35
0
100
200
300
400
500
600

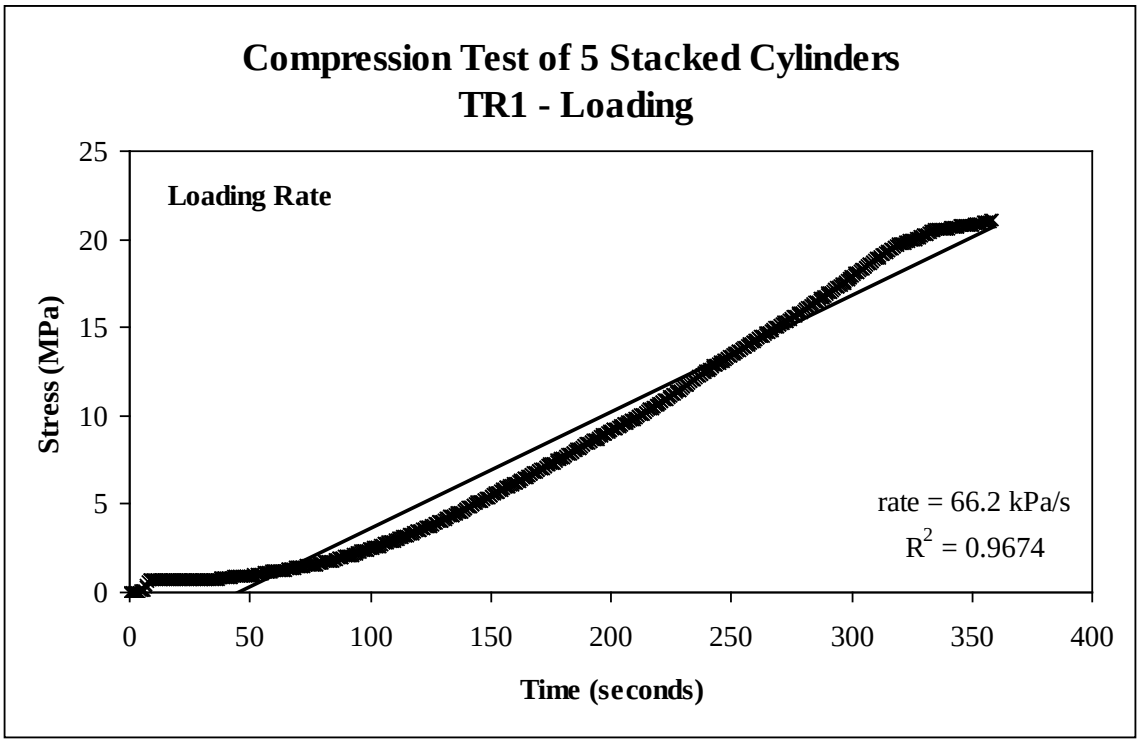

Compression Test of 5 Stacked Cylinders
TR7 - Unloading
Modulus of Elasticity
Stress (MPa)
Strain mm/mm(x10^-6)
E = 56.1 GPa
R2 = 0.9947
0
5
10
15
20
25
30
35
0
100
200
300
400
500
600

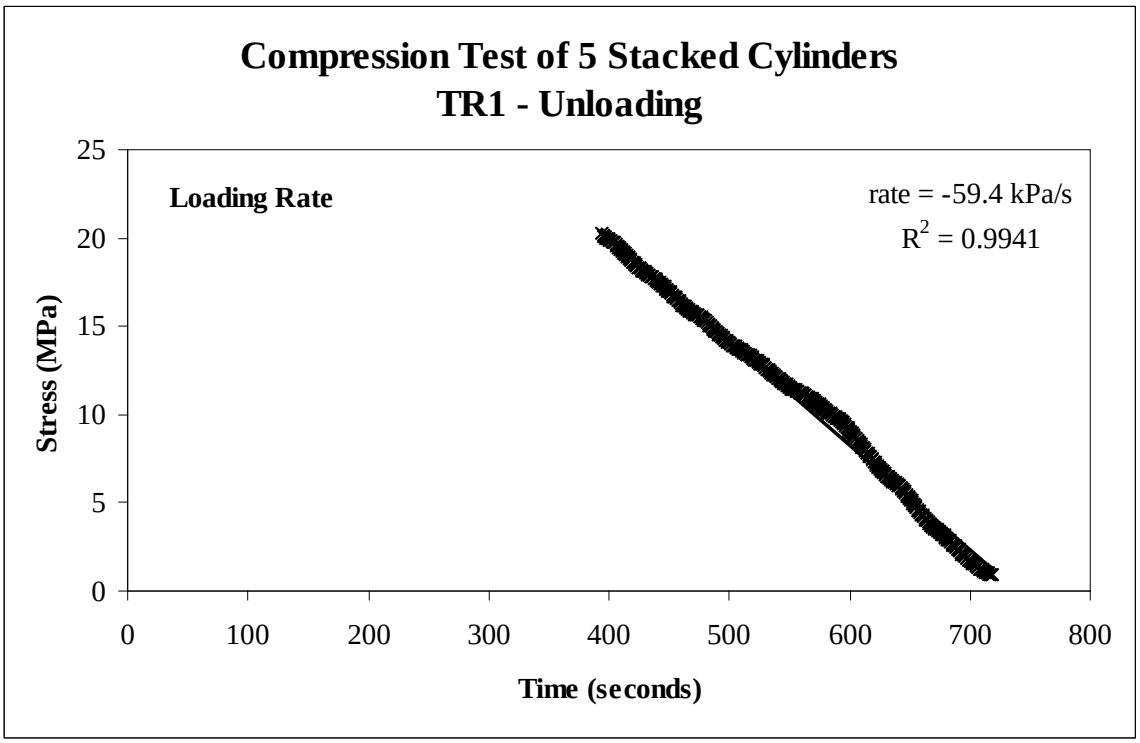

Sample Loading Regime
Compression Tests of 5 Stacked Cylinders
Stress (MPa)
Time (seconds)
Slow Rate
Fast Rate
0
5
10
15
20
25
0
100
200
300
400
500
600
700
800

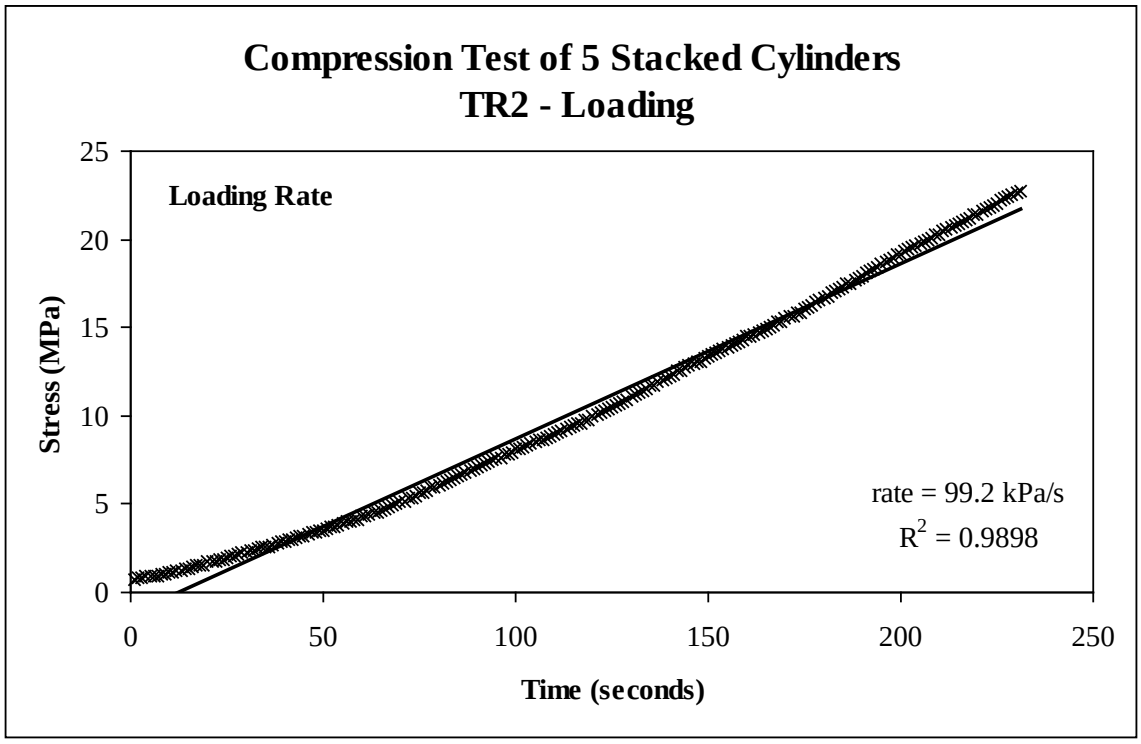

Sample STRESS-STRAIN Plot
Compression Tests of 5 Stacked Cylinders
Loading and Unloading
Stress (MPa)
Strain (x10^-6)
0
50
0
100
200
300
400

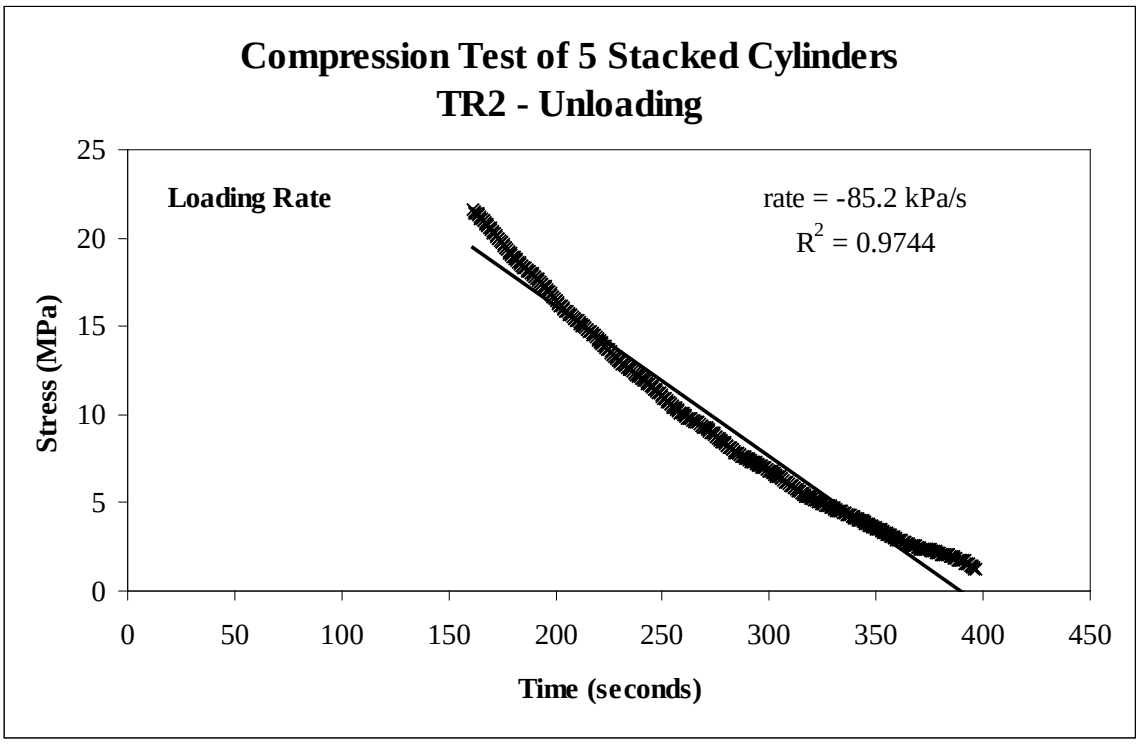
APPENDIX B

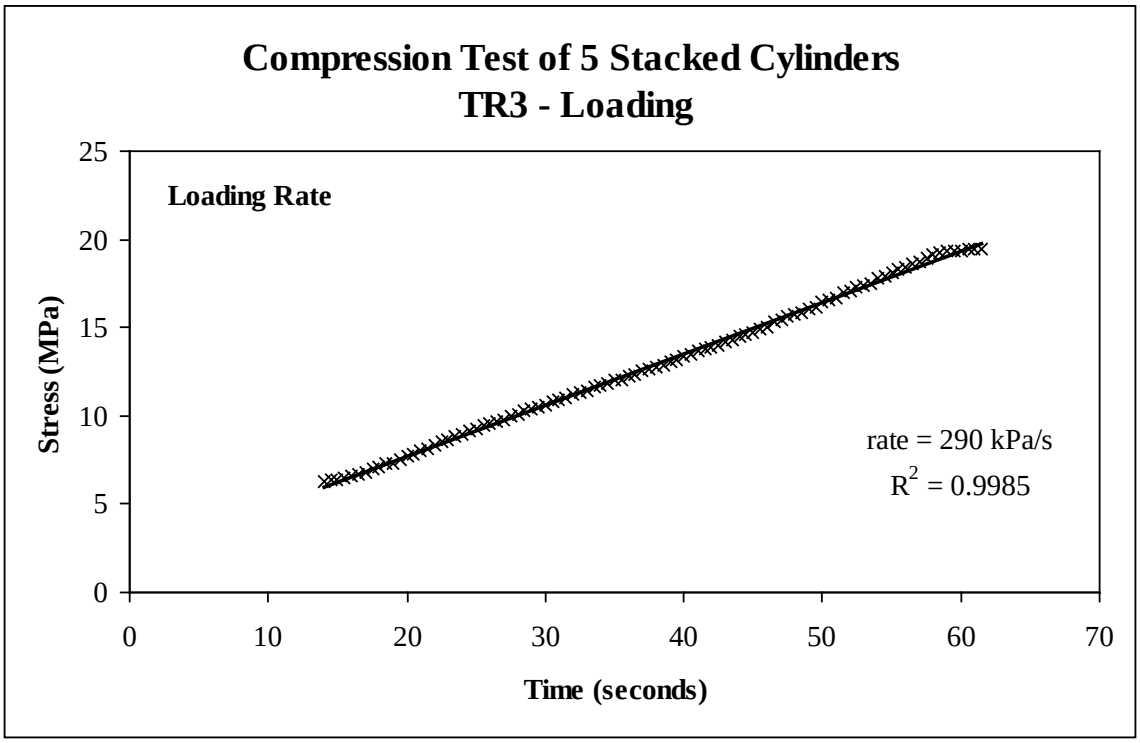
Compression Tests
Loading Rate Curves

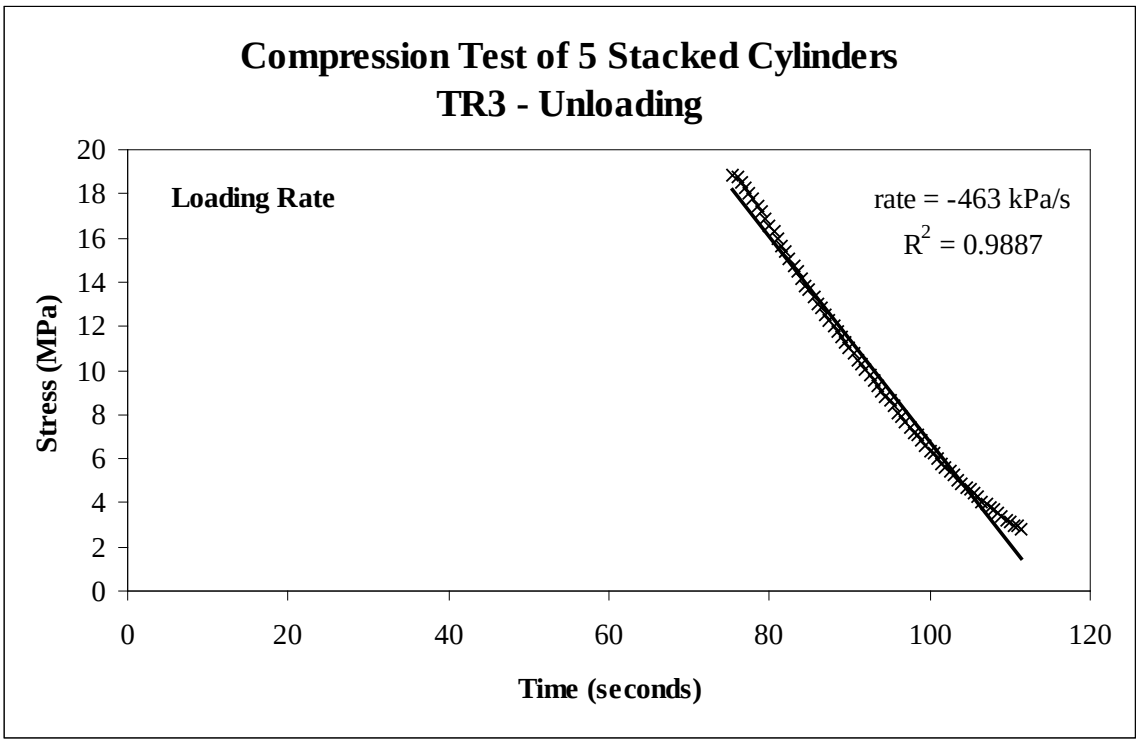

Compression Test of 5 Stacked Cylinders
TR1 - Loading
Loading Rate
rate = 66.2 kPa/s
R2 = 0.9674
Stress (MPa)
Time (seconds)
0
5
10
15
20
25
0
50
100
150
200
250
300
350
400

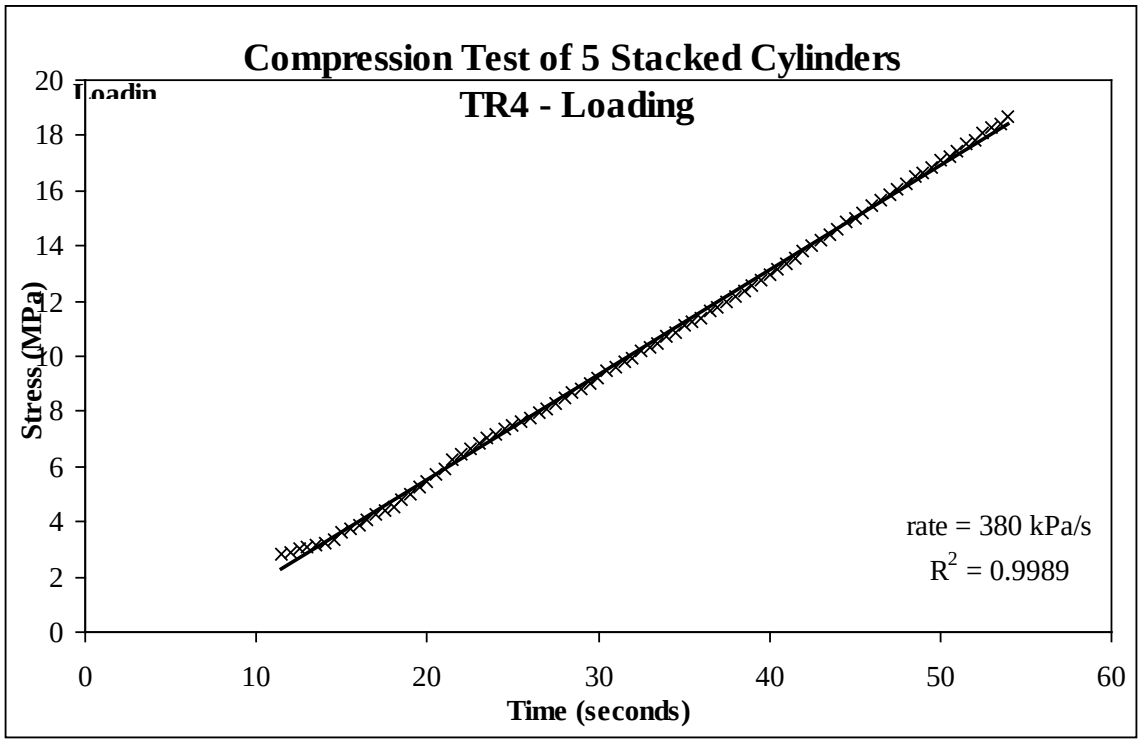

Compression Test of 5 Stacked Cylinders
TR1 - Unloading
Loading Rate
rate = -59.4 kPa/s
R2 = 0.9941
Stress (MPa)
Time (seconds)
0
5
10
15
20
25
0
100
200
300
400
500
600
700
800

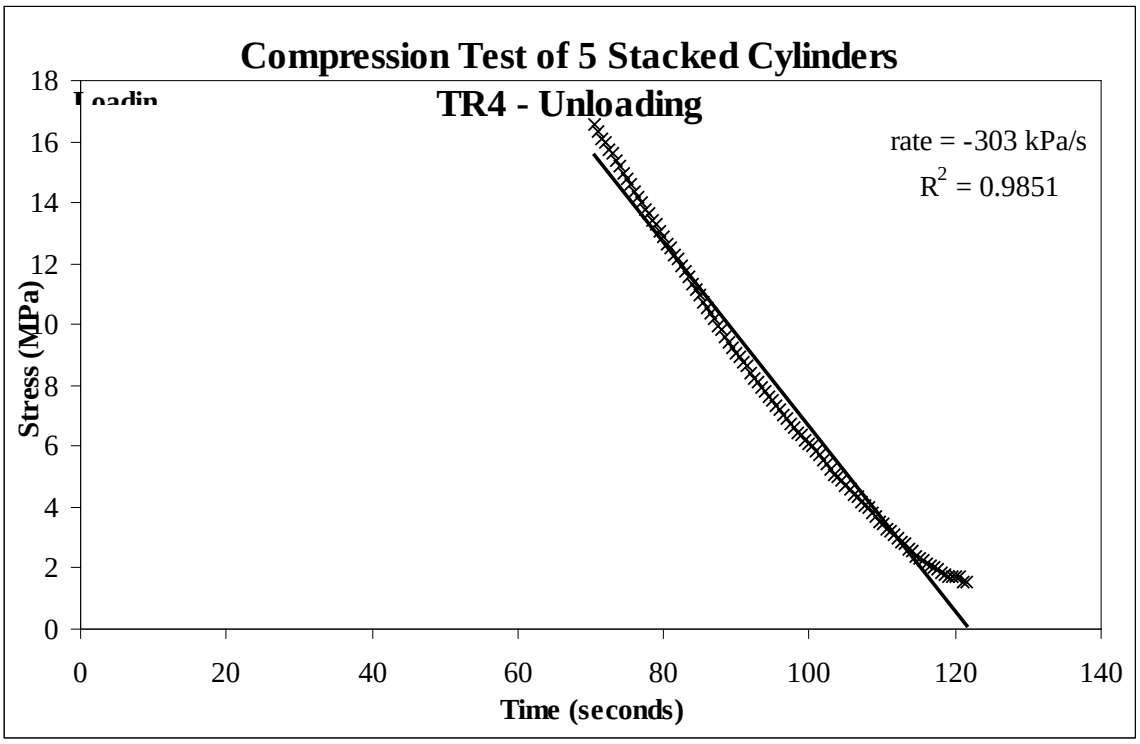

Compression Test of 5 Stacked Cylinders
TR2 - Loading
Loading Rate
rate = 99.2 kPa/s
R2 = 0.9898
Stress (MPa)
Time (seconds)
0
5
10
15
20
25
0
50
100
150
200
250

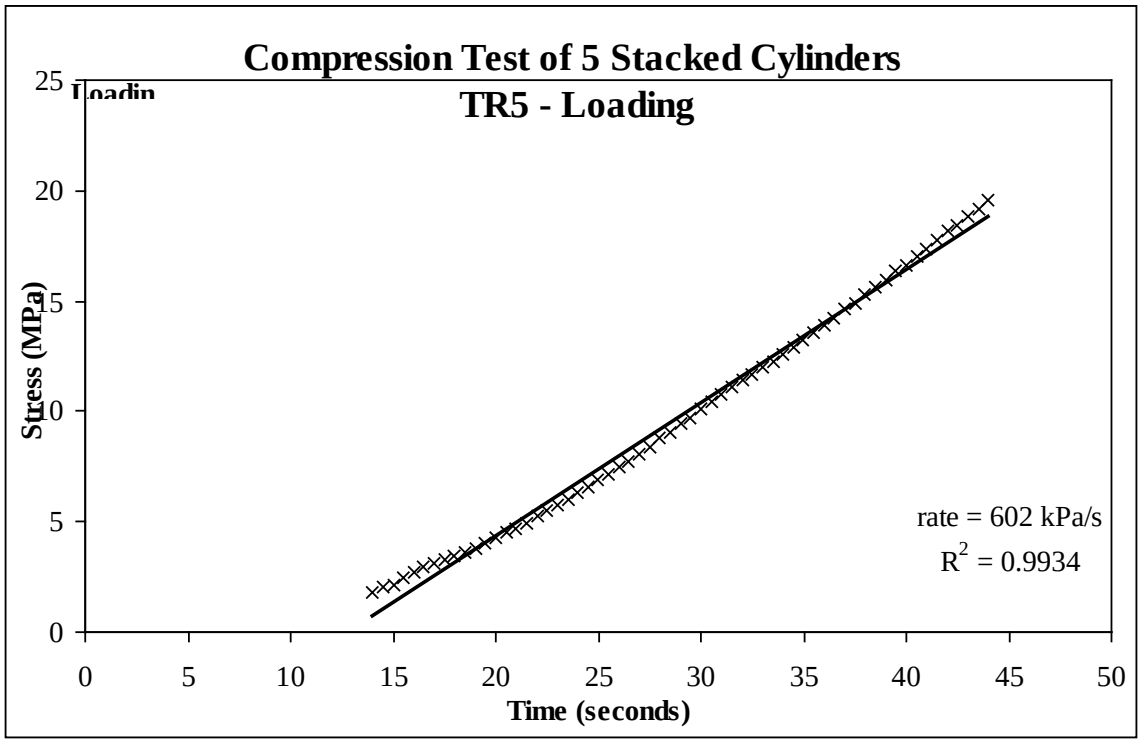

Compression Test of 5 Stacked Cylinders
TR2 - Unloading
Loading Rate
rate = -85.2 kPa/s
R2 = 0.9744
Stress (MPa)
Time (seconds)
0
5
10
15
20
25
0
50
100
150
200
250
300
350
400
450

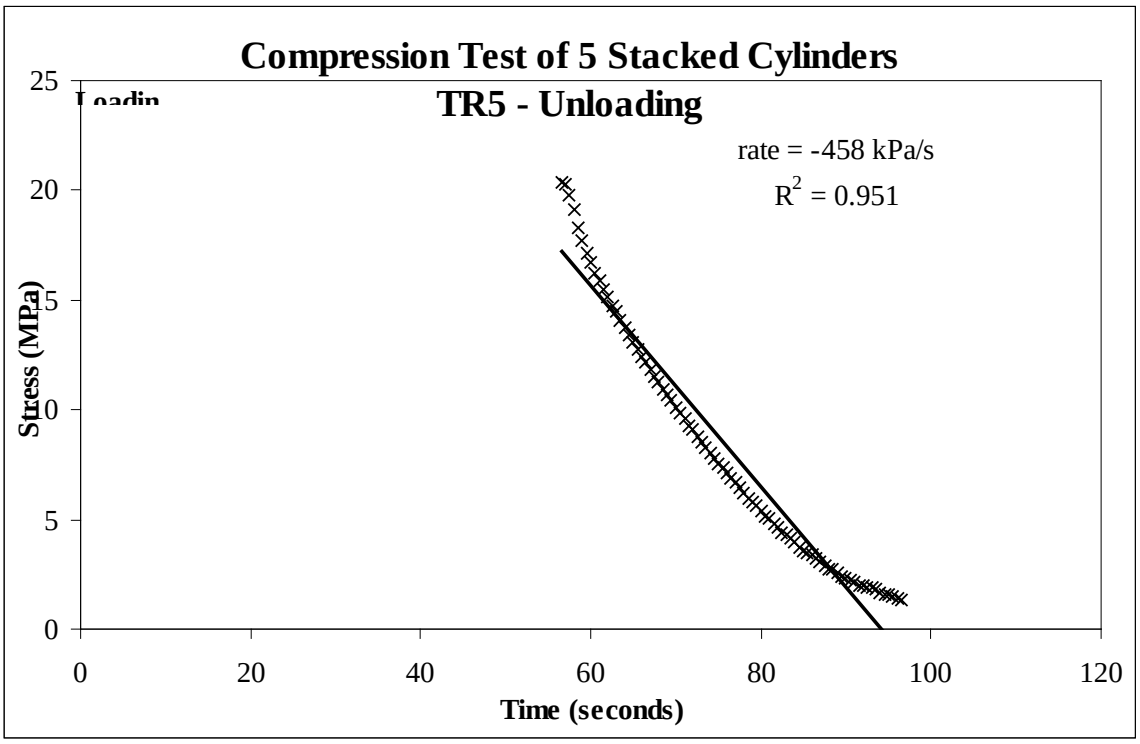

Compression Test of 5 Stacked Cylinders
TR3 - Loading
Loading Rate
rate = 290 kPa/s
R2 = 0.9985
Stress (MPa)
Time (seconds)
0
5
10
15
20
25
0
10
20
30
40
50
60
70

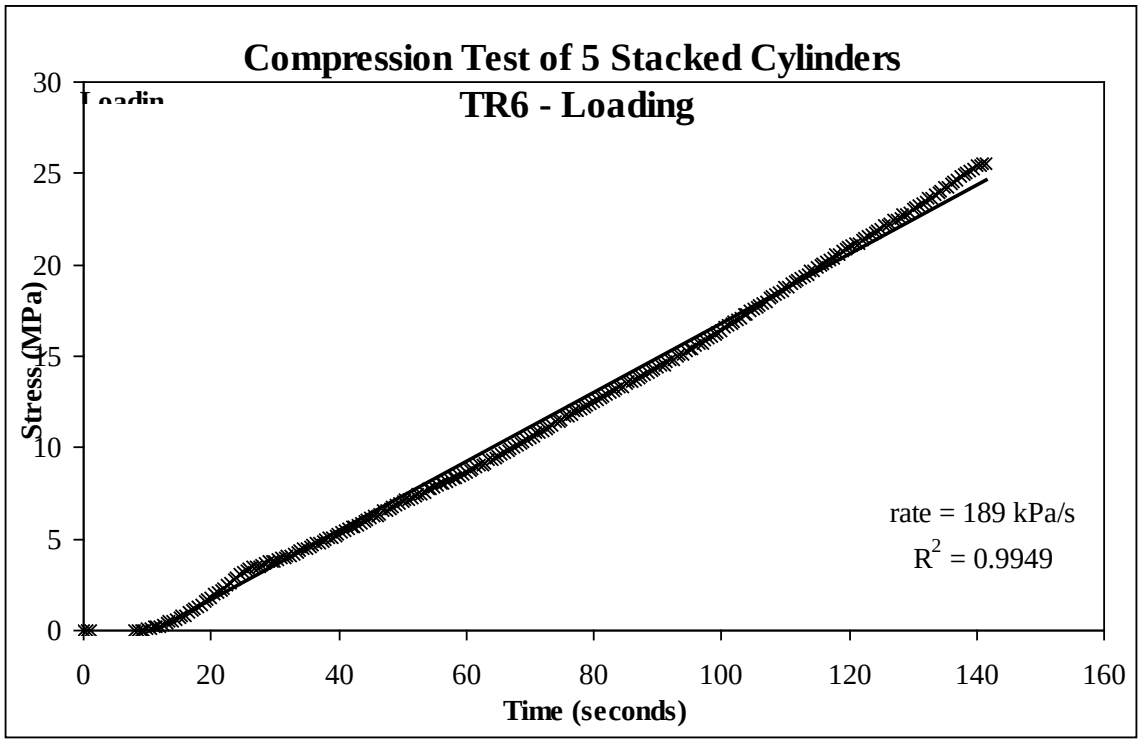

Compression Test of 5 Stacked Cylinders
TR3 - Unloading
Loading Rate
rate = -463 kPa/s
R2 = 0.9887
Stress (MPa)
Time (seconds)
0
2
4
6
8
10
12
14
16
18
20
0
20
40
60
80
100
120

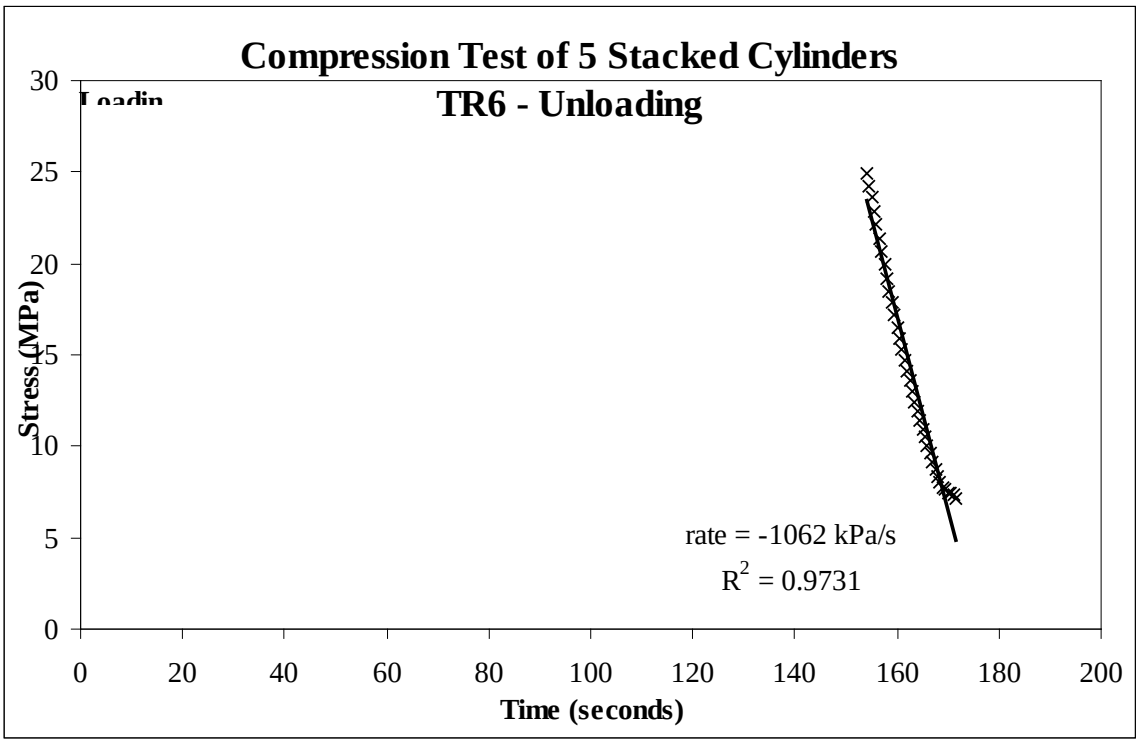

Compression Test of 5 Stacked Cylinders
TR4 - Loading
Stress (MPa)
Time (seconds)
rate = 380 kPa/s
R2 = 0.9989
0
2
4
6
8
10
12
14
16
18
20
0
10
20
30
40
50
60

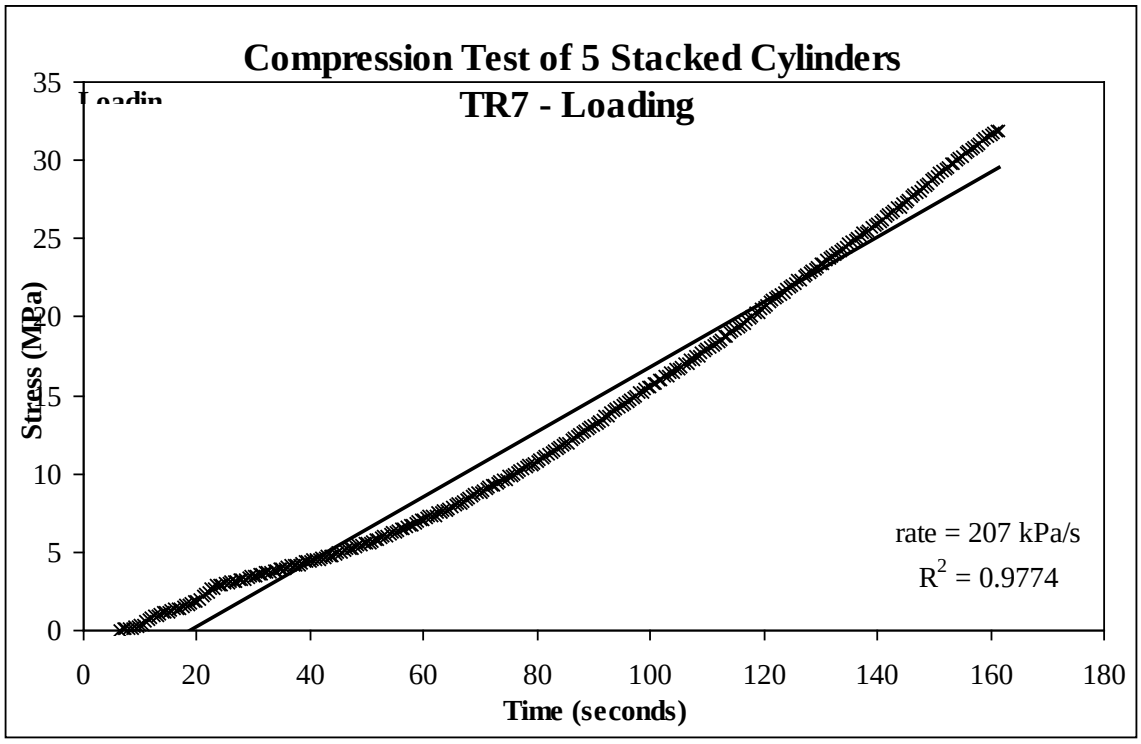

Compression Test of 5 Stacked Cylinders
TR4 - Unloading
Stress (MPa)
Time (seconds)
rate = -303 kPa/s
R2 = 0.9851
0
2
4
6
8
10
12
14
16
18
0
20
40
60
80
100
120
140

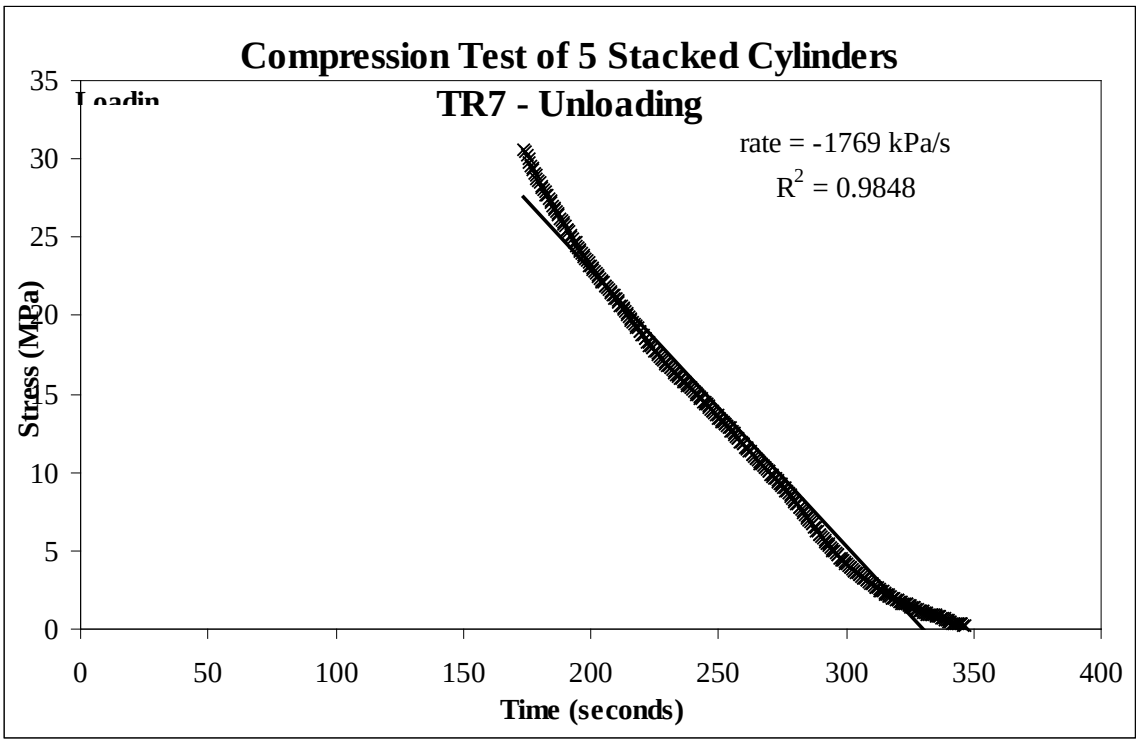

Compression Test of 5 Stacked Cylinders
TR5 - Loading
Stress (MPa)
Time (seconds)
rate = 602 kPa/s
R2 = 0.9934
0
5
10
15
20
25
0
5
10
15
20
25
30
35
40
45
50


Compression Test of 5 Stacked Cylinders
TR5 - Unloading
Stress (MPa)
Time (seconds)
rate = -458 kPa/s
R2 = 0.951
0
5
10
15
20
25
0
20
40
60
80
100
120


Compression Test of 5 Stacked Cylinders
TR6 - Loading
Stress (MPa)
Time (seconds)
rate = 189 kPa/s
R2 = 0.9949
0
5
10
15
20
25
30
0
20
40
60
80
100
120
140
160

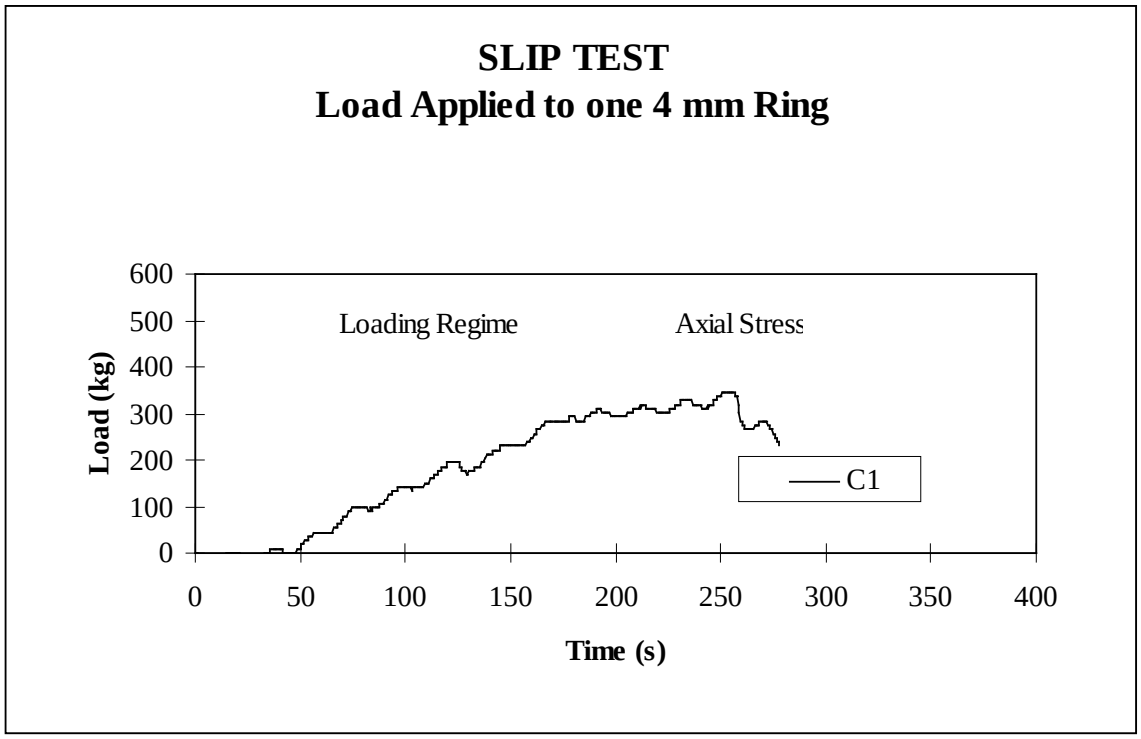

Compression Test of 5 Stacked Cylinders
TR6 - Unloading
Stress (MPa)
Time (seconds)
rate = -1062 kPa/s
R2 = 0.9731
0
5
10
15
20
25
30
0
20
40
60
80
100
120
140
160
180
200


Compression Test of 5 Stacked Cylinders
TR7 - Loading
Stress (MPa)
Time (seconds)
rate = 207 kPa/s
R2 = 0.9774
0
5
10
15
20
25
30
35
0
20
40
60
80
100
120
140
160
180

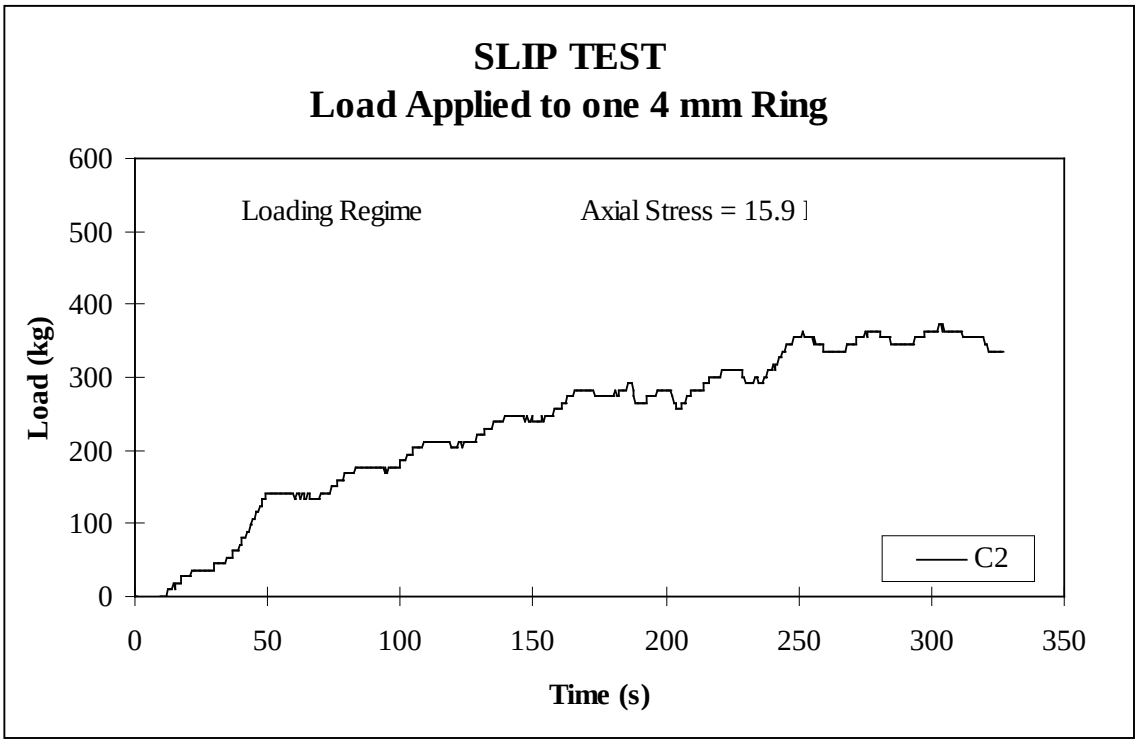

Compression Test of 5 Stacked Cylinders
TR7 - Unloading
Stress (MPa)
Time (seconds)
rate = -1769 kPa/s
R2 = 0.9848
0
5
10
15
20
25
30
35
0
50
100
150
200
250
300
350
400

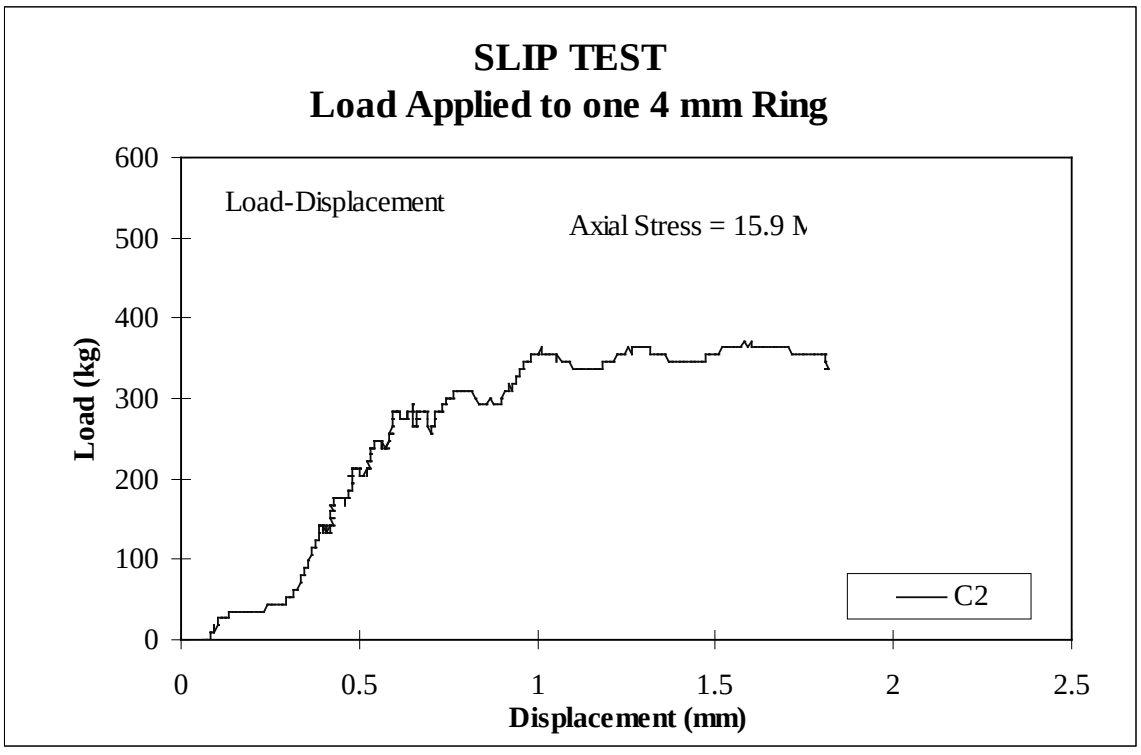
APPENDIX C

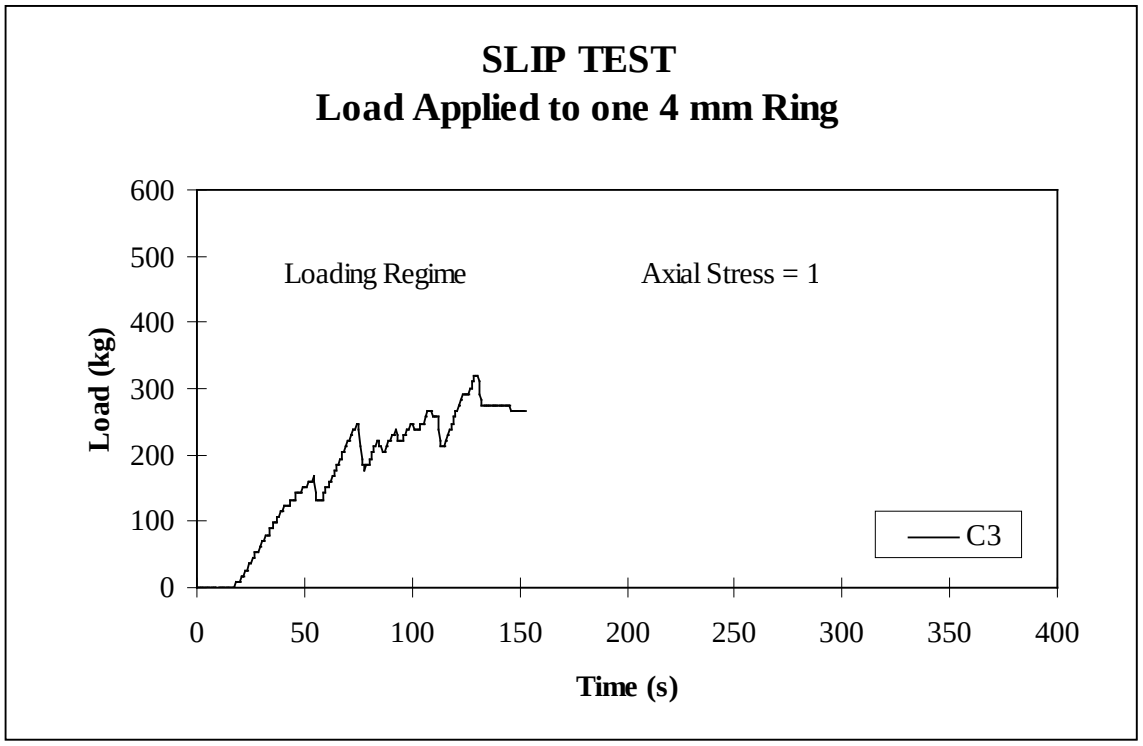
Load-slip Tests
Load and Time, Load-displacement Curves

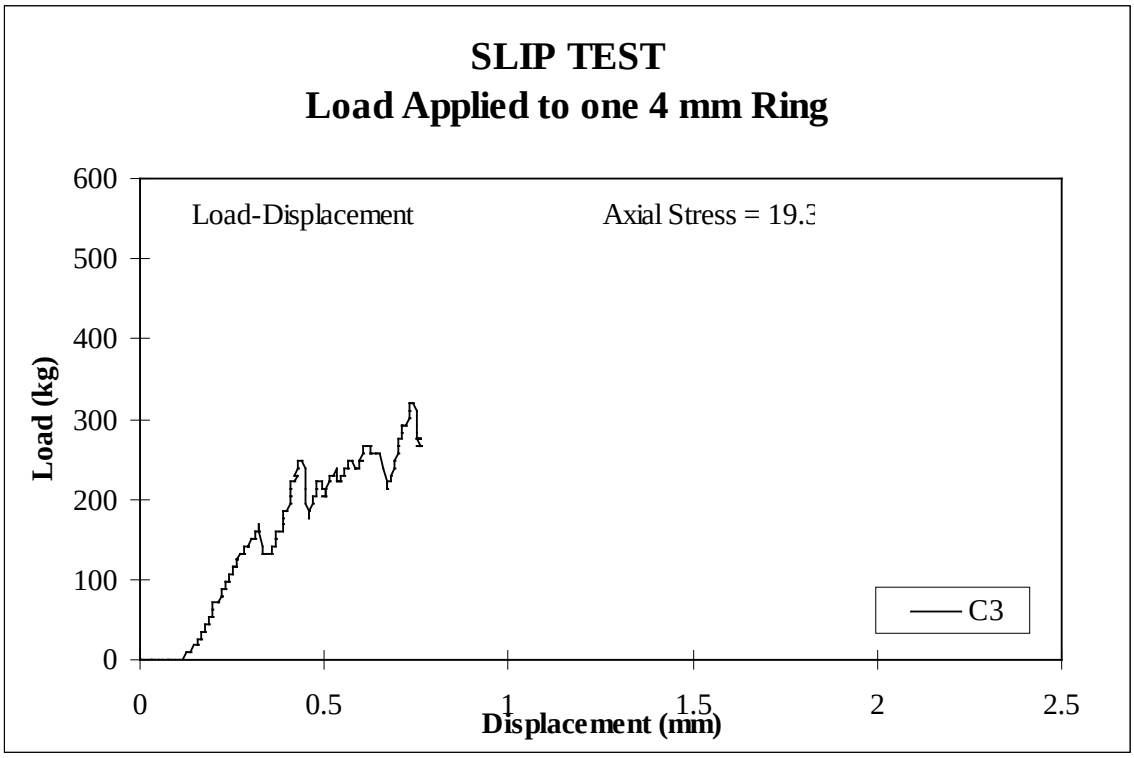


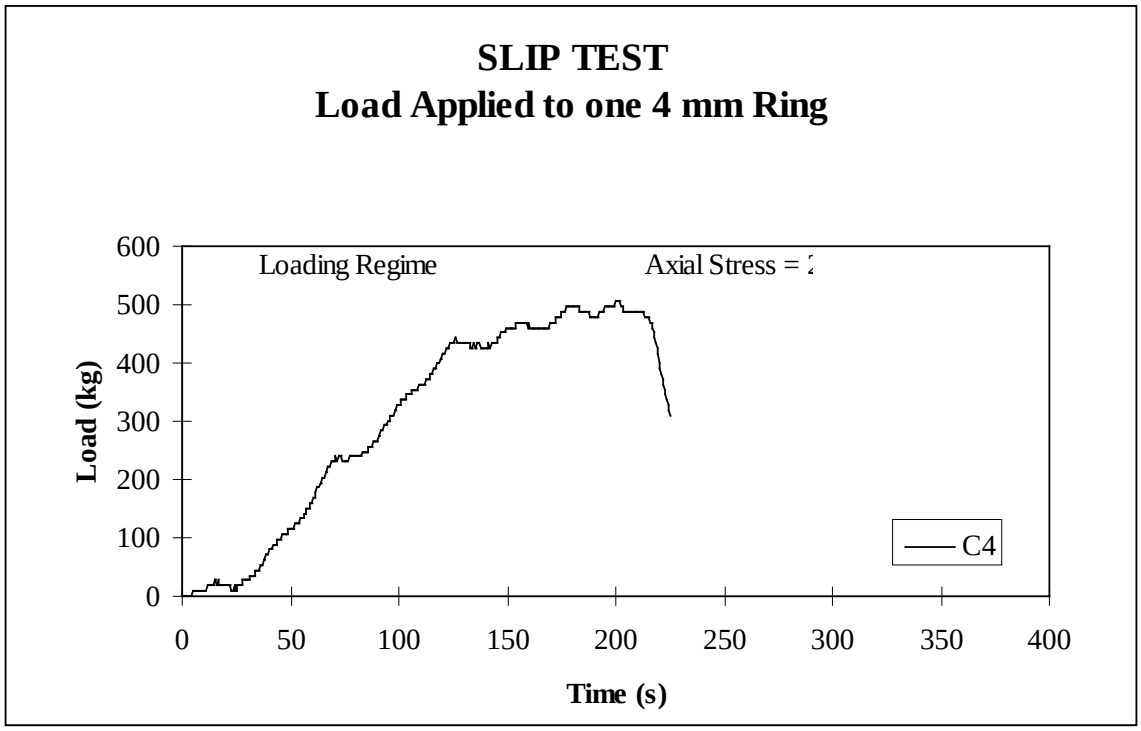
No displacement data
recorded
for this specimen.

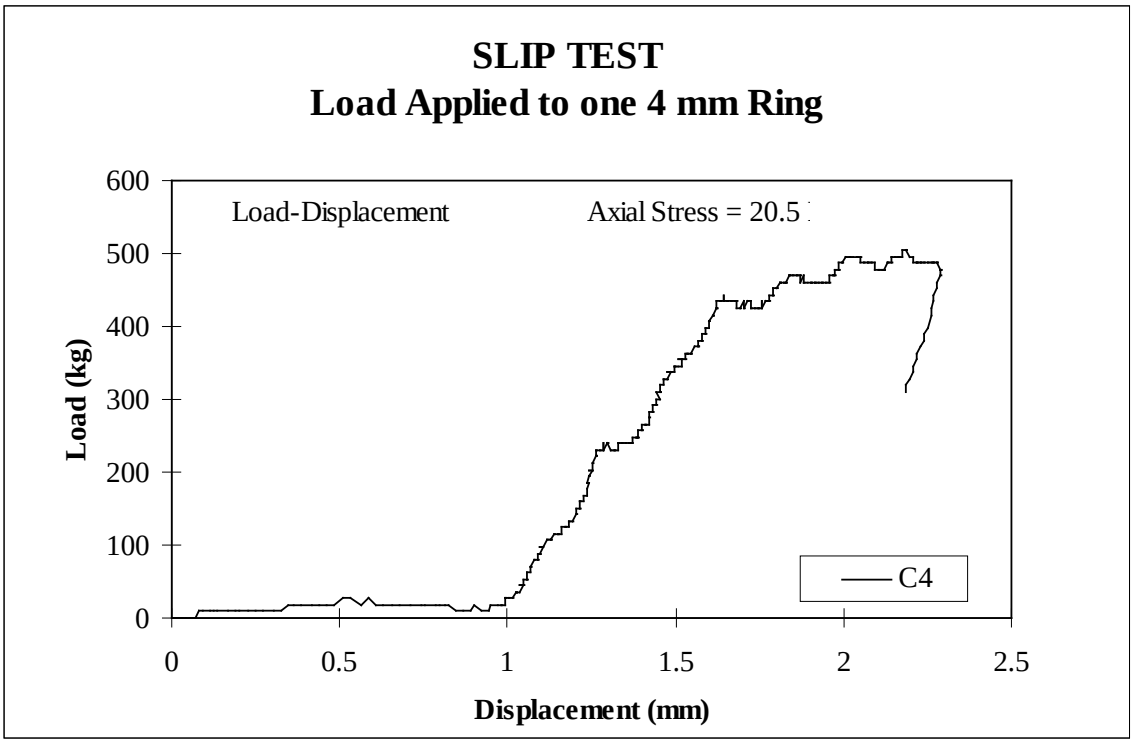

SLIP TEST
Load Applied to one 4 mm Ring
Loading Regime
Axial Stress = 15.9
Load (kg)
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
Time (s)
C2

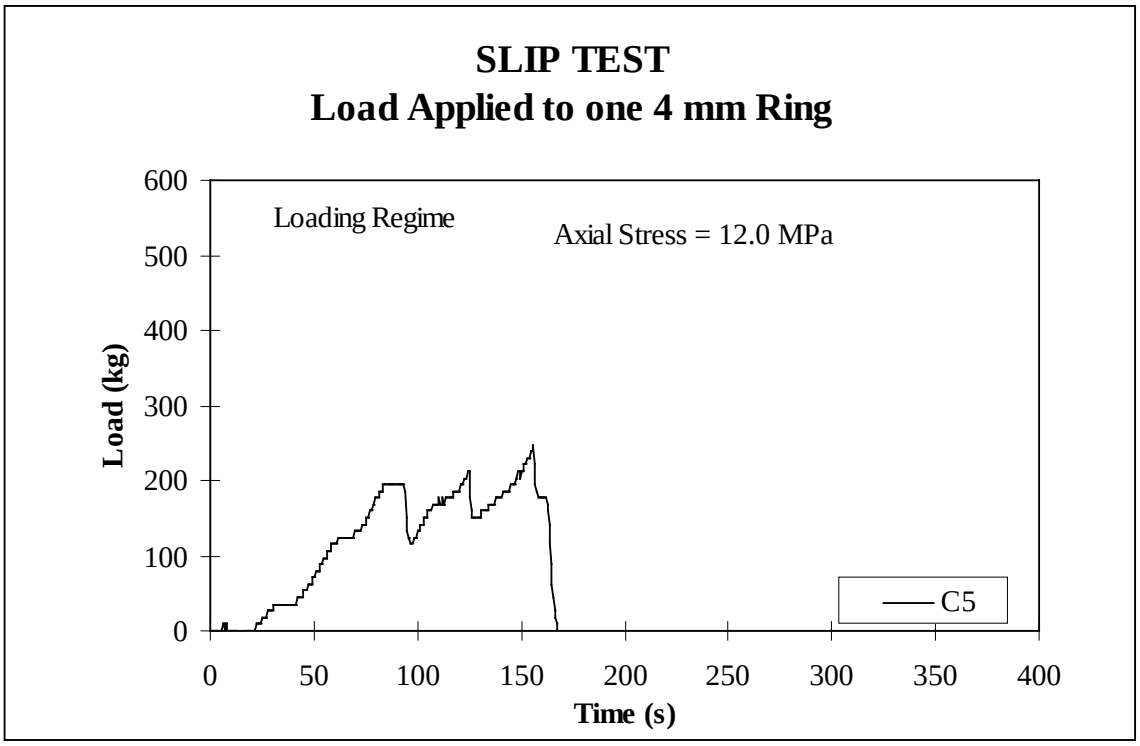

SLIP TEST
Load Applied to one 4 mm Ring
Load-Displacement
Axial Stress = 15.9
Load (kg)
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5
Displacement (mm)
C2

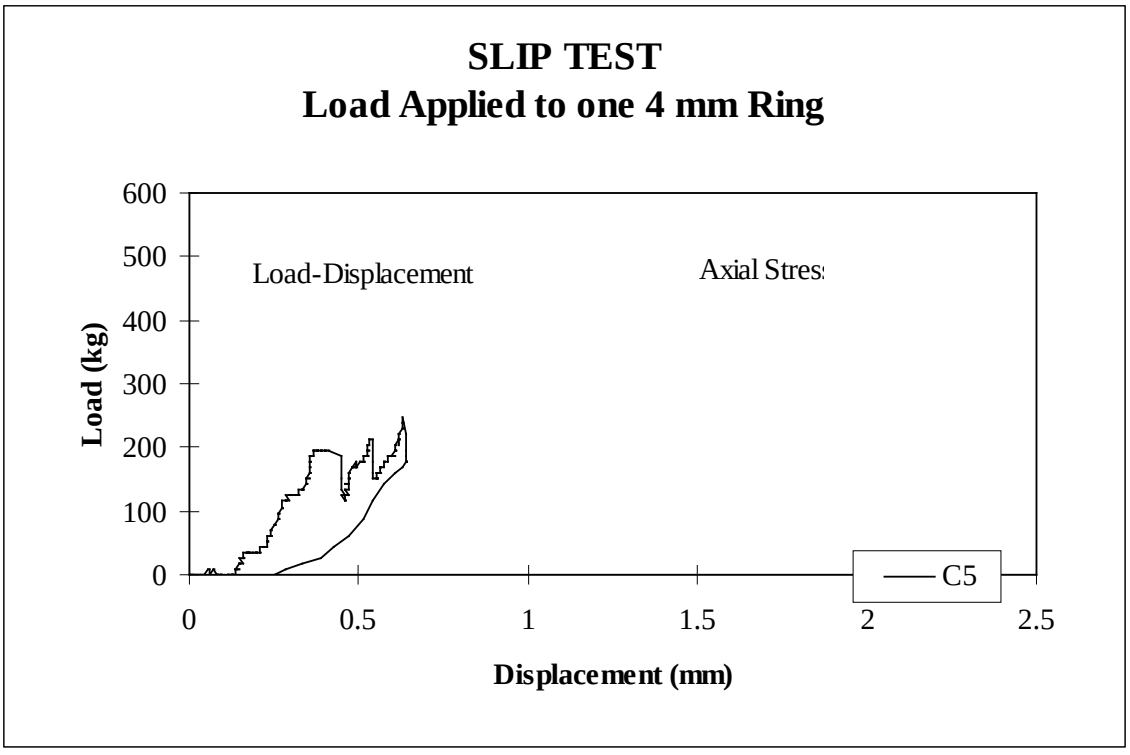

SLIP TEST
Load Applied to one 4 mm Ring
Loading Regime
Axial Stress = 1
Load (kg)
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400
Time (s)
C3

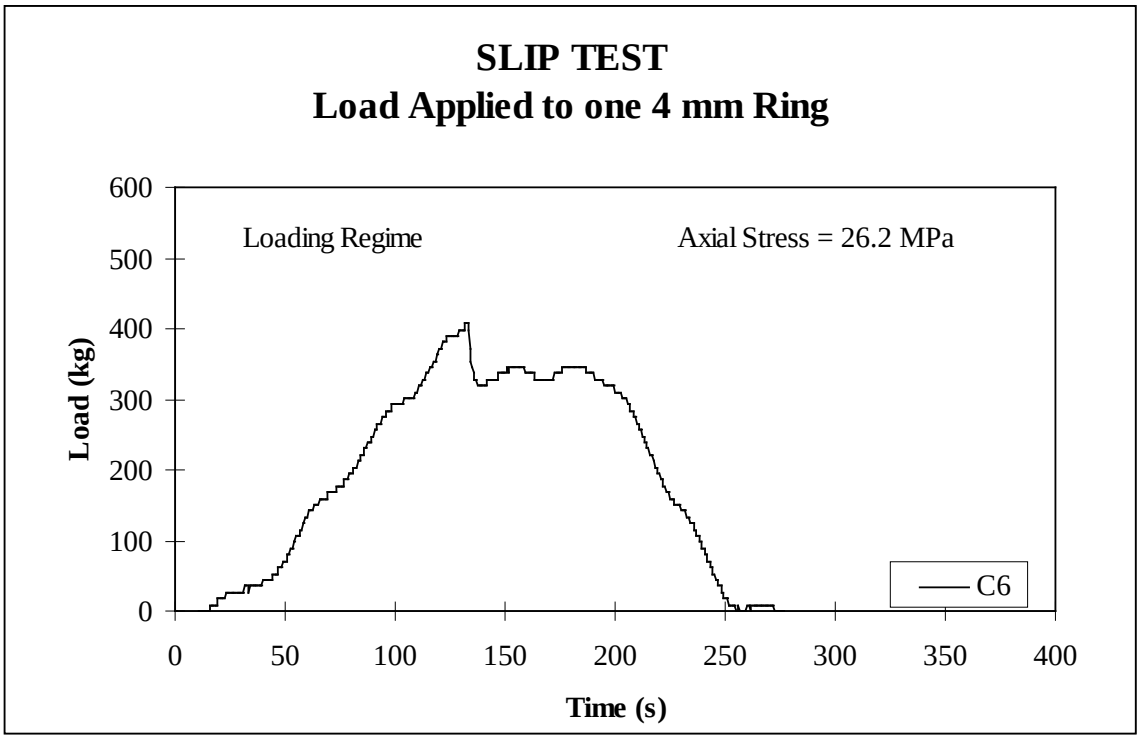

SLIP TEST
Load Applied to one 4 mm Ring
Load-Displacement
Axial Stress = 19.3
Load (kg)
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5
Displacement (mm)
C3

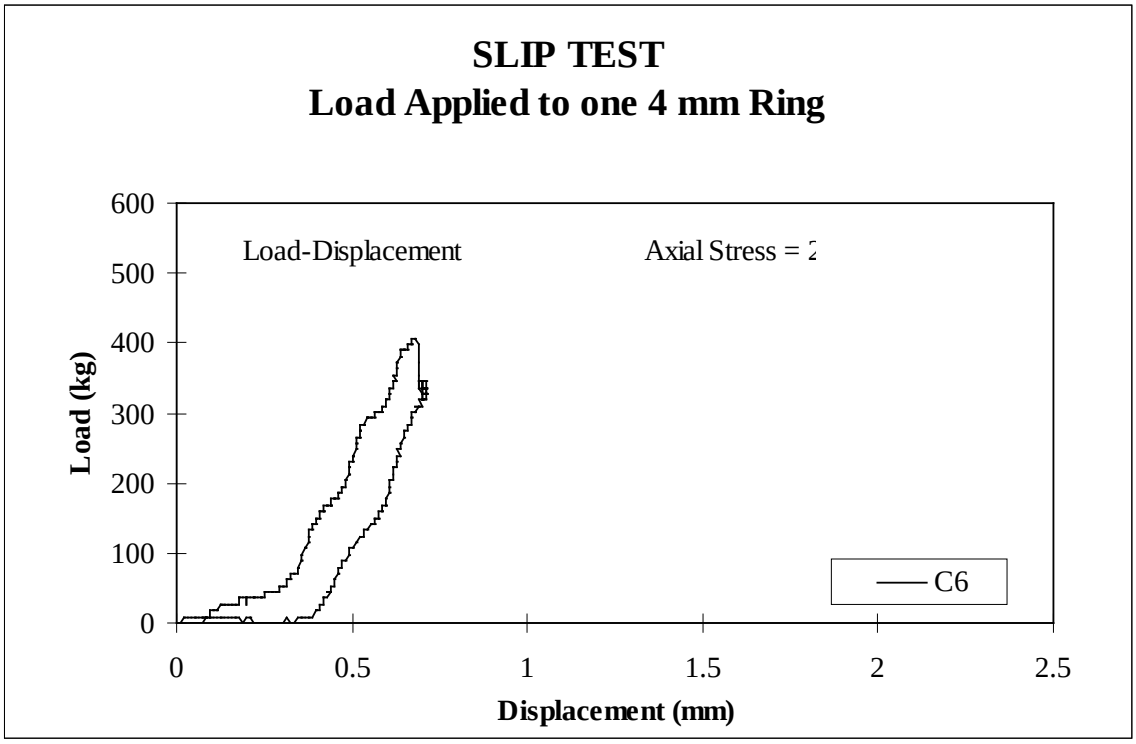

SLIP TEST
Load Applied to one 4 mm Ring
Loading Regime
Axial Stress =
Load (kg)
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400
Time (s)
C4


SLIP TEST
Load Applied to one 4 mm Ring
Load-Displacement
Axial Stress = 20.5
Load (kg)
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5
Displacement (mm)
C4


SLIP TEST
Load Applied to one 4 mm Ring
Loading Regime
Axial Stress = 12.0 MPa
Load (kg)
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400
Time (s)
C5

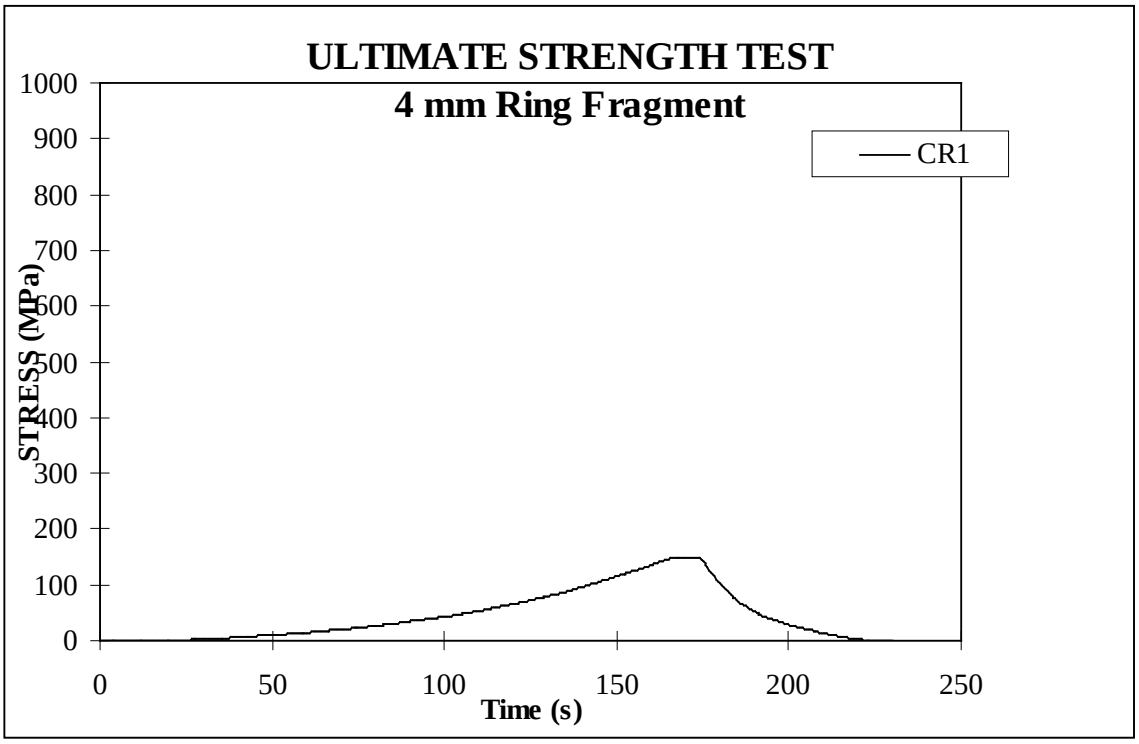

SLIP TEST
Load Applied to one 4 mm Ring
Load-Displacement
Axial Stres
Load (kg)
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5
Displacement (mm)
C5

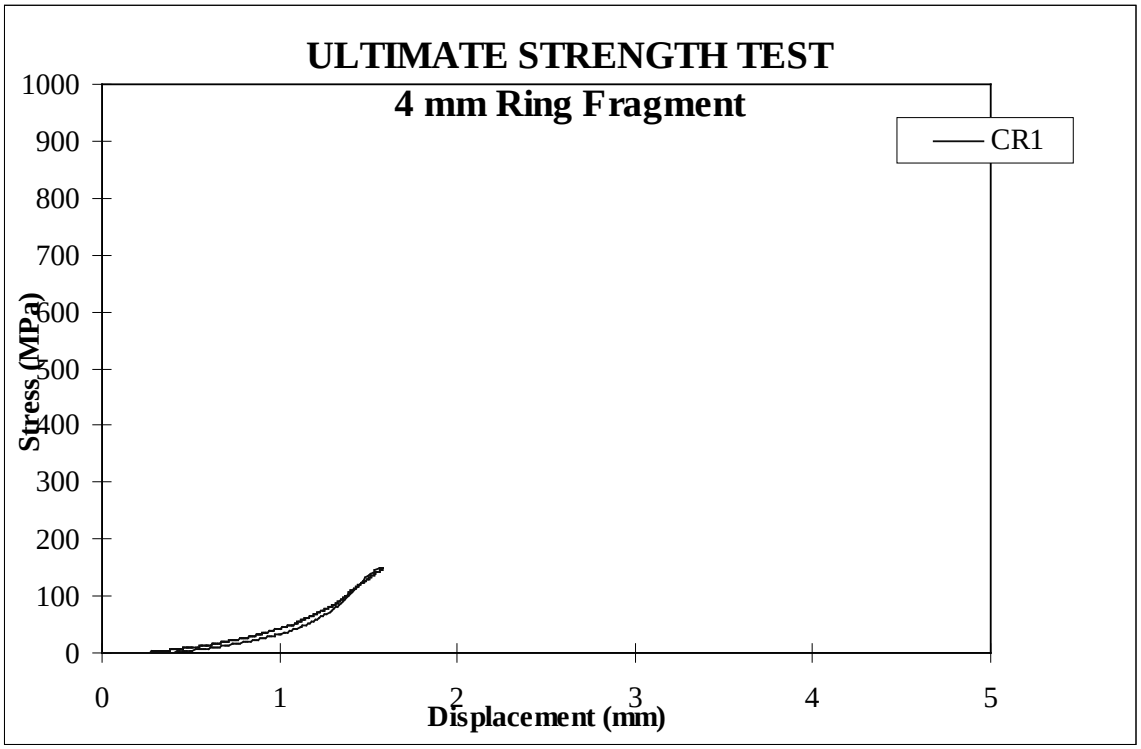

SLIP TEST
Load Applied to one 4 mm Ring
Loading Regime
Axial Stress = 26.2 MPa
Load (kg)
Time (s)
C6
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400

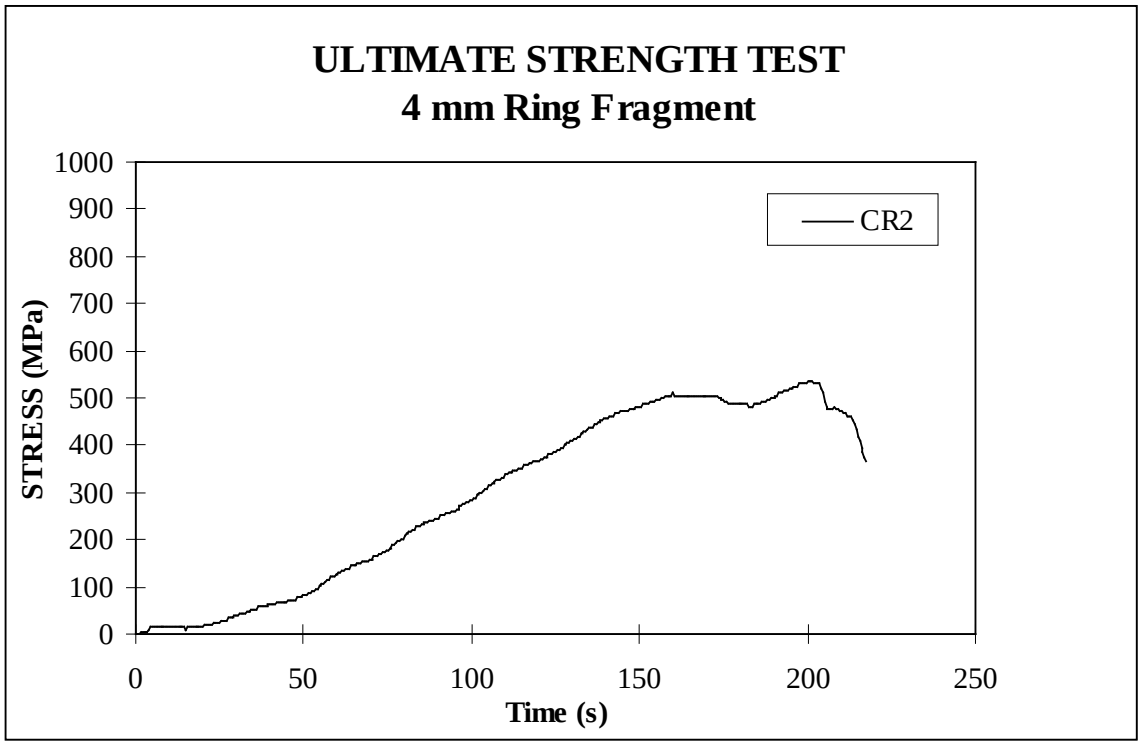

SLIP TEST
Load Applied to one 4 mm Ring
Load-Displacement
Axial Stress = 2
Load (kg)
Displacement (mm)
C6
600
500
400
300
200
100
0
0
0.5
1
1.5
2
2.5

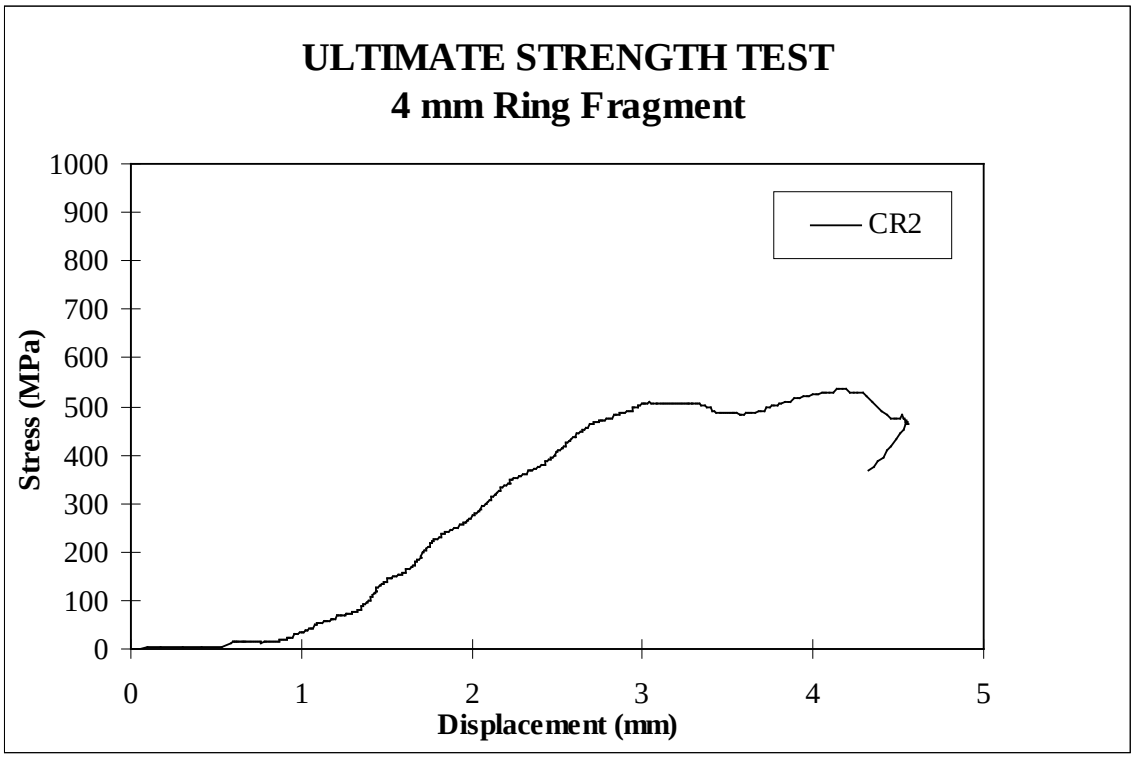
APPENDIX D

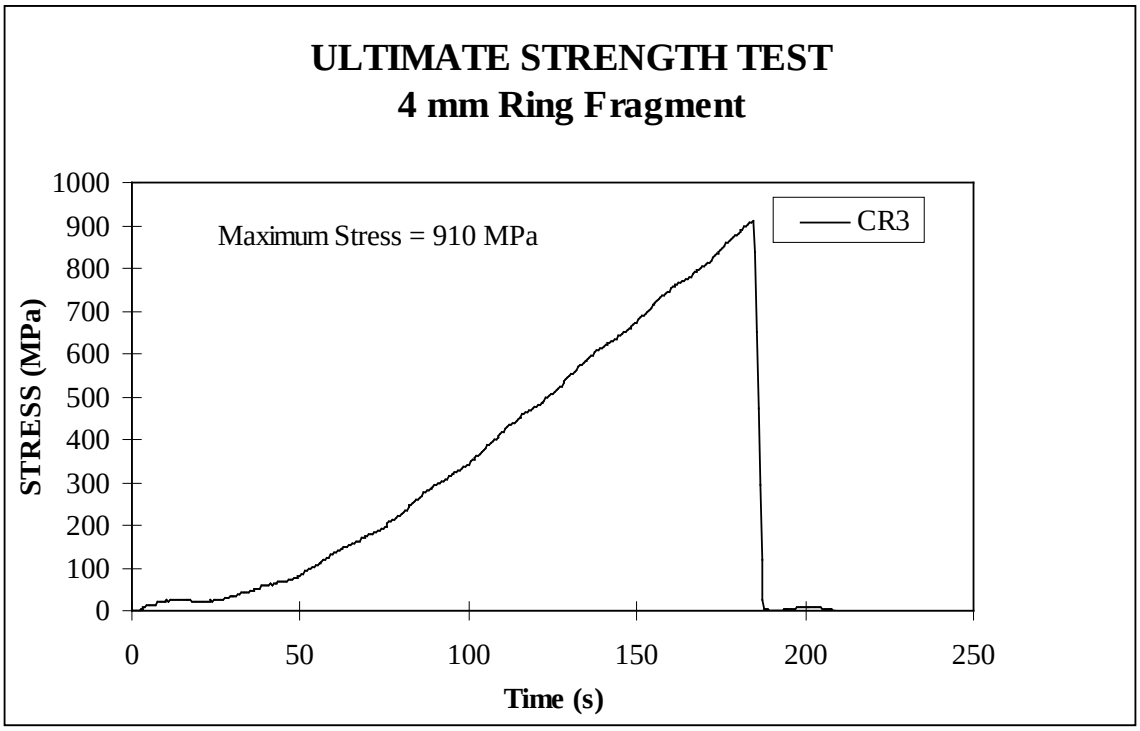
Ultimate Strength Tests

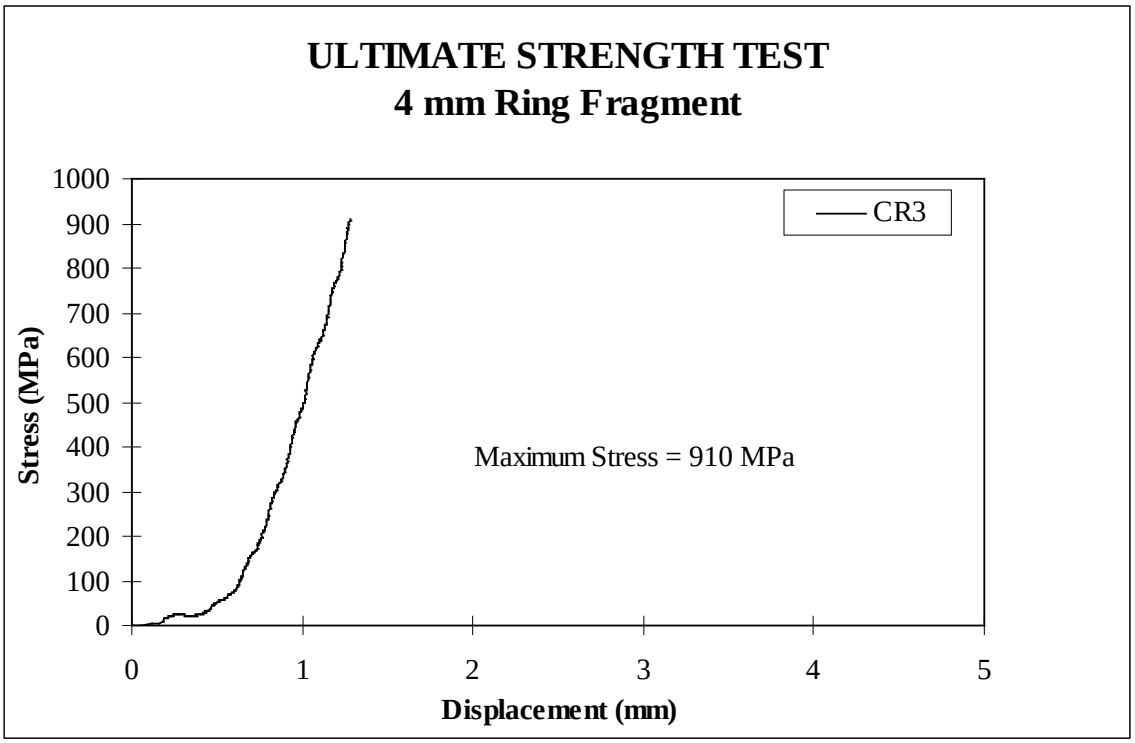

ULTIMATE STRENGTH TEST
4 mm Ring Fragment
CR1
STRESS (MPa)
Time (s)
0
100
200
300
400
500
600
700
800
900
1000
0
50
100
150
200
250


ULTIMATE STRENGTH TEST
4 mm Ring Fragment
CR1
Stress (MPa)
Displacement (mm)
0
100
200
300
400
500
600
700
800
900
1000
0
1
2
3
4
5


ULTIMATE STRENGTH TEST
4 mm Ring Fragment
STRESS (MPa)
Time (s)
CR2
0
100
200
300
400
500
600
700
800
900
1000
0
50
100
150
200
250

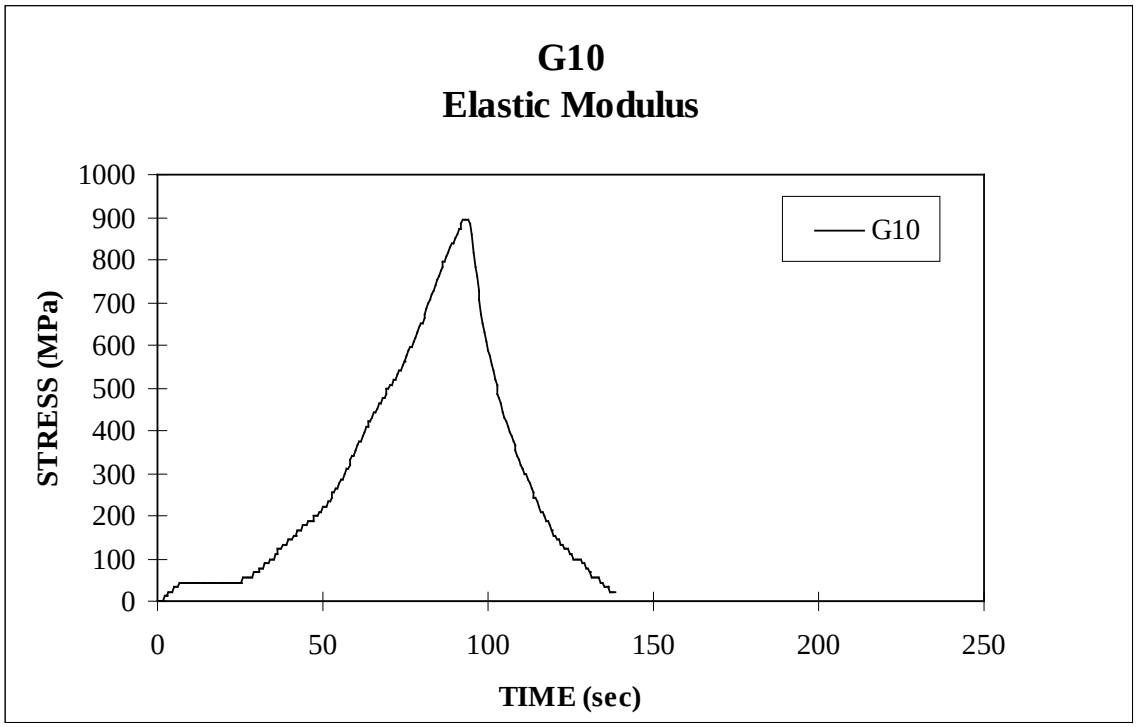

ULTIMATE STRENGTH TEST
4 mm Ring Fragment
Stress (MPa)
Displacement (mm)
CR2
0
100
200
300
400
500
600
700
800
900
1000
0
1
2
3
4
5

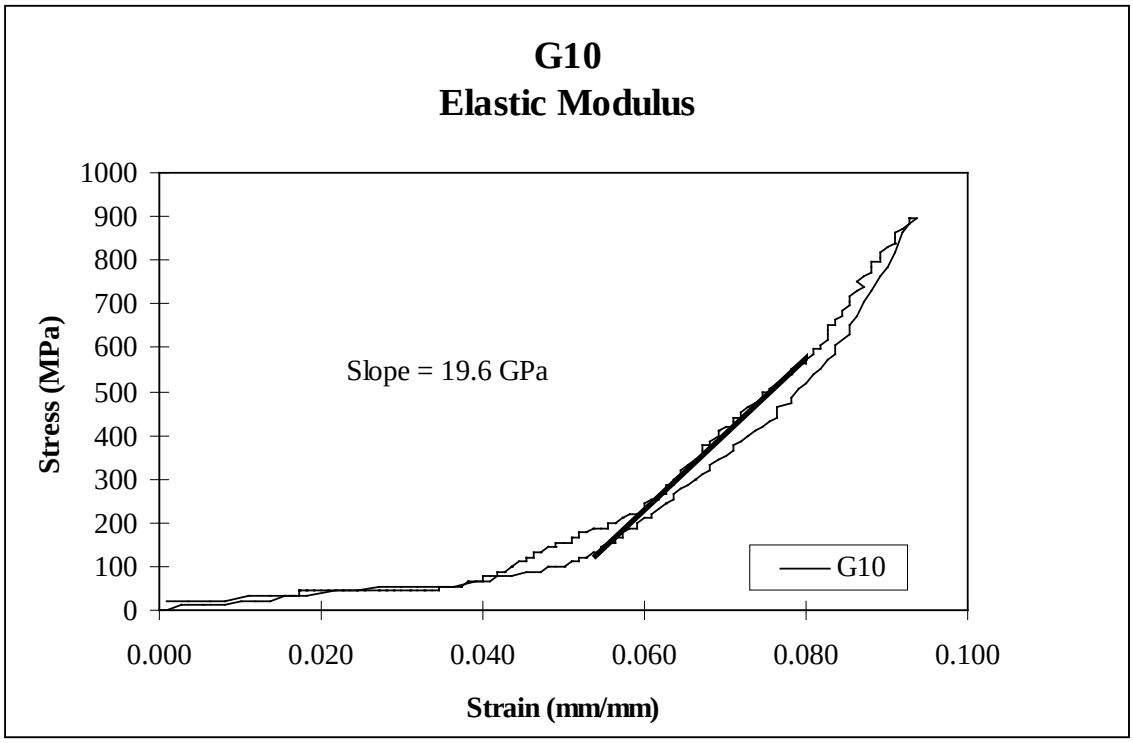

ULTIMATE STRENGTH TEST
4 mm Ring Fragment
Maximum Stress = 910 MPa
CR3
STRESS (MPa)
Time (s)
0
100
200
300
400
500
600
700
800
900
1000
0
50
100
150
200
250


ULTIMATE STRENGTH TEST
4 mm Ring Fragment
CR3
Maximum Stress = 910 MPa
Stress (MPa)
Displacement (mm)
0
100
200
300
400
500
600
700
800
900
1000
0
1
2
3
4
5

APPENDIX E

Compression Tests
G10
Load-displacement Curves


G10
Elastic Modulus
STRESS (MPa)
TIME (sec)
G10
0
100
200
300
400
500
600
700
800
900
1000
0
50
100
150
200
250

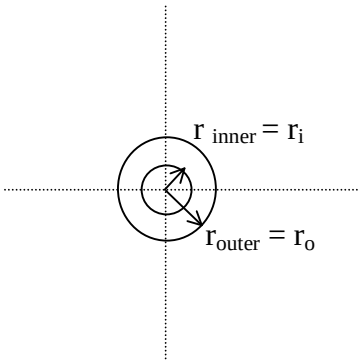

G10
Elastic Modulus
Stress (MPa)
Strain (mm/mm)
Slope = 19.6 GPa
G10
0
100
200
300
400
500
600
700
800
900
1000
0.000
0.020
0.040
0.060
0.080
0.100

APPENDIX F

Calculation of
Moment of Inertia, I
for Various Cross-sections

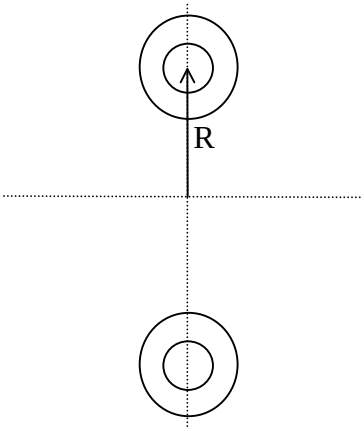
Given:

r_o = 0.025 m (outer radius of one CITAL segment)
r_i = 0.015 m (inner radius of one CITAL segment)
R = 0.26 m (radius of placement of 4 beam structure)



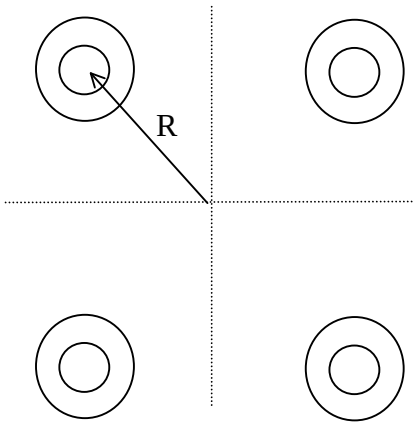
$$I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right) = 2.67\text{x}10^{-7}\,\text{m}^4$$

F.1



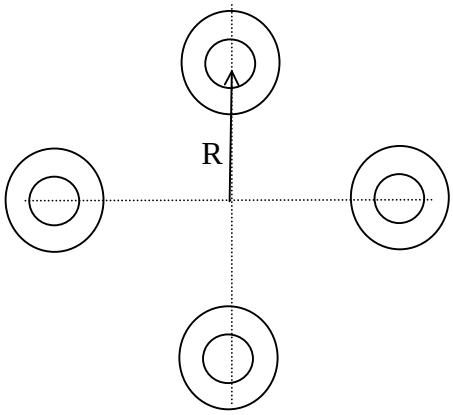
$$I = \pi\left[\frac{r_o^4}{2} - \frac{r_i^4}{2} + 2R^2 r_o^2 - 2Rr_o^3 - 2R^2 r_i^2 + 2Rr_o r_i^2\right]$$
$$= 1.541\text{x}10^{-4}\,\text{m}^4$$

F.2



$$I = \pi\left[r_o^4 - r_i^4 + 2R^2 r_o^2 - 2\sqrt{2}Rr_o^3 - 2R^2 r_i^2 + 2\sqrt{2}Rr_o r_i^2\right]$$
$$= 1.479\text{x}10^{-4}\,\text{m}^4$$

F.3



$$I = \pi\left[r_o^4 - r_i^4 + 2R^2 r_o^2 - 2R^2 r_i^2\right]$$
$$= 1.71\text{x}10^{-4}\,\text{m}^4$$

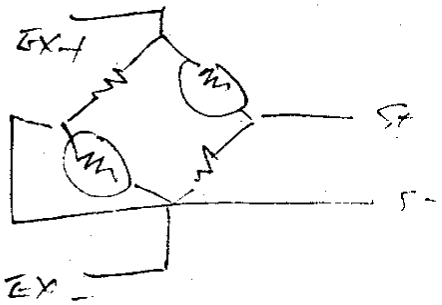
F.4

APPENDIX G

Strain Gauge Schematics

The Software is the Instrument®

$\frac{1}{2}$ BRIDGE



Sensitivity = 0.65 mV/V @ 1000 με

GF = 2.0

Sensitivity = $(0.65)\left(\frac{2.085}{2}\right) = .6776$ mV/V

@ 1000 με

GF = 2.085

* 6.776 μV/με at 10V excitation

scale factor = $\frac{\mu\varepsilon}{6.776\,\mu V}$ = 147579.693 $\frac{\mu\varepsilon}{Volt}$ ~~0.14758 με/Volt.~~

* use gain of 100

then $\frac{\mu\varepsilon}{6.776\,\mu V \times 100}$ = .0147579 $\frac{\mu\varepsilon}{Volt}$

= 1475.797 $\frac{\mu\varepsilon}{Volt}$

NATIONAL INSTRUMENTS®
6504 Bridge Point Parkway
Austin, TX 78730-5039
(512) 794-0100

John Wong, Electrical
Civil Engineering Dept.

ENGINEERING DATA SHEET

THE INFORMATION APPEARING ON THIS SHEET HAS BEEN COMPILED SPECIFICALLY FOR THE GAGES CONTAINED IN THIS PACKAGE. THIS FORM IS PRODUCED WITH ADVANCED EQUIPMENT & PROCEDURES WHICH PERMIT COMPREHENSIVE QUALITY ASSURANCE VERIFICATION OF ALL DATA SUPPLIED HEREIN. SHOULD ANY QUESTIONS ARISE RELATIVE TO THESE GAGES, PLEASE MENTION GAGE TYPE, BATCH NUMBER, AND LOT NUMBER.

H001

Prod OA	Check	Batch
MM 37	EMF	S24801

Micro-Measurements Division
Made in USA

MEASUREMENTS GROUP, INC.
RALEIGH, NORTH CAROLINA

PRECISION STRAIN GAGES

F007

TYPE	LOT NUMBER	QUANTITY	OPTION	RESISTANCE IN OHMS AT 24°C	GAGE FACTOR AT 24°C	CODE
CEA-06-250UW-350	R-A38AD619	5		350.0 ±0.3%	2.085 ±0.5%	993414
CEA-06-250UW-350					(+0.4 ±0.2)%	

GENERAL INFORMATION: CEA-SERIES STRAIN GAGES

GENERAL DESCRIPTION: CEA gages are a general-purpose family of constantan strain gages widely used in experimental stress analysis. The gages are supplied with a fully encapsulated grid and exposed copper-coated integral solder tabs.

TEMPERATURE RANGE: -100° to +400° F (-75° to +205° C) for continuous use in static measurements.

SELF-TEMPERATURE COMPENSATION: See data curve below.

STRAIN LIMITS: Approximately 5% for gage lengths 1/8 in. (3.2 mm) and larger; approximately 3% for gage lengths under 1/8 in. (3.2 mm).

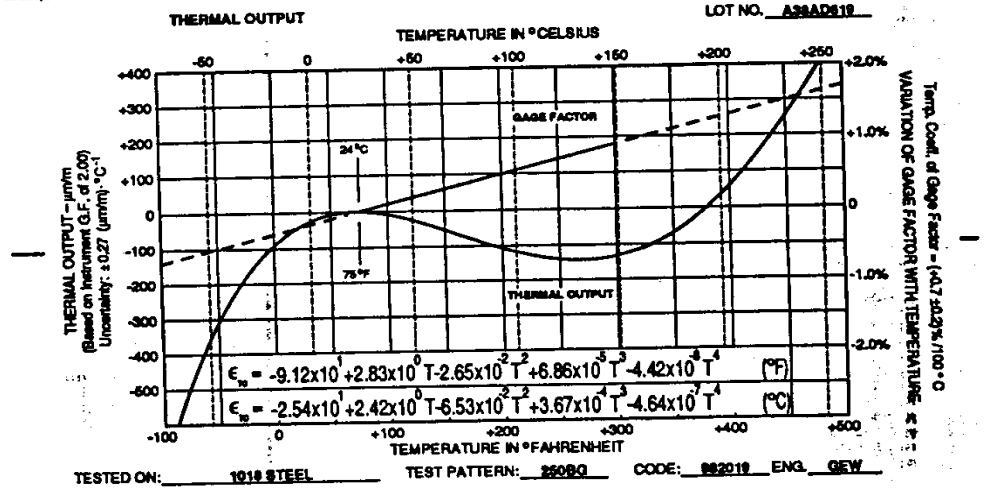
FATIGUE LIFE: Fatigue life is a marked function of solder joint formation. With 30-AWG leads directly attached to gage tabs, fatigue life will be 10^6 cycles at ±1500 μin/in (μm/m) using M-Line 361A solder.

CEMENTS: Compatible with M-M Certified M-Bond 200, but it will normally not provide the greatest strain limit. Micro-Measurements M-Bond AE-10/15, M-Bond GA-2, M-Bond 600, and M-Bond 610 are excellent. M-Bond 610 is the best choice over the entire operating range. Refer to M-M Catalog A-110 for information on bonding agents, and Bulletins B-127, B-130, and B-137 for installation procedures.

SOLDER: If operating temperature will not exceed +300° F (+150° C), M-Line solder 361A (63-37) tin-lead solder may be used for lead attachment. M-Line solder 450 (95-5) tin-antimony is satisfactory to +400° F (+205° C). Refer to M-M Catalog A-110 for further information on solders, and Tech Tip TT-609 for lead attachment techniques.

BACKING: The backing of CEA-Series gages has been specially treated for optimum bond formation with all appropriate strain gage adhesives. No further cleaning is necessary if contamination of the prepared surface is avoided during handling.

G045



TESTED ON: 1018 STEEL TEST PATTERN: 250BG CODE: 992019 ENG. GEW

TEST PROCEDURES USED BY MICRO-MEASUREMENTS

OPTICAL DEFECT ANALYSIS	M-M Procedures and Standards
GAGE RESISTANCE AT 24°C AND 50% RH	M-M Procedure, Direct NBS Traceability on Resistance Standards
GAGE FACTOR AT 24°C & 50% RH (UNIAXIAL STRESS FIELD - POISSON RATIO -0.285)	ASTM E-251 (Constant Stress Cantilever Method)
TEMPERATURE COEFFICIENT OF GAGE FACTOR	ASTM E-251 (Step Deflection Method)
THERMAL OUTPUT	ASTM E-251 (Slow Heating Rate, Continuously Recorded)
TRANSVERSE SENSITIVITY AT 24°C AND 50% RH	ASTM E-251
FATIGUE LIFE	NAS 942 (Modified)
STRAIN LIMITS	NAS 942 (Modified)
GAGE THICKNESS	M-M Procedure
CREEP AND DRIFT	M-M Procedure (Similar to NAS 942 Method)

NOTE: Gage resistance, gage factor, temperature coefficient of gage factor, thermal output, and transverse sensitivity testing and information presentation are in compliance with CEA International Recommendation NO. 68, "Performance characteristics of metallic resistance strain gages". Other tests are not included in LR. NO. 68.

POSITIONING STRAIN GAGES TO MONITOR BENDING, AXIAL, SHEAR, AND TORSIONAL LOADS

In the glossary of the Pressure Reference Section strain is defined as; The ratio of the change in length to the initial unstressed reference length. A strain gage is the element that senses this change and converts it into an electrical signal. This can be accomplished due to the fact that a strain gage changes resistance as it is stretched or compressed, similar to wire. For example, when wire is stretched its cross sectional area decreases, therefore it increases resistance.

The important factors that must be considered before selecting a strain gage are the direction, type, and resolution of strain you wish to measure.

To measure minute strains the user must be capable of measuring minute resistance changes. The Wheatstone Bridge configuration as shown in figure B. is capable of measuring these small resistance changes Note the signs associated with each gage numbered 1 through 4. The total strain is always the sume of the four strains.

$$\epsilon_T = \epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4$$

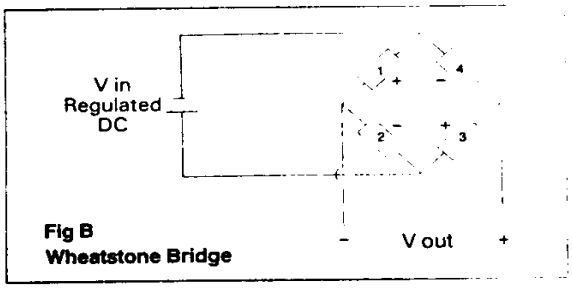
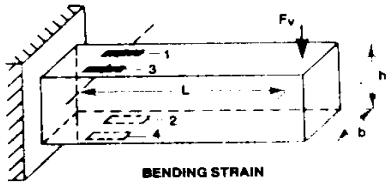
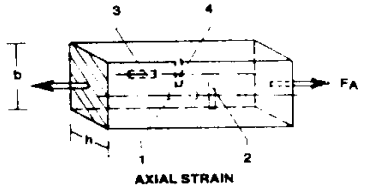


Fig B
Wheatstone Bridge

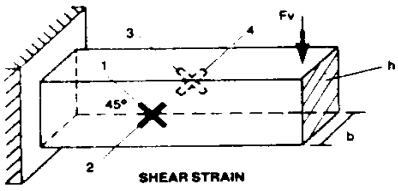
The total strain is represented by a change in V out. If each gage had the same positive strain, the total would be zero and V out would remain unchanged. Bending, Axial, and Shear strain make up the most common types of strain measurements. The actual arrangement of the strain gages will determine the type of strain measured and the output voltage change. See figures C through F.



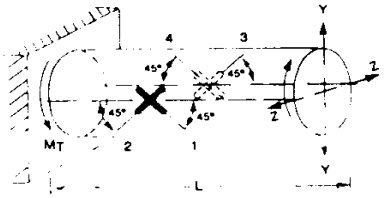
BENDING STRAIN
Fig C



AXIAL STRAIN
Fig D



SHEAR STRAIN
Fig E



TORSIONAL STRAIN
Fig F

For example, if a positive (tensile) strain was on gages 1 and 3, and a negative (compressive) strain was on gages 2 and 4. The total strain would be 4 times the strain on one gage. See figure C.

The total strain being four times the strain of one gage, means that the output will be four times larger. Therefore greater sensitivity and resolution occures when more than one strain gage is used.

The following equations show the relationships between stress, strain, and force for bending, axing, shear, and torsional strain.

1) BENDING STRAIN or moment strain is equal to bending stress divided by Young's Modulus of Elasticity.

$$\epsilon_B = o_B/E \qquad o_B = M_B/Z = F\nu\,(\ell)/Z$$

Moment stress (o_B) equals bending moment ($F\nu \times \ell$) divided by sectional modulus. Sectional modulus (z) is a property of the cross sectional configuration of the specimen. For rectangles only, the sectional modulus is ($bh^2/6$). Strain gages used in the bending strain configuration can be used to determine vertical load ($F\nu$), more commonly referred to as a bending beam load cell.

$$F_\nu = E\,\epsilon_B\,(Z)/\ell = E\,\epsilon_B\,(bh^2/6)/\ell$$

2) AXIAL STRAIN equals axial stress divided by Youngs Modulus.

$$\epsilon_A = o_A/E \qquad o_A = F_A/A$$

Where axial stress (o_A) equals the axial load divided by the cross sectional area. The cross sectional area for rectangles equals (b x d). Therefore strain gages used in axial configurations can be used to determine axial loads (F (axial)).

$$F\,(\text{axial}) = E\,\epsilon_A bh$$

3) SHEAR STRAIN equals shear stress divided by modulus of shear stress.

$$\gamma = \tau/G \qquad \tau = F_\nu \times Q/bI$$

Where shear stress (τ) equals (Q), the moment of area about the neutral axis multiplied by the vertical load (F_ν) divided by the thickness (b) and the moment of inertia (I). Both the moment of area (Q) and the moment of inertia (I) are functions of the specimens cross sectional geometry.

For rectangles only $Q = bh^2/8$ and $I = bh^3/12$

The shear strain (γ) is determined by measuring the strain at 45° angle as shown in Figure E.

$$\gamma = 2 \times \epsilon_{@\ 45°}.$$

The modulus of shear strain (G) = E/2 (1 + μ). Therefore, strain gages used in a shear strain configuration can be used to determine vertical loads (F_ν), more commonly referred to as a shear beam load cell.

$$F_\nu = G(\gamma)\, bI/Q = G(\gamma)\, b\,(bh^3/12/(bh^2/8) = G(\gamma)bh(2/3)$$

4) TORSIONAL STRAIN equals torsional stress (τ) divided by torsional modulus of elasticity (G). See figure F.

$$\gamma = 2 \times \epsilon @\ 45° = \tau/G \qquad \tau = M_t\,(d/2)/J$$

Where torsional stress (τ) equals the torque (M_t) multiplied by the distance from the center of section to outer fiber (d/2), divdied by (J), the polar moment of inertia. The polar moment of inertia is a function of the cross sectional area. For solid circular shafts only ($J = \pi\,(d)^4/32$). The modulus of shear strain (G) has been defined in the preceeding discussion on shear stress. Strain gages can be used to determine torsional moments as shown in the equation below. This represents the principle behind every torque sensor.

$$M_t = \tau\,(J)\,(2/d) = \gamma\,G\,(J)\,(2/d) = \gamma\,G\,(\pi\,d^3/16)$$
$$\varnothing = M_T\,L/G\,(J)$$

STRAIN	BRIDGE TYPE	POSITION OF GAGES Figs. C-F	SENSITIVITY mV/V @ 1000$\mu\epsilon$	OUTPUT PER $\mu\epsilon$ @ 10V EXCITATION	TEMP. COMP.	SUPERIMPOSED STRAIN COMPENSATED
BENDING	¼	1	0.5	5μV/$\mu\epsilon$	No	None
	½	1,2	1.0	10μV/$\mu\epsilon$	Yes	Axial
	Full	All	2.0	20μV/$\mu\epsilon$	Yes	Axial
AXIAL	¼	1	0.5	5μV/$\mu\epsilon$	No	None
	½	1,2	0.65	6.5μV/$\mu\epsilon$	Yes	None
	½	1,3	1.0	10μV/$\mu\epsilon$	No	Bending
	Full	All	1.3	13μV/$\mu\epsilon$	Yes	Bending
SHEAR & TORSIONAL	½	1,2	1.0	10μV/$\mu\epsilon$ @ 45°F	Yes	Axial and Bending
	Full	All	2.0	20μV/$\mu\epsilon$ @ 45°F	Yes	Axial and Bending

Note: Shear and Torsional strain = 2 x ϵ @ 45°

The following table shows how bridge configuration affects output, temperature compensation, and compensation of superimposed strains. This table was based on using a gage factor of 2.0, Poisson's Ratio of 0.3, and disregards the lead wire resistance. This chart is quite useful in determining a required meter sensitivity to read strain values.

Temperature compensation is achieved in many of the above configurations. Temperature compensation means that the gages thermal expansion coefficient does not have to match the specimens thermal expansion coefficient, therefore, any OMEGA strain gage regardless of temperature characteristic can be used with any specimen material. ¼ bridges can have temperature compensation if a dummy gage is used. A dummy gage is a strain gage used in place of a fixed resistor. When this dummy gage is mounted on a small piece of material (same as specimen's) which undergoes temperature changes (same as specimen's) but not strain. Strain temperature compensation is not load (stress) temperature compensation. Strain temperature compensation does not mean load or stress temperature compensation, because Young's Modulus of Elasticity varies with temperature.

END