$\mathcal{TWIST}$
Technical Note TN-57
A Procedure for Fitting $\mathcal{TWIST}$ Spectra using Spectral Basis Sets

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Abstract

A procedure for fitting $\mathcal{TWIST}$ spectra for the extraction of the Michel parameters is described. The procedure has the advantages that:

- Details of the acceptance, of secondary processes, resolution and response, and performance of the tracking are incorporated naturally to the extent that we are able to include these in the simulation.
- The effects of the contributions of each parameter, and of radiative corretions, are isolated. This means that small effects such as the contribution of a non-zero η are not lost in the statistical precision of the simulations.
- The procedure may lend itself to the implementation of a blind analysis.

The Michel distribution, including radiative corrections, can be written (neglecting terms of order m_e^2) as:

$$\frac{d\Gamma}{dx dcos(\theta)} \propto x^2(3-3x+\frac{2}{3}\rho(4x-3)) + x^2 P_\mu \xi cos(\theta)(1-x+\frac{2}{3}\delta(4x-3))$$

$$+3\frac{m_e}{E_{max}}\eta x(1-x) + f_{rc}(x) + g_{rc}P_\mu(x)cos(\theta) \quad (1)$$

This distribution can be broken into polynomials in x and $cos(\theta)$ that are proportional to the parameters ρ, η, $\xi\delta$, and ξ, including the two functions $f_{rc}(x)$ and $g_{rc}(x)$ which describe the radiative corrections. The polynomials are:

- No Michel dependence:

$$t_o = 3x^2(1-x) + f_{rc} + P_\mu g_{rc} cos(\theta) \quad (2)$$

- Proportional to ρ:

$$t_\rho = x^2(\frac{8x}{3} - 2) \quad (3)$$

[1]Incorporating ideas from Carl Gagliardi

- Proportional to η:
$$t_\eta = 3\frac{m_e}{E_{max}}x(1-x) \tag{4}$$
- Proportional to $P_\mu \xi\delta$:
$$t_{\xi\delta} = \frac{2}{3}x^2(4x-3) \tag{5}$$
- Proportional to $P_\mu \xi$:
$$t_\xi = x^2(1-x) \tag{6}$$

Therefore, the Michel distribution can be written as

$$\frac{d\Gamma}{dx dcos(\theta)} \propto t_o + \rho t_\rho + \eta t_\eta + f_{rc} + P_\mu(\xi\delta t_{\xi\delta} + \xi t_\xi + g_{rc})cos(\theta) \tag{7}$$

It is worth noting that the expression is linear in each of the Michel parameters. There are no higher order terms, so a Taylor series expansion in the Michel parameters is exact[2]. Further, a Taylor series in deviations of the Michel parameters from the Standard Model is identical to a Taylor series in deviations of the Michel parameters from zero. $\mathcal{TWIST}$ data can be fit to a linear combination of expressions 2 through 6 plus contributions of radiative corrections.

Some of these expressions go through zero. This means that there are kinematic regions that are enhanced by ρ (for example), and other regions that see a decrease in flux due to the ρ term. This makes sense, but won't work for a generator. Therefore, the generator should be the absolute value of each of the expressions (2) through (6). We will need to keep track of the zero crossings in building the associated spectra for recombination into the simulated data.

The proposed generators, therefore, are:

$$t_o = |3x^2(1-x) + f_{rc} + P_\mu g_{rc} cos(\theta)| \tag{8}$$
$$t_\rho = |x^2(\frac{8x}{3} - 2)| \tag{9}$$
$$t_\eta = 3|\frac{m_e}{E_{max}}x(1-x)| \tag{10}$$
$$t_{\xi\delta} = |\frac{2}{3}x^2(4x-3)cos(\theta)| \tag{11}$$
$$t_\xi = |x^2(1-x)cos(\theta)| \tag{12}$$

The weight given to each spectrum is given by the integral of the generator over all $(x, cos(\theta))$. It is important that each of the generators be a different shape in (x,$cos(\theta)$). Similar shapes are more highly correlated. The shapes of the distributions are shown at the end of this document.

[2]For the purposes of discussion, one may wish to define a new parameter "$\xi\delta$". This parameterization has in fact been used in some of the theoretical work[1]. Of course, δ can be recovered as $\xi\delta/\xi$.

The OPAL collaboration has made use of a technique similar to that above to fit the Michel spectrum from tau decays[2]. They chose a slighty different basis set, however, so that it was combinations of the Michel parameters which were varied in the fit rather than the Michel parameters themselves. The choice doesn't really matter, aside from the fact different basis sets will result in fits with differing correlations. The OPAL fits were done with generators equivalent to:

$$t_0 + f_{rc} + g_{rc}P_\mu cos(\theta) + \frac{3}{4}t_\rho + (t_\xi + \frac{3}{4}t_{\xi\delta})P_\mu cos(\theta) \tag{13}$$

$$t_0 + f_{rc} + g_{rc}P_\mu cos(\theta) + t_\rho \tag{14}$$

$$t_0 + f_{rc} + g_{rc}P_\mu cos(\theta) + t_\xi P_\mu cos(\theta) \tag{15}$$

$$t_0 + f_{rc} + g_{rc}P_\mu cos(\theta) + \frac{3}{4}t_\rho + \frac{1}{2}t_\eta \tag{16}$$

$$t_0 + f_{rc} + g_{rc}P_\mu cos(\theta) \tag{17}$$

Note that because the OPAL generators have $t_0+f_{rc}+P_\mu g_{rc}cos(\theta)$ in common, they may have more correlations than the set we've proposed above. It is likely that the OPAL basis set was chosen due to the simplicity of using the existing generator of Michel decays with various distributions created by simply setting some of the Michel parameters to zero. We can do better than this - in terms of reducing correlations and optimizing the use of the statistics in each data set - by writing separate generators. In any case, we can make our choice of basis set later. For the moment, either set of generators (8-12 or 13-17) will work. Indeed we may wish to try some other basis set built of a different combination of the generators 8-12. The shapes of the OPAL generators are plotted at the end of this document.

It may be possible to use this fitting procedure to introduce parameters that allow for a blind analysis. A blind analysis will be extremely important to our credibility in the reporting of our final results. The idea is that we will need to validate our Monte Carlo independent of the final physics result. That is, we need to argue for the validity of our assessment of the effect of items such as

- hardware efficiency,
- software efficiency,
- resolution,
- hard scatters,
- energy loss,
- multiple scattering,

- energy calibrations and stopping distribution,
- delta rays, and
- beam related backgrounds,

prior to our knowledge of the agreement or disagreement with the Standard Model. Our arguments for the validation of the simulation of these effects cannot be based on the Michel spectrum (aside from the endpoint energy calibration), and the validation must be suitable for publication prior to the comparison of the spectrum shape with the theoretical spectrum.

One way to do this would be to modify the "Michel independent" generator (Eq. 8), introducing a small and unknown contribution from each of the other generators. In this case, Eq. 8 is replaced by

$$t_0^{prime} = |t_0 + t_\rho \Delta\rho + t_\eta \Delta\eta + t_{\xi\delta}\Delta(\xi\delta) + t_\xi \Delta\xi| \tag{18}$$

The fit may take the form[3]

$$data(x, cos(\theta)) = constant * (t_0^{prime} + \rho_{fit} t_\rho + \eta_{fit} t_\eta + (\xi\delta)_{fit} t_{\xi\delta} + \xi_{fit} t_\xi) \tag{19}$$

in terms of the fitting parameters ρ_{fit}, η_{fit}, $(\xi\delta)_{fit}$, ξ_{fit}. The physical parameters are then given by $\rho = \rho_{fit} + \Delta\rho$, $\eta = \eta_{fit} + \Delta\eta$, $\xi\delta = \xi\delta_{fit} + \Delta(\xi\delta)$, $\xi = \xi_{fit} + \Delta\xi$.

The small contributions of $\Delta\rho$, $\Delta\eta$, and so on will need to be hidden from the collaboration to keep the final result of the analysis "blind". A procedure will need to be developed by which these deviations are not available until the full validation of the Monte Carlo is ready for publication.

References

[1] W. Fetscher and H.-J. Gerber. Precision tests of the standard model. In P. Langacker, editor, *Precision tests of the standard electroweak model*, pages 657–705. World Scientific, 1993.

[2] K. Ackerstaff et al. Measurement of the michel parameters in leptonic tau decays. *Eur. Phys. J.*, C8:3–21, 1999.

[3] The OPAL collaboration used a binned maximum likelihood fit. That choice doesn't alter the philosophy of these basis sets, and can be studied for possible adoption.

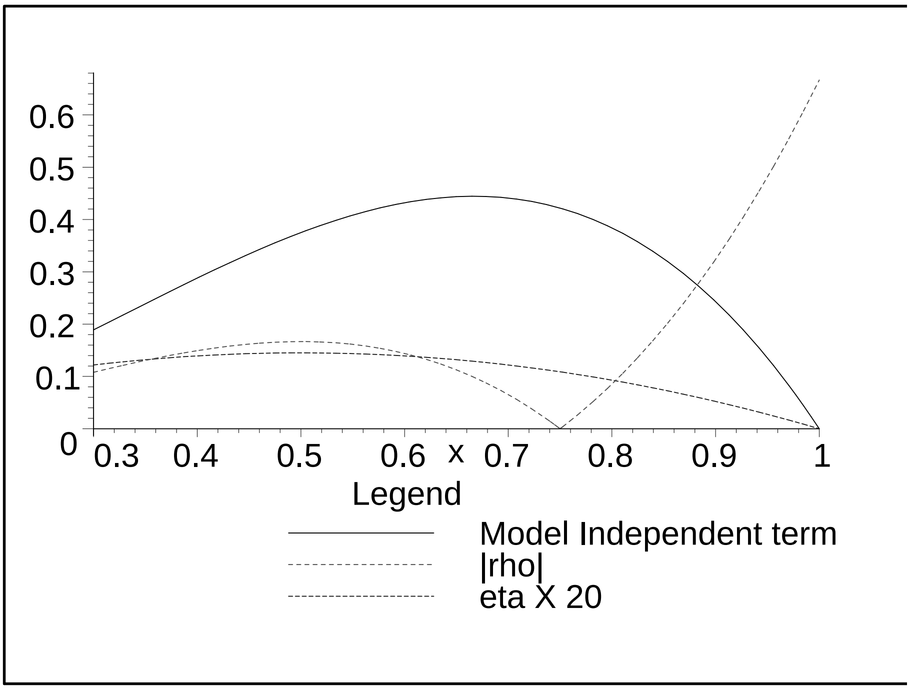

Figure 1: Proposed generator functions - $cos(\theta)$ independent terms. Note: the radiative corrections have been omitted from the plot.

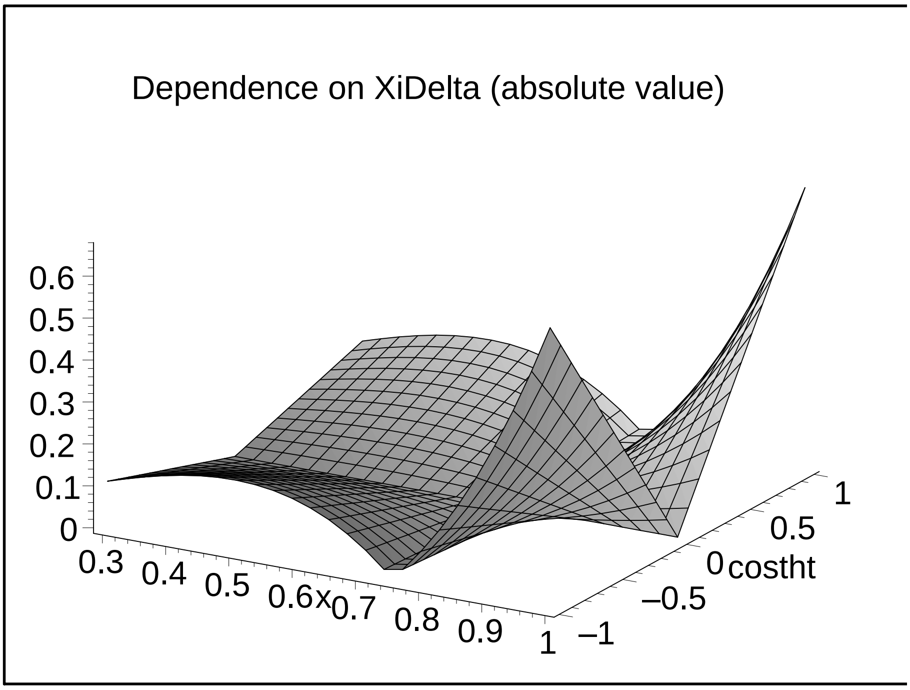

Figure 2: Proposed generator - term proportional to $\xi\delta$

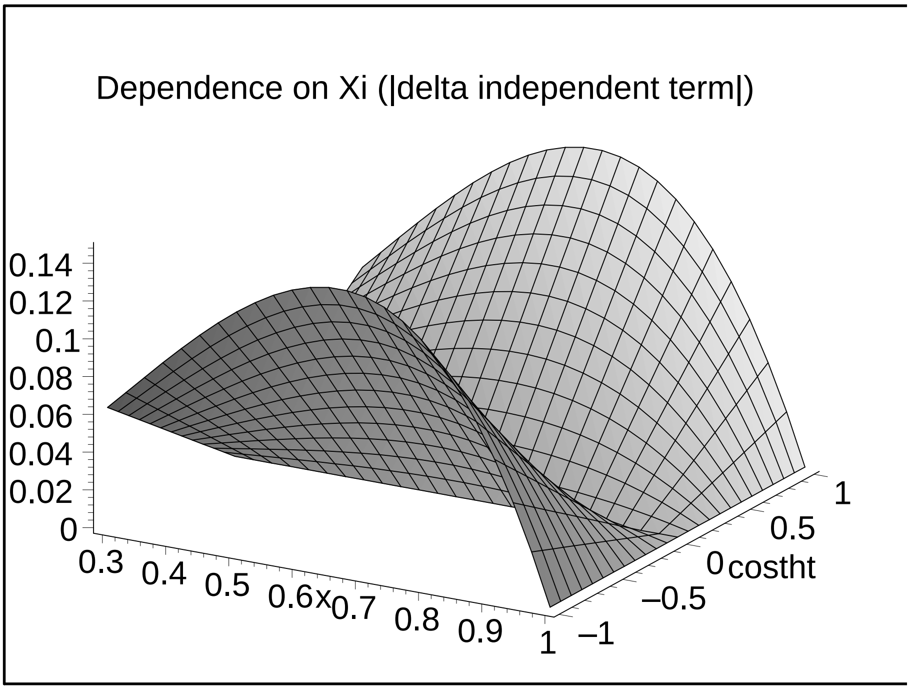

Figure 3: Proposed generator - term proportional to ξ

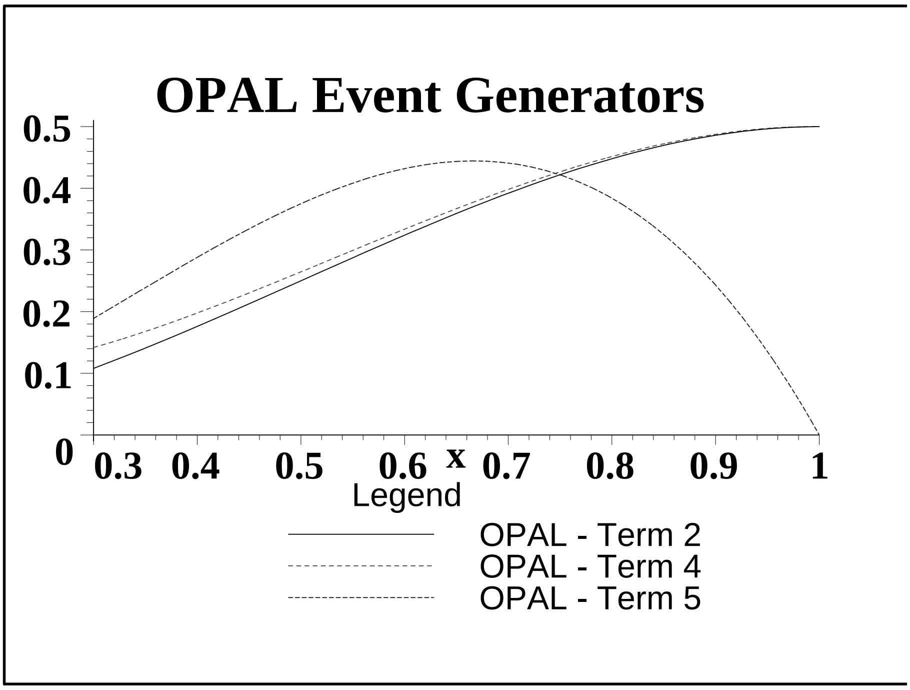

Figure 4: OPAL Event Generators. Radiative corrections need to be applied to every distribution.

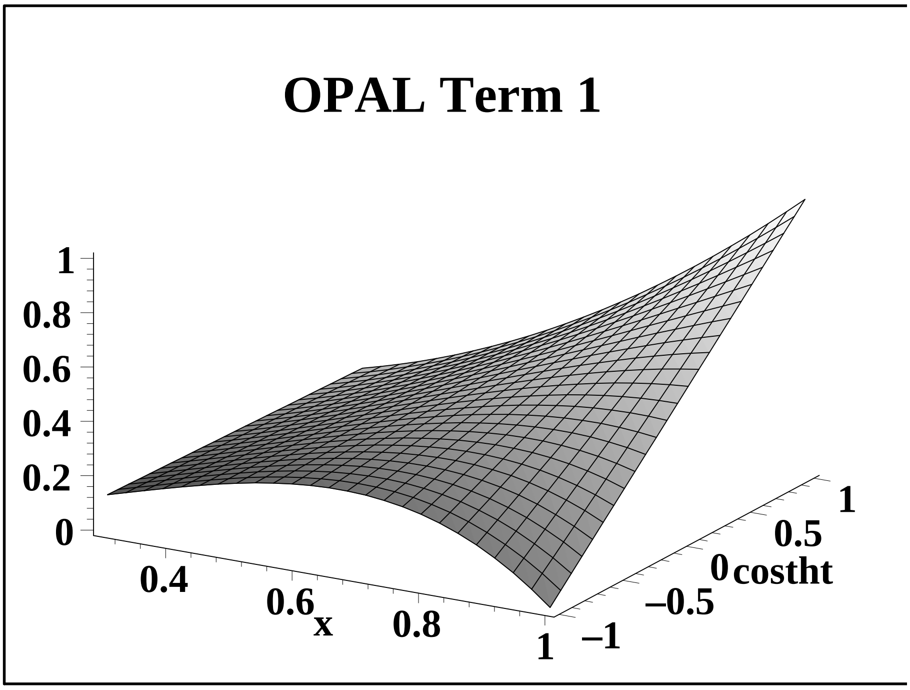

Figure 5: OPAL Event Generators. Radiative corrections need to be applied to every distribution.

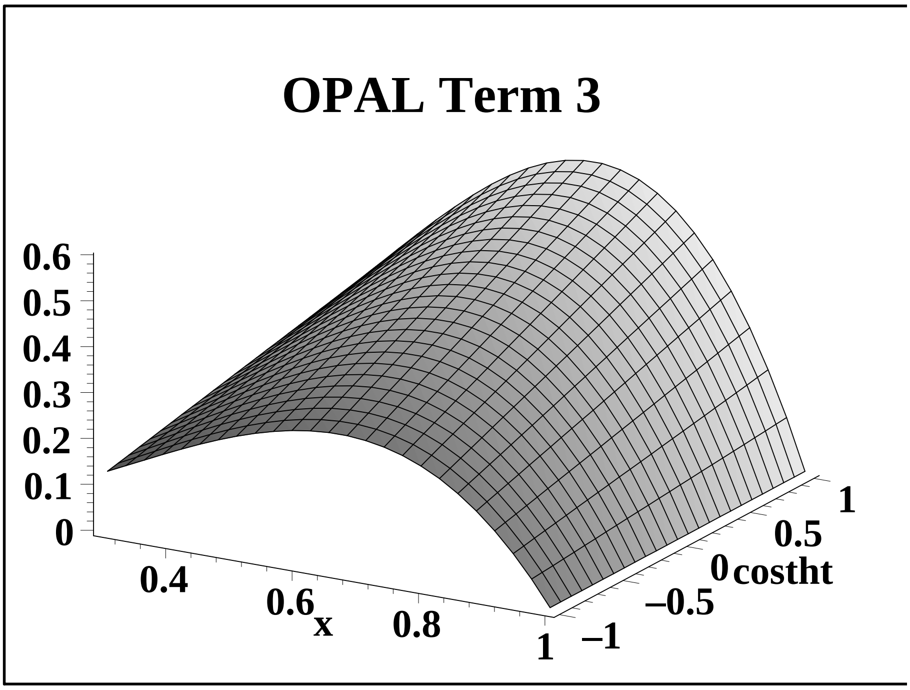

Figure 6: OPAL Event Generators. Radiative corrections need to be applied to every distribution.