$\mathcal{TWIST}$

Technical Note TN-62
Positron energy scale calibration.

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This note discusses a systematic uncertainty caused by an error in the positron energy scale. Sensitivities of all the four Michel parameters to the energy scale are obtained from fits to a generated 2-d spectrum. Stabilities of the sensitivities with respect to a choice of the fit region are shown. A method of determining an energy scale is described and a statistical part of the energy scale uncertainty is obtained.

Introduction

This note deals with a distortion of the reconstructed positron spectrum in the form $E \longrightarrow (1+\beta)E$. The parameter $1+\beta$ is often referred to as the "energy scale". For example a mismatch between the absolute value of a magnetic field used by the reconstruction program and the actual value of the magnetic field inside the detector can cause such a distortion.

An energy calibration procedure can be used to determine the parameter β of the reconstructed spectrum. Then a correction can be applied on an event per event basis either to the experimental spectrum to eliminate the distortion or, alternatively, to simulated spectra being used in fitting for Michel parameters [1]. Technically calibrations for the upstream and downstream parts of the spectrum will be done separately. That gives us two parameters $\beta_{\mathtt{up}}$ and $\beta_{\mathtt{dn}}$ instead of a single global one.

Error on a Michel parameter a caused by an energy scale uncertainty $\Delta\beta$ (where $\beta = \beta_{\mathtt{up}}$ or $\beta = \beta_{\mathtt{dn}}$) can be written as

$$\Delta a = \frac{\partial a}{\partial \beta}\,\Delta\beta \tag{1}$$

The derivative is called sensitivity of the Michel parameter to the corresponding energy scale parameter.

Sensitivities of the four Michel parameters ρ, η, ξ, and δ to $\beta_{\mathtt{up}}$ and $\beta_{\mathtt{dn}}$ are determined in part 1 of the note. Part 2 describes the energy calibration procedure and determines a statistical part of $\Delta\beta_{\mathtt{up}}$ and $\Delta\beta_{\mathtt{dn}}$. A conclusion summarizes the results of these two parts and estimates errors on Michel parameters for a case of energy calibration performed using a physics data set.

A Michel spectrum function which includes the first order radiative corrections was used in the study. Muon polarization was assumed to be 100%.

All histogramming and fitting was done in the ROOT framework [2]. ROOT fitting subroutines are based on the well known MINUIT tool [3]. Maximum log likelihood fits were employed. All simulations used a ROOT implementation of the Mersenne Twistor random number generator [4] with the period $2^{19937} - 1$.

1 Sensitivity of Michel parameters to the energy scale.

"Events" $\{x, \cos(\theta)\}$ were produced by sampling the Michel spectrum using the acceptance-rejection method. Standard Model values of Michel parameters were used in the generation. A data set consisted of 10^9 events in the whole spectrum has been generated. The events were filled into a 2-dimensional histogram with 500 bins in the the [0,1] interval of fractional energy and 200 bins in the [-1,1] $\cos(\theta)$ interval. The fractional energy x is a dimensionless variable defined as $x = E/E_{\mathtt{max}}$ where E is the positron energy and $E_{\mathtt{max}}$ is the upper kinematic limit on E, $E_{\mathtt{max}} \approx 52.83$MeV.

Then the histogram was fitted with a distorted spectrum. Values of $\beta_{\mathtt{up}}$ and $\beta_{\mathtt{dn}}$ were fixed during the fit, while ρ, η, ξ, δ and a global normalization factor were free fit parameters. The fits were done in a restricted region of the phase space $\{x_{\mathtt{min}} \leq x \leq x_{\mathtt{max}}\} \times \{c_{\mathtt{min}} \leq |\cos(\theta)| \leq c_{\mathtt{max}}\}$. A "nominal" region was chosen as: $x_{\mathtt{min}} = 0.4$, $x_{\mathtt{max}} = 0.97$, $c_{\mathtt{min}} = 0.5$, $c_{\mathtt{max}} = 0.98$.

Figure 1 shows deviations of Michel parameters obtained as a result of the fit from their original values for $\beta_{\mathtt{dn}} = 0$ and $\beta_{\mathtt{up}}$ varying from $-30\cdot^{-4}$ to $+30\cdot^{-4}$. The slope P1 of the straight line is the sensitivity of the corresponding Michel parameter to $\beta_{\mathtt{up}}$. Note, that different points on a graph are correlated since they were obtained from the same data set. Therefore a fit error on P1 is meaningless and not displayed. Sensitivities to $\beta_{\mathtt{dn}}$ can be seen on Fig. 2.

Dependence of the sensitivities on the fit region has been studied. One of the region boundaries was varied while the others were set to the "nominal" values. An analysis similar to the one above was performed and the sensitivities obtained for different regions. The results are shown on Figs. 3, 4. Note that the η sensitivity is scaled down on the plots by a factor of 10 to make a common vertical scale for all parameters. The sensitivities do not change much with the fit region change, except for the η sensitivity which changes from 5 to more than 20 in one case.

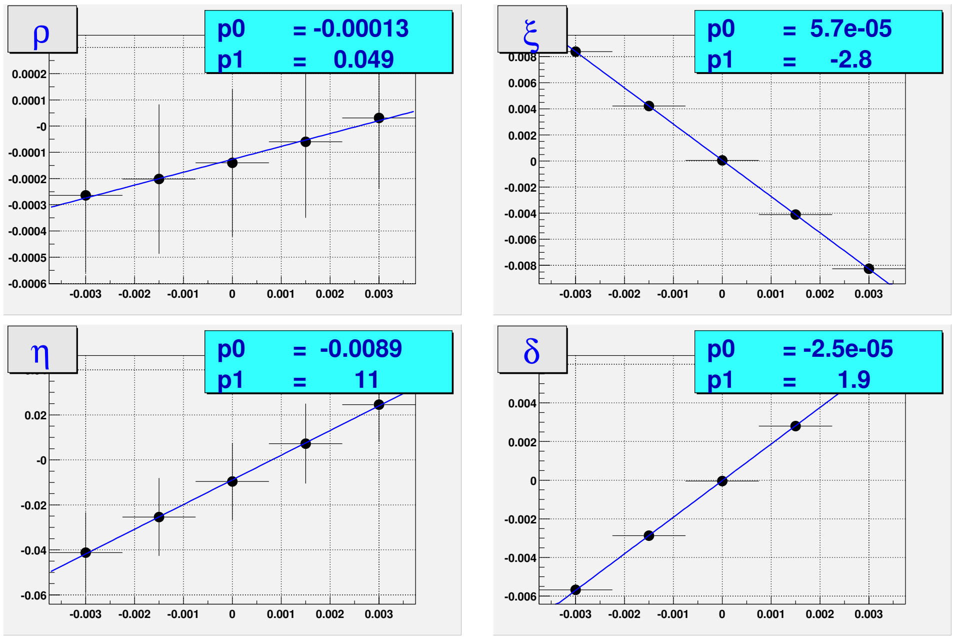

Figure 1: Dependence of Michel parameters on the upstream energy scale $\beta_{\mathtt{up}}$.

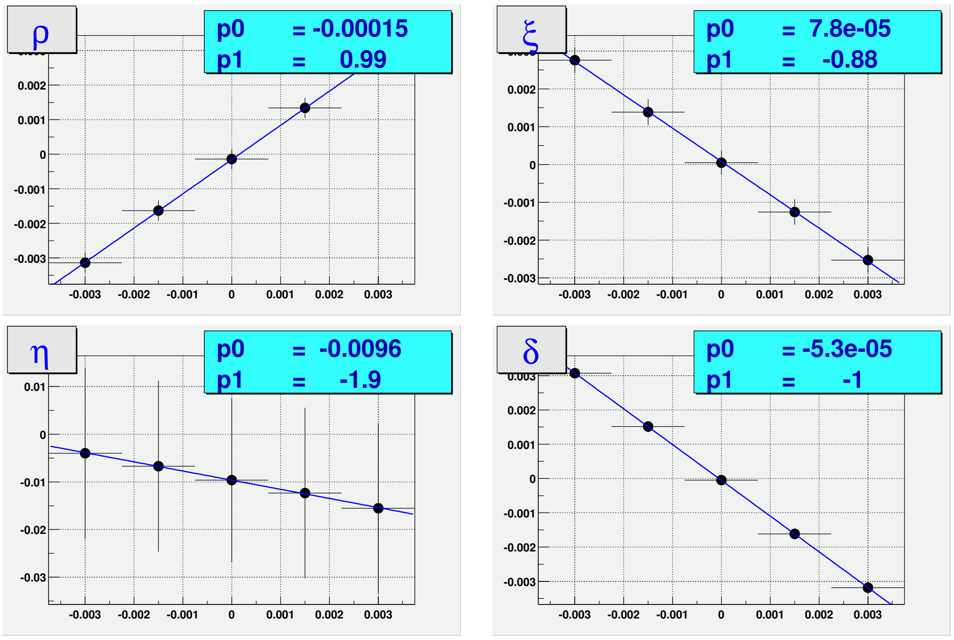

Figure 2: Dependence of Michel parameters on the downstream energy scale $\beta_{\mathtt{dn}}$.

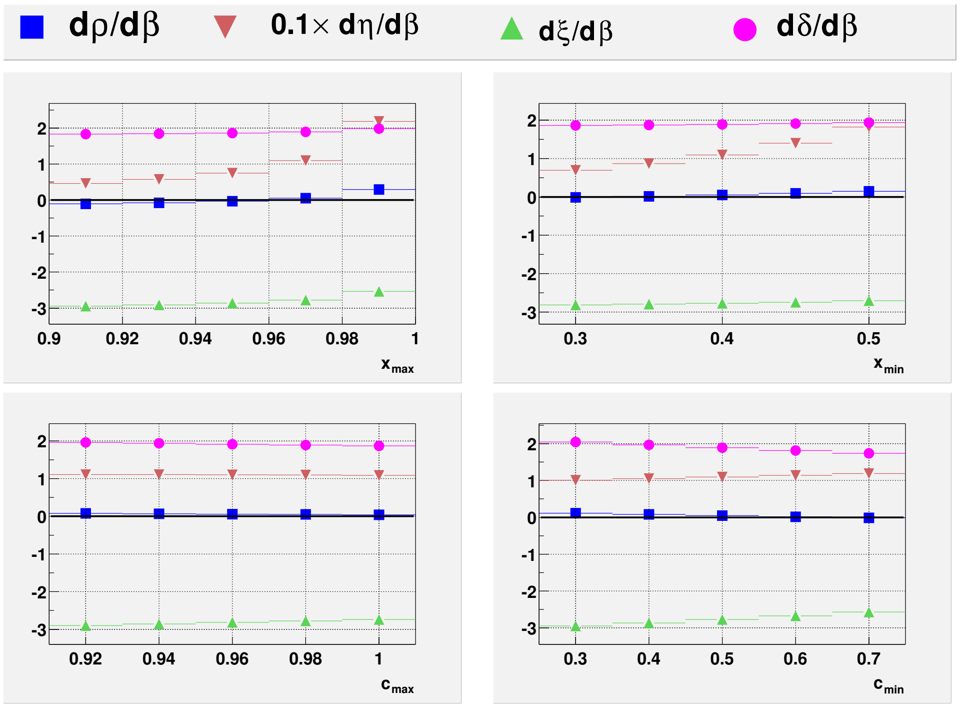

Figure 3: Dependence of the sensitivities to β_{up} on the fit region.

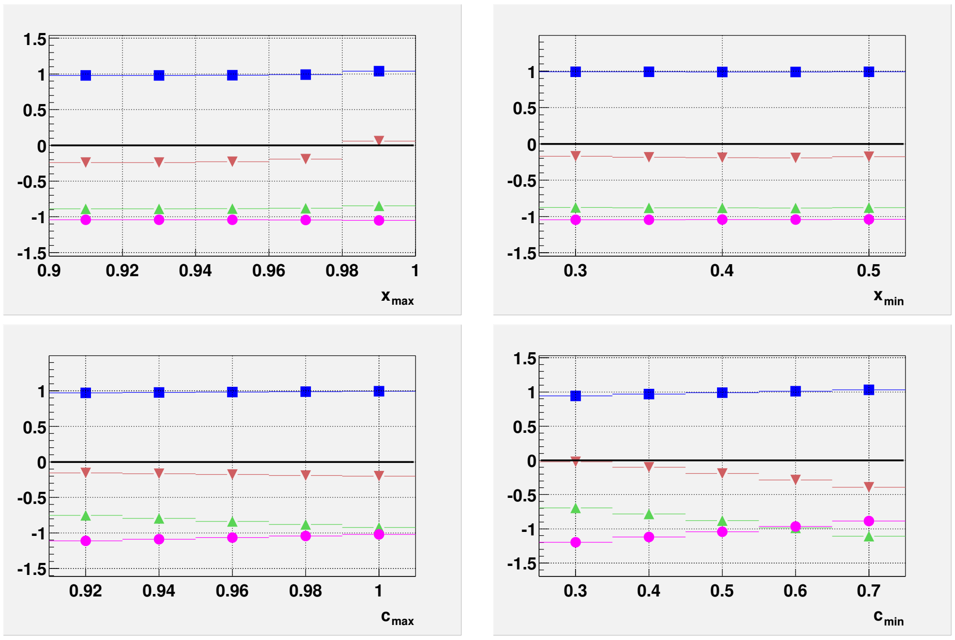

Figure 4: Dependence of the sensitivities to β_{dn} on the fit region.

2 The energy calibration procedure.

The sharp end of the positron energy spectrum at the upper kinematic limit provides a natural calibration point. However positrons lose energy traveling in the detector material. Due to this process a systematic shift in the end point of the reconstructed spectrum should be expected.

A procedure to calibrate the energy scale taking into account the energy loss is described in previous TWIST documents, see for example [5]. It's based on the fact that a mean energy loss for a positron with some fixed initial energy is linear in $1/|\cos(\theta)|$. This simple dependence is rigorously valid for the planar detector geometry. Applying this fact to the positrons emitted with the maximum possible energy we get for the edge of the reconstructed spectrum

$$E_{\text{edge}} = (1+\beta)\left(E_{\text{max}} - \frac{\alpha}{|\cos(\theta)|}\right) \qquad (2)$$

In the dimensionless energy units used $E_{\text{max}} \longrightarrow x_{\text{max}} \equiv 1$.

So, if we determine x_{edge} for different angles θ we can fit (2) and find the energy scale β and the energy loss constant α.

Unlike fitting the inner part of the spectrum for Michel parameters, an exact theoretical spectrum shape cannot be used to fit the spectrum near the edge. In this region shape of the reconstructed spectrum is defined mainly by resolution effects. It has been shown in [6] that a simple step function $\Theta(x_{\text{edge}} - x)$, imitating the sharp spectrum edge, convoluted with a Gaussian, can be used to fit the edge of a simulated spectrum in some cases. In the current study a more precise description was used: the theoretical spectrum shape, including the first order radiative corrections, was convoluted with a Gaussian. The resulting function was fitted to simulated data. The convolution can not be done analytically, so a numerical integration was used. An adaptive ROOT integration procedure utilizing the 8-point and 16-point Gaussian quadrature approximations [7, 2] was employed to calculate the fitting function.

A test data sample consisting of $1.88658 \cdot 10^8$ $\{x, \cos(\theta)\}$ points the [0.9, 1.] dimensionless energy range was produced by sampling the Michel spectrum. This corresponds to 10^9 decays in total. To imitate resolution a Gaussian random number with $\sigma = 0.005$ was added to the generated x. I.e. constant energy resolution 0.26MeV [8, 9] was assumed. Twelve angular bins of 4° each were defined for the downstream part of the spectrum between 10° and 58°, and symmetric twelve bins for the upstream part. For each angular bin a histogram with 150 bins in x in the [0.95, 1.05] interval was defined and filled. Then each of those histograms was fitted with the convoluted Michel spectrum. Position of the edge, resolution σ and overall normalization were the 3 fit parameters. Maximum log likelihood method was used in the fits. $\cos(\theta)$, which is a parameter of the Michel spectrum function, was fixed at the mean value $(\cos(\theta_1) + \cos(\theta_2))/2$ for a bin between θ_1 and θ_2. Since Michel spectrum is linear in $\cos(\theta)$ this gives an exact result. Results of some of the fits are shown on Fig. 5.

One can see that this method works even for small angles in the downstream part of the spectrum, where the step at the edge is very small. Typical error on

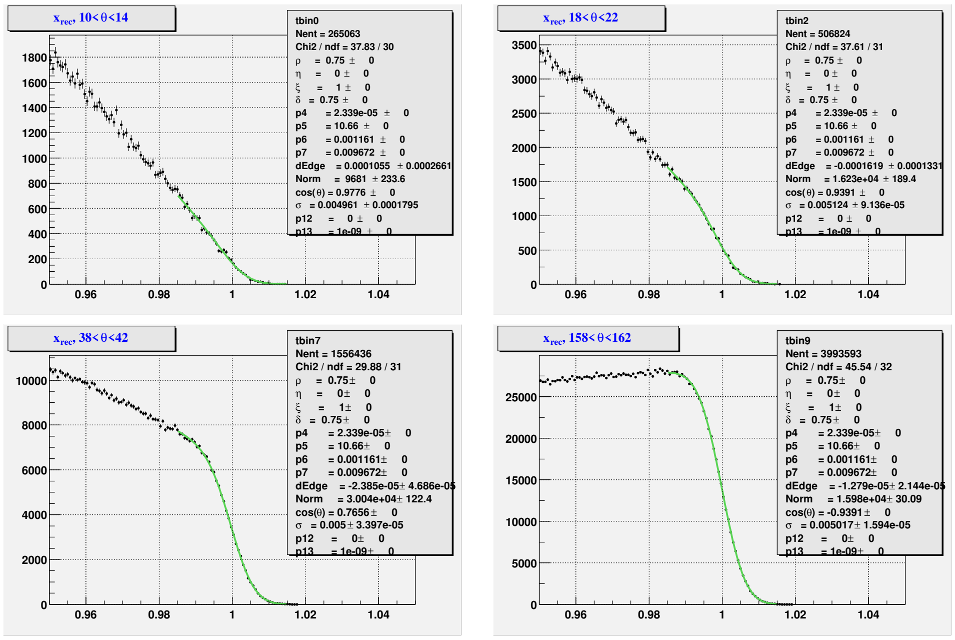

Figure 5: Examples of the end point fitting. Nominal edge position is 1. Deviation from 1 is the dEdge shown on the plots. The first 3 plots correspond to bins 1, 3, 8 on Fig. 6. The fourth plot corresponds to bin 10 on Fig. 7.

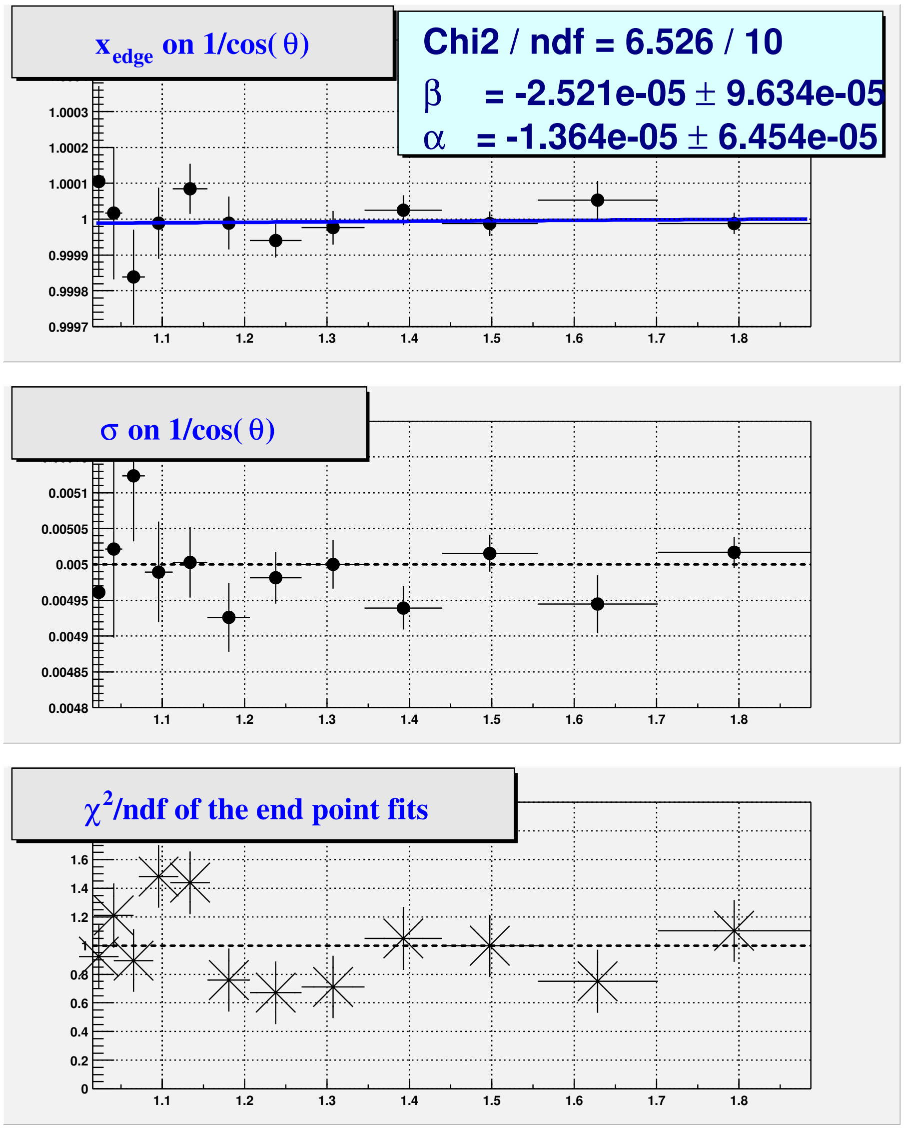

Figure 6: Downstream energy calibration. Horizontal scale is $1/\cos(\theta)$.

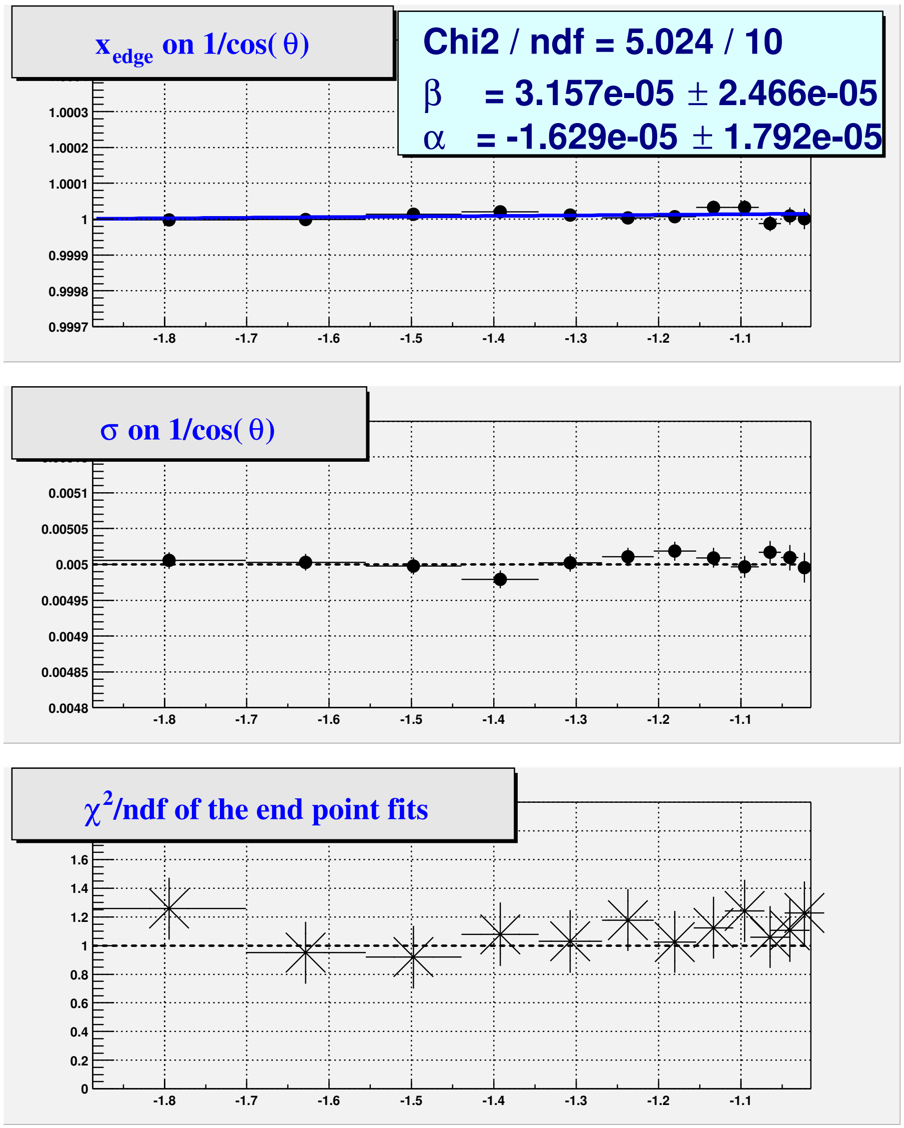

Figure 7: Upstream energy calibration. Horizontal scale is $1/\cos(\theta)$.

the edge is less than 10^{-4} for the downstream part, as can be seen on Fig.6. For the upstream part of the spectrum, where the edge is high and the statistics is bigger, the errors are even smaller see Fig. 7.

Results of straight line fits (2) to the obtained x_{edge} points are shown on Fig.6–7. Since the energy loss was not simulated and no distortion of the energy scale was made, nominal values for α and β are zero. Values obtained from the fits are consistent with that, and the errors are $0.96 \cdot 10^{-4}$ and $0.25 \cdot 10^{-4}$ for $\beta_{\texttt{dn}}$ and $\beta_{\texttt{up}}$ correspondingly.

Note that the errors on $\beta_{\texttt{up}}$ and $\beta_{\texttt{dn}}$ were obtained assuming a 100% polarization of the decaying muons. An actual data set in addition to the surface muons will contain cloud muons in an amount equivalent to 5% of oppositely polarized particles. From the cloud muon subsample $\beta_{\texttt{dn}}$ can be determined with the precision $\sqrt{\frac{1}{0.05}} \times 0.25 \cdot 10^{-4} \approx 1.1 \cdot 10^{-4}$. This gives for a "combined" measurement on the whole data set $\Delta\beta_{\texttt{dn}} = 0.72 \cdot 10^{-4}$, $\Delta\beta_{\texttt{up}} = 0.25 \cdot 10^{-4}$.

These numbers can be looked at as a statistical part of the energy scale uncertainty. Systematics corresponding, for example, to different choices of the spectrum edge fitting functions has to be studied separately.

3 Conclusion.

Combining the sensitivities of Michel parameters from part 1 for the "nominal" fit region with the uncertainties obtained in part 2 we get

	$\partial/\partial\beta_{\texttt{up}}$	$\partial/\partial\beta_{\texttt{dn}}$	$\sqrt{(0.25 \cdot \partial/\partial\beta_{\texttt{up}})^2 + (0.72 \cdot \partial/\partial\beta_{\texttt{dn}})^2}$
ρ	0.05	1.	0.72
η	11	-1.9	3.3
ξ	-2.8	-0.9	1.1
δ	1.9	-1	1.1

Table 1: Sensitivities of Michel parameters to the upstream and downstream energy scales and the final error estimate. The errors are in the 10^{-4} units.

where the last column gives the error on Michel parameters due to the statistical part of the energy scale uncertainty.

So, it may be possible to make an energy calibration with about the 10^{-4} precision using the same data sample of polarized muon decays as for the Michel parameter fitting. Note that the statistics of 10^9 muon decays in the whole spectrum is *smaller* than 10^9 good reconstructed decays because of the acceptance.

A more precise downstream energy scale calibration can be obtained using an unpolarized muon sample. For δ and ξ that can be an "external" calibration only, and another systematics corresponding to a stability of the running conditions arises.

One more option is to utilize the fact that $\alpha_{\tt up} + \alpha_{\tt dn} = \texttt{const}$ since the target thickness does not change. The sum can be determined from a dedicated run. Then on a physics data sample $\beta_{\tt up}$ and $\alpha_{\tt up}$ can be determined from a two-parameter straight line fit leaving a single unknown parameter $\beta_{\tt dn}$. A quick test has showed that an error on $\beta_{\tt dn}$ goes down by a factor of 6 if $\alpha_{\tt dn}$ is fixed during the fit instead of being a free parameter.

References

[1] N. Rodning and A. Gaponenko, "A Procedure for Fitting $\mathcal{TWIST}$ Spectra using Spectral Basis Sets", $\mathcal{TWIST}$ Technical Note TN-57.

[2] `http://root.cern.ch`

[3] F. James. "MINUIT—Function Minimization and Error Analysis" CERN Program Library entry D506.

[4] M. Matsumoto and T. Nishimura, "Mersenne Twistor: A 623-dimensionally equidistributed uniform pseudorandom number generator" ACM Transactions on Modeling and Computer Simulation, Vol. 8, No. 1, January 1998, pp 3–30. See also `http://www.math.keio.ac.jp/~matumoto/emt.html`

[5] MIG proposal, current proposal.

[6] A.Gaponenko, "Energy calibration", Presentation at TWIST Collaboration meeting 7 June 2001. See the old bulletin board at `http://stoney.phys.ualberta.ca/~e614/`

[7] Sigfried Kölbig, "Adaptive Gaussian Quadrature", CERN Program Library entry D103 (1966, 1994).

[8] A.A. Khrutchinsky, Yu. Yu. Lachin, V.I. Selivanov, "A Monte-Carlo study of a precision magnetic spectrometer with planar geometry.", Nuclear Instruments and Methods in Physics Research A 396 (1997) 135–146.

[9] D.H. Wright, "Monte Carlo Study of E614 Spectrometer Resolution and Acceptance.", Technical Note 2 (1997), see the old bulletin board. (The document calls itself TN#4, not 2.)