$\mathcal{TWIST}$ Physics Motivation

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Abstract

A discussion paper is presented outlining the motivation for $\mathcal{TWIST}$. The role of the Michel parameters in defining the muon decay spectrum is presented along with their dependence on the weak coupling parameters. Of particular importance is the model-independent observable Q_R^μ, which determines the weak couplings to right-handed muons. $\mathcal{TWIST}$ will be the first experiment to make an independent measurement of Q_R^μ. The $\mathcal{TWIST}$ sensitivity to left/right symmetric models is discussed for both the more general case, and for the case of "manifest" left/right symmetry. Mention is made of possible SUSY sensitivity.

1 A general description of Muon Decay

The most general derivative free interaction describing muon decay can be written in the form

$$M = 4\frac{G_F}{\sqrt{2}} \sum_{\substack{\gamma = S,V,T \\ \epsilon,\mu = R,L}} g^\gamma_{\epsilon\mu} < \bar{e}_\epsilon \mid \Gamma^\gamma \mid \nu_e >< \bar{\nu}_{muon} \mid \Gamma_\gamma \mid \mu_\mu > . \tag{1}$$

The strength of the interaction is contained in the Fermi coupling constant, G_F. The labels γ indicate the type of interaction (Γ^S = scaler, Γ^V = Vector, Γ^T = Tensor) and the subscripts ϵ and μ define the chirality of the spinors of the charged leptons, ϵ for the electron and μ for the muon.

There exist ten unique complex couplings[1]. Making the choice that g^V_{LL} is real defines an overall arbitrary phase, leaving 19 free parameters. These are normalized such that

$$\begin{array}{lllll} & \frac{1}{4} \mid g^S_{RR} \mid^2 & + \ \frac{1}{4} \mid g^S_{RL} \mid^2 & + \ \frac{1}{4} \mid g^S_{LR} \mid^2 & + \ \frac{1}{4} \mid g^S_{LL} \mid^2 \\ + & \mid g^V_{RR} \mid^2 & + \ \mid g^V_{RL} \mid^2 & + \ \mid g^V_{LR} \mid^2 & + \ \mid g^V_{LL} \mid^2 \\ & & + \ 3 \mid g^T_{RL} \mid^2 & + \ 3 \mid g^T_{LR} \mid^2 & = 1 \end{array} \tag{2}$$

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[1] g^T_{LL} and g^T_{RR} are not unique, as they can be constructed using combinations of the other operators

The muon decay spectrum is described by the four Michel parameters[1][2], which are bilinear combinations of these weak coupling constants. These can be written[2] as

$$
\begin{array}{rcccccccc}
\rho & = & \frac{3}{16} \mid g^S_{RR} \mid^2 & + & \frac{3}{16} \mid g^S_{RL} \mid^2 & + & \frac{3}{16} \mid g^S_{LR} \mid^2 & + & \frac{3}{16} \mid g^S_{LL} \mid^2 \\
 & & +\frac{3}{4} \mid g^V_{RR} \mid^2 & & & & & + & \frac{3}{4} \mid g^V_{LL} \mid^2 \\
 & & & + & \frac{3}{4} \mid g^T_{RL} \mid^2 & + & \frac{3}{4} \mid g^T_{LR} \mid^2 & & \\
 & & +\frac{3}{4} Re[g^{S*}_{RL} g^T_{RL} & + & g^S_{RL} g^{T*}_{RL} & + & g^{S*}_{LR} g^T_{LR} & + & g^S_{LR} g^{T*}_{LR}] \\
 & & & & & & & & \\
\xi\delta & = & -\frac{3}{16} \mid g^S_{RR} \mid^2 & + & \frac{3}{16} \mid g^S_{RL} \mid^2 & - & \frac{3}{16} \mid g^S_{LR} \mid^2 & + & \frac{3}{16} \mid g^S_{LL} \mid^2 \\
 & & -\frac{3}{4} \mid g^V_{RR} \mid^2 & & & & & + & \frac{3}{4} \mid g^V_{LL} \mid^2 \\
 & & & + & \frac{3}{4} \mid g^T_{RL} \mid^2 & - & \frac{3}{4} \mid g^T_{LR} \mid^2 & & \\
 & & +\frac{3}{4} Re[g^{S*}_{RL} g^T_{RL} & + & g^S_{RL} g^{T*}_{RL} & - & g^{S*}_{LR} g^T_{LR} & - & g^S_{LR} g^{T*}_{LR}] \\
 & & & & & & & & \\
\xi & = & -\frac{1}{4} \mid g^S_{RR} \mid^2 & + & \frac{1}{4} \mid g^S_{RL} \mid^2 & - & \frac{1}{4} \mid g^S_{LR} \mid^2 & + & \frac{1}{4} \mid g^S_{LL} \mid^2 \\
 & & - \mid g^V_{RR} \mid^2 & - & 3 \mid g^V_{RL} \mid^2 & + & 3 \mid g^V_{LR} \mid^2 & + & \mid g^V_{LL} \mid^2 \\
 & & & - & 5 \mid g^T_{RL} \mid^2 & + & 5 \mid g^T_{LR} \mid^2 & & \\
 & & -\frac{1}{2} Re[g^{S*}_{RL} g^T_{RL} & + & g^S_{RL} g^{T*}_{RL} & - & g^{S*}_{LR} g^T_{LR} & - & g^S_{LR} g^{T*}_{LR}]
\end{array}
\tag{3}
$$

$$\eta = \frac{1}{2} Re\{g^V_{LL} g^{S*}_{RR} + g^V_{RR} g^{S*}_{LL} + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) + g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})\} \tag{4}$$

The decay spectrum for positrons from μ^+ decay can be written in terms of the Michel parameters as

$$\frac{d^2\Gamma}{dx dcos(\theta_e)} = \frac{m_\mu}{4\pi^3} W^4_{e\mu} G^2_F \sqrt{x^2 - x^2_o}[F_{IS}(x) + P_\mu cos(\theta_e) F_{AS}(x)] \tag{5}$$

where $W_{e\mu} = (m^2_\mu + m^2_e)/(2m_\mu)$, $x = E_e/W_{e\mu}$, and $x_o = m_e/W_{e\mu}$. The isotropic and anisotropic functions are given by

$$
\begin{aligned}
F_{IS}(x) &= x(1-x) + \tfrac{2}{9}\rho(4x^2 - 3x - x^2_o) + \eta x_o(1-x) \\
F_{AS}(x) &= \tfrac{1}{3}\xi\sqrt{x^2 - x^2_o}[1 - x + \tfrac{2}{3}\delta(4x - 3 - (1 - \sqrt{1 - x^2_o}))]
\end{aligned}
\tag{6}
$$

The spectrum is shown in Fig. 1, including the projection in energy (integrated over all angles) and the projection in $cos(\theta)$ (integrated over energy). The $\mathcal{TWIST}$ experiment will measure the decay spectrum over an acceptance of approximately $0.35 < |cos(\theta)| < 0.95$ and for $x > 0.3$, with several data sets of 10^9 events each within the acceptance. The Michel parameters will be extracted from the shape of the spectrum.

In addition to their importance in defining the decay spectrum and as constraints on the weak couplings, the Michel parameter η is important in determining the strength of the Fermi Coupling constant,[3] which can be written as

$$G^2_F = \frac{1}{\tau_\mu} \frac{192\pi^3}{m^5_\mu} \frac{1}{1 + 4\eta(m_e/m_\mu)} \tag{7}$$

[2] It is important to realize that because of the normalization condition (Equation 2), various equivalent expressions can be written for the Michel parameters. These forms may differ in appearance due to the elimination of the explicit dependance on one or more couplings. Other sources [3] don't show the cross term involving $g^{S*}_{\epsilon\mu}$.

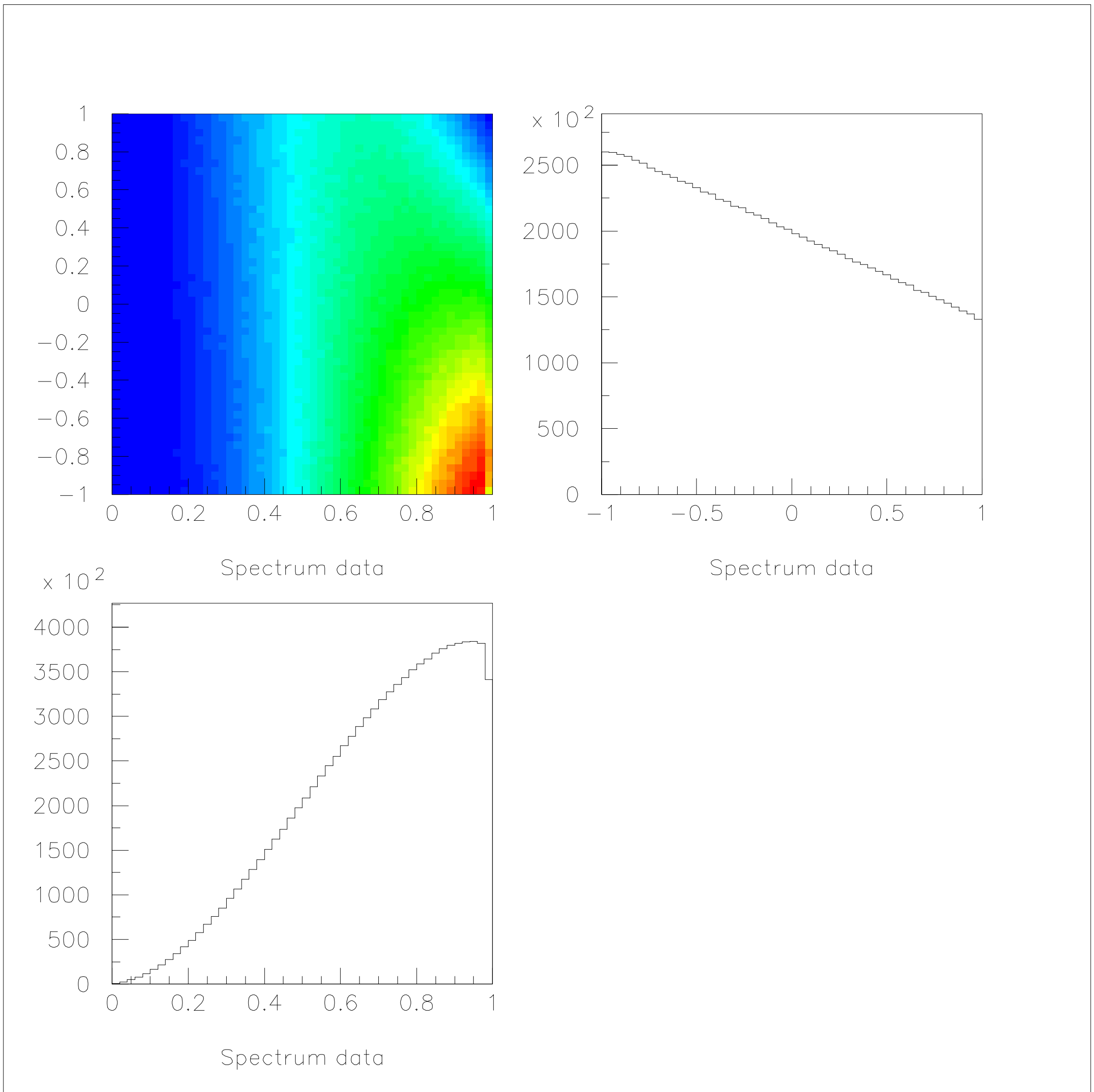

Figure 1: The Michel Spectrum including radiative corrections and the resolution anticipated from $\mathcal{TWIST}$. The spectrum at the upper left shows the spectrum as a function of $cos(\theta_e)$ on the vertical axis and the reduced positron energy x on the horizontal. The spectrum at the upper right shows the number of counts as a function of $cos(\theta_e)$ for the muon polarization oriented toward $cos(\theta_e) = -1$. The figure at the lower left shows the spectrum vs. the reduced energy x.

Decay parameter	Standard Model value	Measured value	Anticipated $\mathcal{TWIST}$ precision
ρ	0.75	0.7518 $\pm$ 0.0026	0.00031
η	0.0	-0.007 $\pm$ 0.013	$>$ 0.01
δ	0.75	0.7486 $\pm$ 0.0026 $\pm$ 0.0028	0.00015
$P_\mu\xi$	1	1.0027 $\pm$ 0.0079 $\pm$ 0.0030	0.00017
Q_R^μ	0	$<$ 0.0077	0.0002
$P_\mu\xi\delta/\rho$	1	$\geq$ 0.99682	not evaluated
lifetime		2.19703 $\pm$ 0.00004 μs	–
ξ'	1	1.00 $\pm$ 0.04	–

Table 1: Current values of some of the parameters describing μ decay. The values are those accepted by the Particle Data Group[4]. For details on the anticipated $\mathcal{TWIST}$ precision, see Table 2

Michel parameter	systematic uncertainty[5] ($x10^{-4}$)	statistical uncertainty[5] ($x10^{-4}$)	uncertainty in the radiative corrections[6] ($x10^{-4}$)	sum (in quadrature) ($x10^{-4}$)
ρ	0.7	0.5	3	3.1
η	not evaluated	not evaluated	100	$>$ 100
$P_\mu\xi$	1	1	1	1.7
δ	0.7	0.8	1	1.5

Table 2: Contributions to the anticipated uncertainty of the $\mathcal{TWIST}$ measurements.

where τ_μ is the muon lifetime The effect of the uncertainty in η on the value of G_F can be seen by writing

$$\begin{aligned} \tfrac{\Delta G_F}{G_F} \mid_{\eta=0} &= -0.2\Delta\eta \\ &= 0.00025 \end{aligned} \tag{8}$$

for $\Delta\eta = 0.013$. Given the accepted value[4] of $G_F/(\hbar c)^3 = 1.16639(1)GeV^{-2}$, considered accurate to ≈ 1 in 10^5, the uncertainty in η contributes an uncertainty 30x larger than the total uncertainty quoted in G_F.

2 Anticipated $\mathcal{TWIST}$ results

Experimental knowledge of the Michel parameters, along with measured values of other observables in muon decay are summarized along with the $\mathcal{TWIST}$ anticipated precision in Table 1.

The "anticipated $\mathcal{TWIST}$ precision" shown in Table 1 has been calculated using values found in Table 2. It has been assumed that the statistical, systematic, and theoretical uncertainties are uncorrelated.

3 Model-independent tests of the Weak Interaction in Muon Decay

In any physical process, the coupling constants enter in bilinear combinations. In many cases, the observables available involve a sum over several positive definite terms. Because a zero sum of several positive definite terms requires that each of the terms be zero, it is possible to constrain the 19 coupling constants with far less than 19 physical measurements. It has been shown that the Lorentz structure of the weak interaction in muon decay can be constrained by only five judiciously chosen experiments.[3][7] One choice for the five measurements is

- The muon lifetime which determines the magnitude of the Fermi coupling constant,
- the Michel parameter δ,
- the Michel parameter ξ,
- the outgoing electron polarization, and
- the rate of absorption of ν_e from muon decay.

This reduction to five required measurements is extremely powerful, and greatly increases the importance of the $\mathcal{TWIST}$ measurements. Of the items on the list of five proposed experiments, $\mathcal{TWIST}$ will contribute two measurements.

Taken by themselves, the $\mathcal{TWIST}$ measurements also offer powerful tools for the study of the weak interaction in muon decay. First of all, of course deviations of any of the Michel parameters from their Standard Model values would signal new physics.

The Michel parameters ξ and δ can be used in a search for right-handed currents and other muon decay modes outside of the standard model. The muon decay rate can be broken into four pieces describing

- The decay of a left-handed muon into a left-handed electron,
- the decay of a left-handed muon into a right-handed electron,
- the decay of a right-handed muon into a left-handed electron,
- the decay of a right-handed muon into a right-handed electron.

Each of these probabilities can be expressed in terms of the coupling constants, as follows

$$\begin{array}{lcllll} Q_{LL} & = & \frac{1}{4}\mid g^S_{LL}\mid^2 & + & \mid g^V_{LL}\mid^2, & \\ Q_{LR} & = & \frac{1}{4}\mid g^S_{LR}\mid^2 & + & \mid g^V_{LR}\mid^2 & + \; 3\mid g^T_{LR}\mid^2 \\ Q_{RL} & = & \frac{1}{4}\mid g^S_{RL}\mid^2 & + & \mid g^V_{RL}\mid^2 & + \; 3\mid g^T_{RL}\mid^2 \\ Q_{RR} & = & \frac{1}{4}\mid g^S_{RR}\mid^2 & + & \mid g^V_{RR}\mid^2 & \end{array} \tag{9}$$

Note that $0 \leq Q_{\epsilon\mu} \leq 1$ and that $\sum_{\epsilon\mu} Q_{\epsilon\mu} = 1$. $Q_{\epsilon\mu}$ is the probability for the decay of a muon of chirality μ into an electron of chirality ϵ.

We can write

$$Q^\mu_R \equiv Q_{RR} + Q_{LR}, \tag{10}$$

which is the relative rate of the decay of right-handed muons into either right- or left-handed electrons. Using Equations 9, we get

$$Q^\mu_R = \frac{1}{4}\mid g^S_{RR}\mid^2 + \frac{1}{4}\mid g^S_{LR}\mid^2 + \mid g^V_{RR}\mid^2 + \mid g^V_{LR}\mid^2 + 3\mid g^T_{LR}\mid^2, \tag{11}$$

which can be written in terms of the Michel parameters ξ and δ using Equations 3, as

$$Q_R^\mu = \frac{1}{2}(1 + \frac{1}{3}\xi - \frac{16}{9}\xi\delta). \tag{12}$$

The absence of right-handed currents in the standard model requires that this quantity be zero. If this single observable is found to be zero, that would demonstrate that all of the weak couplings to right-handed muons are zero. This would constitute a proof that the weak coupling acts exclusively on the left-handed component of the muon. If Q_R^μ is non-zero, then a weak coupling to the right-handed component of the muon would have to exist (one of the $g_{\epsilon R}^\gamma \neq 0$), indicating physics beyond the standard model.

Using accepted values[4] for $P_\mu\xi$[8] and δ[9], we can infer that $\xi > 0.994$. The allowed region for δ and the region inferred for ξ are shown in Fig. 2. The line defining $Q_R^\mu = 0$ passes through the allowed region, indicating that the results are consistent with the absence of right-handed currents in muon decay.

To the extent[3] that $\mathcal{TWIST}$ is able to determine or limit ξ and δ, we will obtain a model-independent constraint on right-handed currents that does not rely upon other measurements. We expect to determine δ and $P_\mu\xi$ at the level of ~ 0.0002. The $\mathcal{TWIST}$ result for $P_\mu\xi$ will be in the form

$$P_\mu\xi \geq limit \tag{13}$$

from which we can conclude that

$$\xi \geq limit \tag{14}$$

(where "limit" refers to the same numerical value in Eq 13).

The $\mathcal{TWIST}$ limit on Q_R^μ will be much stronger then the existing limits. A preliminary result with an accuracy ≈ 0.001 is anticipated for both $P_\mu\xi$ and for δ. The allowed region about the standard model is shown in Figure 3. Using the anticipated $\mathcal{TWIST}$ accuracy from Table 1, the anticipated final $\mathcal{TWIST}$ limits on δ and ξ are shown as well. The figure assumes that the standard model holds, and that no new depolarization phenomena decrease the measured asymmetry.

4 Tests of Left-right Symmetric Models

Additional strength can be gained from the $\mathcal{TWIST}$ measurements by studying specific models. In particular, there is considerable interest in testing the validity of left/right symmetric models. Specifically, consider the case in which the scaler and tensor couplings are taken to be zero, and $g_{RL}^V = g_{LR}^V$ without additional assumptions.

The right-handed sector is introduced through mixing of mass eigenstates M_1 and M_2[11].

$$\begin{aligned} M_L &= M_1 cos(\zeta) - M_2 sin(\zeta) \\ M_R &= e^{i\omega} M_1 sin(\zeta) + e^{i\omega} M_2 cos(\zeta). \end{aligned} \tag{15}$$

M_1 is the eigenvalue of one mass eigenstate ($M_1 \approx M_L$), M_2 is the mass of a new weak boson, ζ is the left/right mixing parameter, and ω is a CP-violating phase. For the

[3]in the end, our knowledge of P_μ will be limited by our ability to limit or correct for muon depolarization. Because a polarimeter more sensitive than $\mathcal{TWIST}$ will not be available for the study of these depolarizing effects, $\mathcal{TWIST}$ will determine a lower limit for $P_\mu\xi$.

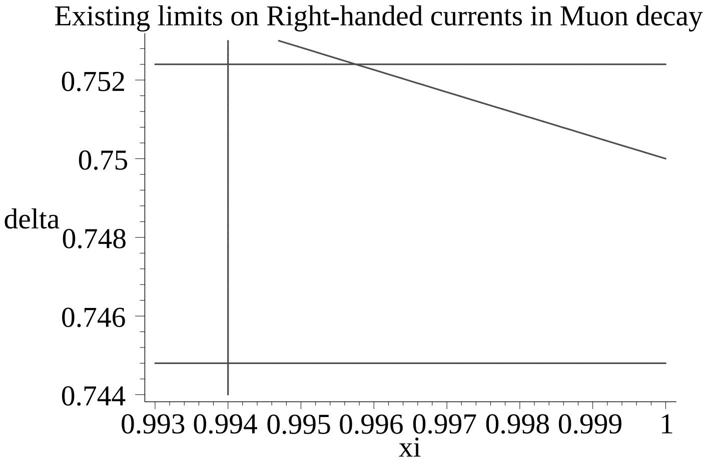

Figure 2: Existing limits on δ and ξ. The diagonal line defines the condition $Q_R^\mu = 0$. The observation that this curve passes through the region allowed by the measurements of δ and ξ demonstrates that the measurements are consistent with the absence of right-handed currents in muon decay.

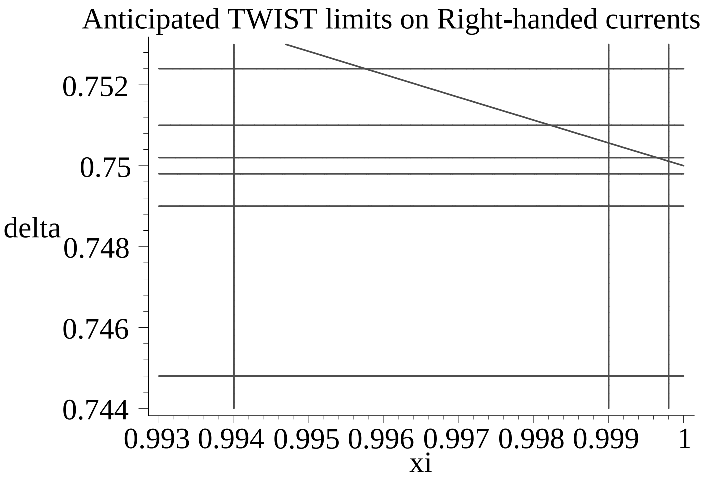

Figure 3: Anticipated $\mathcal{TWIST}$ limits on δ and ξ. The sensitivity is plotted for both preliminary and final results, along with the existing exclusion limits shown taken from Figure 2. If the diagonal line defining $Q_R^\mu = 0$ passes through that region, that would indicate the absence of right-handed currents in muon decay. If the diagonal does not pass through the region constrained by $\mathcal{TWIST}$ that would indicate the existence of right-handed currents and a violation of the Standard Model. The existing limits, shown in Fig. 2 are included to put the anticipated $\mathcal{TWIST}$ precision into context.

case of manifest left/right symmetry and assuming no CP violation, measurements of ρ and ξ fully determine the two parameters of the model. In addition, measurements of η and δ can be used as constraints on the model.

Using the definitions of the Michel Parameters (from Equations 3), together with the normalization condition given by Equation 2, it is possible to write the exact identities[4]

$$\begin{aligned}
\rho - \tfrac{3}{4} \equiv\ & -\tfrac{3}{4} \mid g^V_{RL} \mid^2 - \tfrac{3}{4} \mid g^V_{LR} \mid^2 - \tfrac{3}{2} \mid g^T_{RL} \mid^2 - \tfrac{3}{2} \mid g^T_{LR} \mid^2 \\
& \tfrac{3}{4} Re[g^{S*}_{RL} g^T_{RL} + g^S_{RL} g^{T*}_{RL} + g^{S*}_{LR} g^T_{LR} + g^S_{LR} g^{T*}_{LR}] \\
\\
\xi - 1 \equiv\ & -\tfrac{1}{2} \mid g^S_{RR} \mid^2 - \tfrac{1}{2} \mid g^S_{LR} \mid^2 \\
& -2 \mid g^V_{RR} \mid^2 - 4 \mid g^V_{RL} \mid^2 + 2 \mid g^V_{LR} \mid^2 - 8 \mid g^T_{RL} \mid^2 + 2 \mid g^T_{LR} \mid^2 \\
& -\tfrac{1}{2} Re[g^{S*}_{RL} g^T_{RL} + g^S_{RL} g^{T*}_{RL} - g^{S*}_{LR} g^T_{LR} - g^S_{LR} g^{T*}_{LR}]
\end{aligned} \tag{16}$$

Assuming only vector couplings, and $g^V_{LR} \approx g^V_{RL}$, the Michel parameters take the simple forms

$$\begin{aligned}
&\rho = \tfrac{3}{4}[1 - 2 \mid g^V_{LR} \mid^2] = \tfrac{3}{4}[1 - 2\zeta^2] \\
&\eta = 0 \\
&\delta = \tfrac{3}{4} \\
&\xi = 1 - 2 \mid g^V_{RR} \mid^2 - 2 \mid g^V_{LR} \mid^2 = 1 - 2(\epsilon^2 + \zeta^2),
\end{aligned} \tag{17}$$

where the parameters ϵ and ζ take the approximate values

$$\begin{aligned}
&\epsilon = (\tfrac{M_L}{M_R})^2 \approx g^V_{RR} << 1 \\
\\
&g^V_{LR} = g^V_{RL} \approx \zeta << 1.
\end{aligned} \tag{18}$$

The polarization of the muons used by $\mathcal{TWIST}$ as those muons are produced at rest is predicted by the standard model to be -1. In the left-right symmetric models, the muon polarization can be written as

$$P_\mu = 1 - 2[\epsilon \frac{cos(\theta_R)}{cos(\theta_L)} + \zeta]^2. \tag{19}$$

The quotient $cos(\theta_R)/cos(\theta_L)$ is the quotient of matrix elements in the left- and right-handed CKM matrices. Some authors express this as V_R/V_L, or as V^{ud}_R/V^{ud}_L. The generalization to $cos(\theta_R)/cos(\theta_L)$ allows for the specific cases of muons from pion decay and from kaon decay, which depend upon the ud and us mixing respectively. The muon polarization can be written

$$\begin{aligned}
&P^\pi_\mu = 1 - 2[\epsilon \tfrac{V^{ud}_R}{V^{ud}_L} + \zeta]^2 \\
\\
&P^K_\mu = 1 - 2[\epsilon \tfrac{V^{us}_R}{V^{us}_L} + \zeta]^2.
\end{aligned} \tag{20}$$

The left-right symmetric models can be separated into those that make no assumptions about the right-handed CKM matrix, and those that assume that the right-handed CKM matrix is the same as the left-handed matrix. The latter is described

[4]The cross terms involving $g^{S*}_{\epsilon\mu}$ are not found in Reference [3].

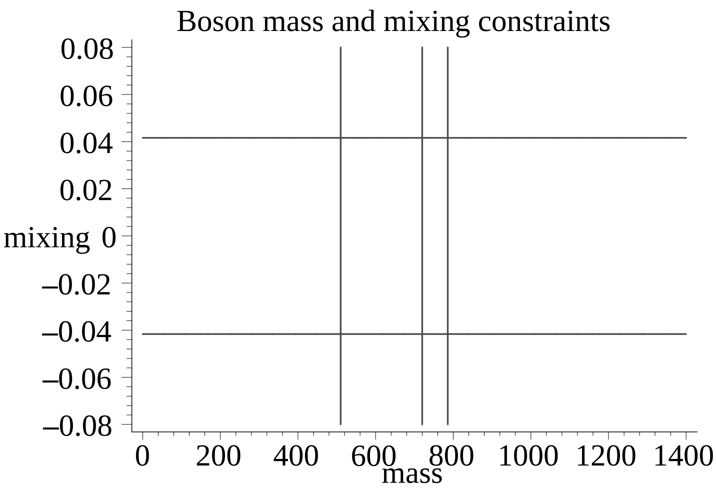

Figure 4: Existing limits on W_R mass and mixing. The W_R mass limits are determined from direct searches at the Tevatron, and are therefore sensitive to assumptions about the right-handed CKM matrix. The D0 collaboration excludes W_R masses less than 720 GeV under the assumption of left-right symmetry, but when $V_R^{us} = 1$, that limit is reduced to 510 GeV.[13] The CDF limit at 786 GeV assumes manifest left-right symmetry[12]. Limits on left-right mixing are taken from a measurement of ρ in muon decay by Derenzo,[4][14][15][16] which excludes mixing angles with magnitude greater than 0.042

as manifest left-right symmetry.[11] $\mathcal{TWIST}$ has some sensitivity to the assumptions made about the right-handed CKM matrix, but the direct searches are much more sensitive to these assumptions. Searches for a W boson at mass scales greater than the mass of the known W have been undertaken at the Tevatron, where a W could be produced in pp collisions. In this case, the elements of the right-handed CKM matrix are of direct importance. Tevatron results from both CDF[12] and D0[13] are shown in 4. The results exclude W_R masses less than 786 GeV and 720 GeV respectively, under the assumption of manifest left-right symmetry. When V_R^{us} is set to unity in the right-handed CKM matrix, the D0 limit drops to 510 GeV, indicating the sensitivity of these searches to assumptions about the right-handed CKM matrix.

The direct search results are insensitive to the left/right mixing angle. A result obtained by Derenzo[4][14][15][16] for ρ has been used to set a limit on the left/right mixing which is shown in Fig. 4.

In the experiment done by Strovink et al. at TRIUMF[17][18], the quantity $P_\mu \xi \delta/\rho$ was measured. The result was used to obtain limits on the W_R mass and mixing under the assumption of manifest left-right symmetry. Using 19 under the assumption that $cos(\theta_R) = cos(\theta_L)$ and the expressions for ρ, δ, and ξ in Eq. 17, to second order in ϵ and ζ

$$P_\mu \xi \frac{\delta}{\rho} = 1 - 2(2\epsilon^2 + 2\epsilon\zeta + \zeta^2) \tag{21}$$

The Strovink result is shown in Fig. 5 along with results from Derenzo and the Tevatron taken from Figure 4.

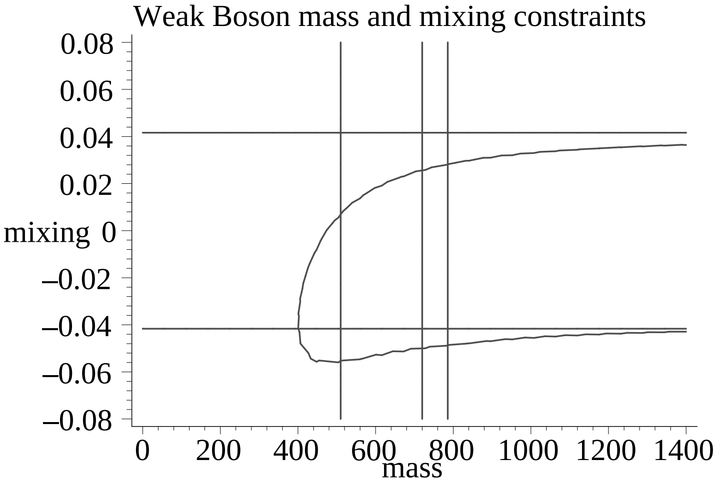

Figure 5: The same curves plotted in Figure 4 are reproduced here, along with a curve defined by the Strovink result $P_\mu\xi\delta/\rho \geq 0.99682$ without the assumption of left/right symmetry.

The Strovink result can also be analyzed without invoking manifest left/right symmetry, that is, without assuming that $cos(\theta_R) = cos(\theta_L)$ in Eq. 19. In that case, again to second order in ϵ and ζ, we obtain

$$P_\mu\xi\frac{\delta}{\rho} = 1 - 2\epsilon^2[\frac{cos(\theta_R)}{cos(\theta_L)}]^2 - 4\epsilon\zeta\frac{cos(\theta_R)}{cos(\theta_L)} - 2\epsilon^2 - 2\zeta^2, \tag{22}$$

which reduces to Eq. 21 when $cos(\theta_R) = cos(\theta_L)$.

A range of assumptions for the right-handed CKM matrix is tested by setting $cos(\theta_R)/cos(\theta_L) = 0$. In this case, the exclusion limits provided by the Strovink measurement shift slightly, but the results are not affected as much as the D0 results shown in Fig. 4. The two loci,

$$\begin{aligned} P_\mu\xi\tfrac{\delta}{\rho} &= 1 - 2(2\epsilon^2 + 2\epsilon\zeta + \zeta^2) \geq 0.99682 \\ P_\mu\xi\tfrac{\delta}{\rho} &= 1 - 2\epsilon^2 - 2\zeta^2 \geq 0.99682 \end{aligned} \tag{23}$$

are shown in Fig. 6.

4.1 Testing Left/Right Symmetric Models with $\mathcal{TWIST}$

For a general test of left/right symmetric models, without assumptions about the right-handed CKM matrix, the curves shown in Fig. 6 representing work done by Strovink et al., Derenzo, and the D0 collaboration define the state of the art. These exclusion limits are included on figures that follow to keep the $\mathcal{TWIST}$ anticipated results in context.

It should be noted that the exclusion regions determined from the Strovink measurement, plotted in Figs. 5 and 6, involve only the measurement of $P_\mu\xi\delta/\rho$. The

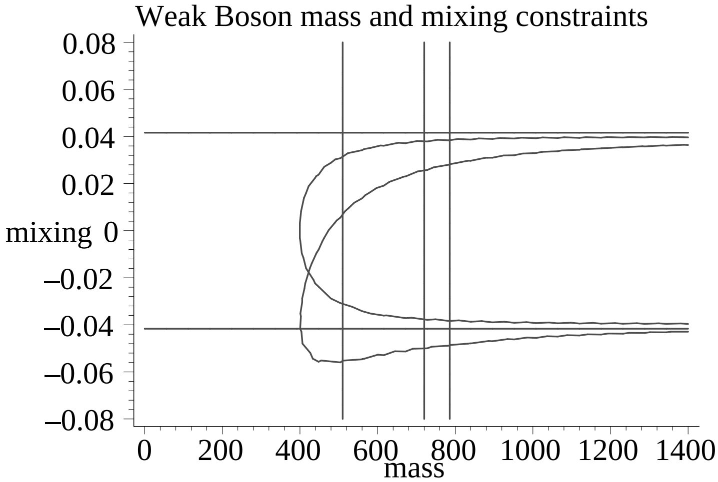

Figure 6: Limits on the W_R mass and left/right mixing obtained by the Strovink measurement of $P_\mu\xi\delta/\rho \geq 0.99682$. The two elliptical loci are obtained using Equations 23, the lower of the two assuming manifest left-right symmetry, and the more nearly cetered locus obtained assuming that $cos(\theta_R)/cos(\theta_L) = 0$. The horizontal and vertical lines are those plotted in Figure 4.

$\mathcal{TWIST}$ measurements will provide stronger constraints not only due to the increased systematic and statistical precision, but also because multiple parameters will be measured. For example, both ρ and $P_{mu}\xi$ will be measured. To a partial extent, these two measurements will be independent. Certainly they will be statistically independent, and the systematic uncertainties will be largely independent since those related to the measurement of $P_\mu\xi$ are dominated by polarization related uncertainties while uncertainties in ρ are dominated by effects of correlations with the poorly determined η and radiative corrections.

The constraint on left/right mixing from the ρ measurement comes from Equations 17. A constraint on the W_R mass and the left/right mixing can be obtained from a limit on $P_\mu\xi$. Using Equations 17 and 19, to second order in ϵ and η, we obtain

$$P_\mu\xi = 1 - 2\epsilon^2 - 2\zeta^2 - 2\epsilon^2[\frac{cos(\theta_R)}{cos(\theta_L)}]^2 - \epsilon\zeta\frac{cos(\theta_R)}{cos(\theta_L)} - 2\zeta^2. \tag{24}$$

$\mathcal{TWIST}$ anticipates making preliminary measurements of ρ and $P_\mu\xi$ with a precision ≈ 0.001. In this case, assuming that $cos(\theta_R) = cos(\theta_L)$, the limits shown in Fig 7 are obtained. The results are shown in 7, along with the results from Strovink, D0, and Derenzo which are obtained without assuming manifest left/right symmetry. It is expected that the preliminary $\mathcal{TWIST}$ results will be sensitive beyond the present limits. Note that the $\mathcal{TWIST}$ result shown provides a consistency check, as the mass/mixing constraint provided by the $\mathcal{TWIST}$ $P_\mu\xi$ measurement has not been combined with the constraint on mixing from the $\mathcal{TWIST}$ ρ measurement.

The sensitivity of the anticipated final precision of the $\mathcal{TWIST}$ measurements (taken from Table 1) is shown in Fig. 8.

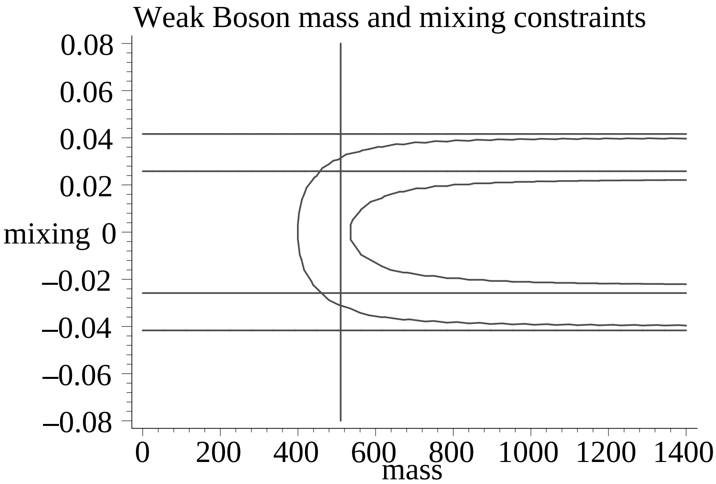

Figure 7: Constraints anticipated from preliminary $\mathcal{TWIST}$ measurements of ρ and $P_\mu\xi$ at an accuracy of ± 0.001. The limits have been obtained without invoking manifest left/right symmetry. Preliminary results will be sensitive to W_R masses less than $\approx$ 500 GeV and to mixing greater than 0.026.

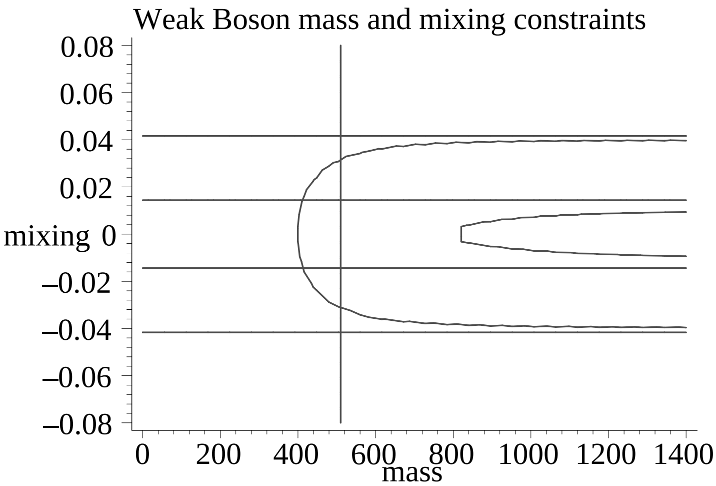

Figure 8: Constraints anticipated from the final $\mathcal{TWIST}$ measurements of ρ and $P_\mu\xi$. A $\mathcal{TWIST}$ measurement of ρ with a precision of ± 0.0003 will be sensitive to mixing in left/right symmetric models with mixing angles greater than ≈ 0.014, and to W_R masses less than $\approx$ 900 GeV. Again, the limits have been obtained without invoking manifest left/right symmetry.

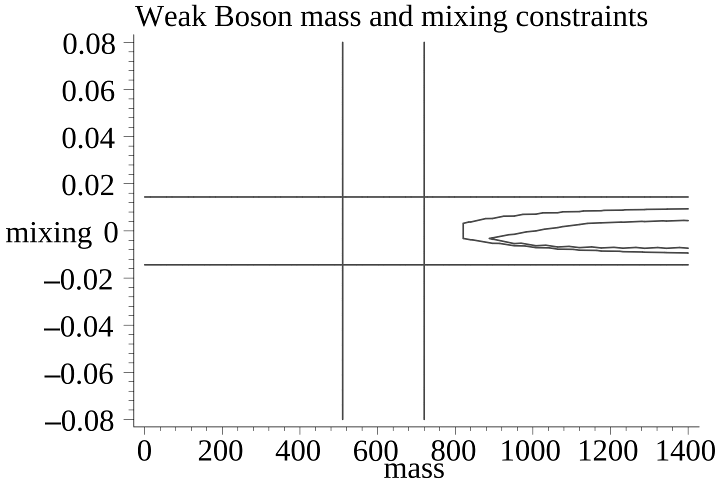

Figure 9: Constraints anticipated from the final $\mathcal{TWIST}$ measurements of ρ and $P_\mu\xi$. The two elliptical loci demonstrate the small sensitivity of the $\mathcal{TWIST}$ measurements to assumptions about the right-handed CKM matrix, the larger ellipse having been corresponding to the case given by Equation 24 under the assumption $cos(\theta_L)/cos(\theta_R) = 1$, and the weaker limit corresponding to the case $cos(\theta_R)/cos(\theta_L) = 0$. The two vertical lines are obtained by D0 under corresponding assumptions regarding the right-handed CKM matrix.

Unlike the Tevatron results, the $\mathcal{TWIST}$ result is largely insensitive to assumptions about the right-handed CKM matrix. The limits shown in Figure 9 demonstrate this lack of sensitivity. The tighter locus was calculated using Equation 24, under the assumption $cos(\theta_L)/cos(\theta_R) = 1$. The weaker limit is obtained using Equation 24, for the case $cos(\theta_R)/cos(\theta_L) = 0$. In contrast, the two limits obtained by D0 under the same assumptions are shown to have considerable sensitivity to the assumption made about the right-handed CKM matrix.

5 Possible tests of Supersymmetric Theories with $\mathcal{TWIST}$

In the preceding sections, we have discussed model-independent tests of the (V-A) interaction and searches for left/right mixing in left/right symmetric models. In the left/right symmetric models, the Michel parameters η and δ maintain their standard model values. It was pointed out that deviations of these parameters may signal a flaw in the left/right symmetric model being tested.

It has been suggested that the muon decay spectrum may be sensitive to R-parity violating super-symmetric theories. The R-parity violating interactions $\lambda_{311}L_L^{(3)}L_L^1$-$E_R^{(1)} + \lambda_{322}L_L^{(3)}L_L^2E_R^{(2)}$ (where the index denotes the lepton generation) alter the Michel parameters as follows[19]:

$$
\begin{aligned}
\Delta\rho &= \tfrac{3\epsilon^2}{16} \\
\Delta\eta &= \tfrac{\epsilon}{2} \\
\Delta\xi &= \tfrac{-\epsilon^2}{4} \\
\Delta\delta &= 0 \\
\epsilon &\equiv \lambda_{311}\lambda_{322}/4\sqrt{2}G_F\tilde{m}^2_{e_l^{(3)}}
\end{aligned}
\tag{25}
$$

In this case, because $\Delta\eta$ enters in first order in the SUSY coupling, it may be possible to look for the relative deviations in the Michel parameters. That task is made difficult, however, due to the smaller sensitivity of the spectrum to η.

While any chance of seeing evidence of SUSY is speculative, it is interesting that it is the set of deviations in the Michel parameters that could signal SUSY. If the SUSY coupling parameter ϵ is determined by $\Delta\eta$, then

$$
\begin{aligned}
\Delta\rho &\approx \tfrac{3}{4}\Delta\eta^2 \\
\Delta\xi &\approx -\Delta\eta^2
\end{aligned}
\tag{26}
$$

At this stage, $\mathcal{TWIST}$ sensitivity to $\Delta\eta$ has not been fully evaluated, nor is it clear what magnitude to expect for plausible ϵ, but it is expected that $\mathcal{TWIST}$ will have increased sensitivity to η over previous work. As well, it is clear that determining the set of four Michel parameters at increased precision in a single experiment will be an advantage.

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