Technote 78
Asymmetry Measurement in TWIST

Blair Jamieson

University of British Columbia

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1 Introduction

This note is meant to summarize the current method used to calculate asymmetry of muon decays with TWIST analysis package (MOFIA). Methods of comparing these plots to expected asymmetries to estimate the average polarization of the muon beam is presented.

2 Method of Finding Asymmetry

The asymmetry (G) calculated in mofia is defined as the difference between the count of forward decays (F) and the backward decays(B) divided by their sum as in Equation 1.

$$G = \frac{F - B}{F + B} \tag{1}$$

In MOFIA, there are currently two types of asymmetry plots made. The first is an asymmetry as a function of decay time, and the other is asymmetry as a function of capacitive probe time (TCAP). The plots in mofia are made for all clean and time clean events, using the window classification to decide on the direction of the decay.

Unfortunately, the acceptance of events which come from the event classification is not well defined right now. This makes it difficult to estimate the average muon polarization with these plots. To make a simple comparison, plots of asymmetry for events cut on fit $\cos\theta$ and fit momentum (actually on reduced energy x) were made. By choosing a region of fit $\cos\theta$ and x that has nearly unity reconstruction efficiency, a comparison to the asymmetry of the Michel distribution with the same cuts in $\cos\theta$ and x should give a reasonable guess at the average muon polarization. Plots of asymmetry with these cuts on them were done using information from the ntuples of the most recent production analysis. The plot of forward counts, backward counts, and asymmetry versus TCAP is shown in Figure 2.

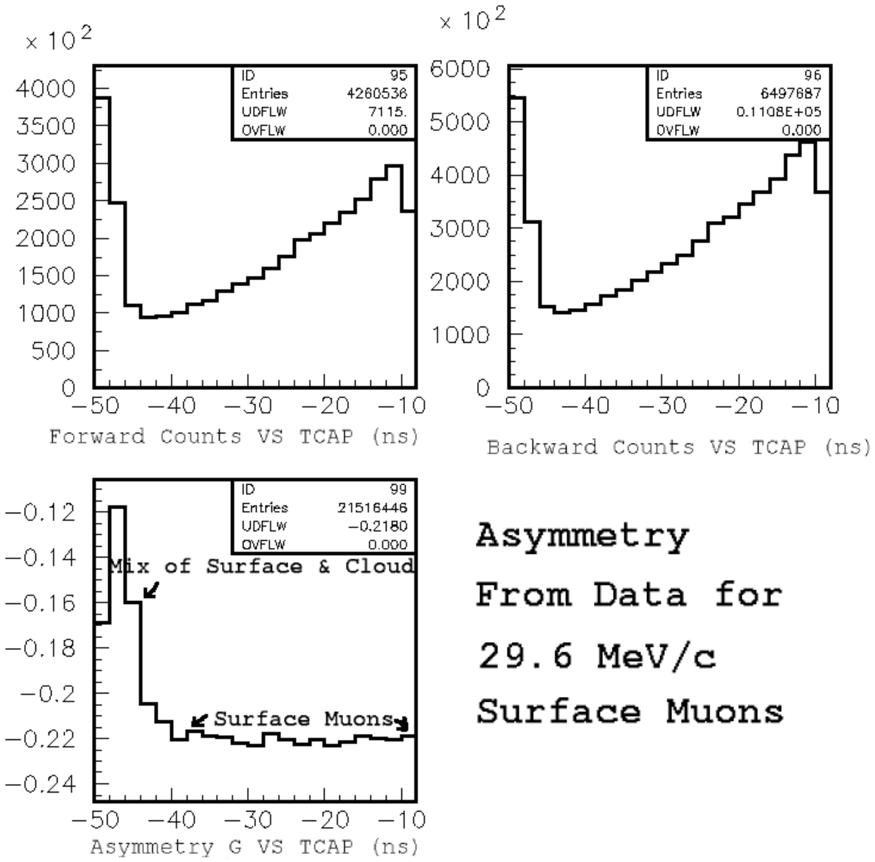

Figure 1: Asymmetry from data for 29.6 MeV/c surface muon runs. The assymetry of -0.22 shown in this figure is with cuts on the fiducial volume: $0.3 < x < 1.0$, and $0.5 < abs(\cos\theta) < 0.97$.

Comparing to analysis of surface muon decays from GEANT data, with the same cuts on event type and fiducial volume, the asymmetry is found to be -0.2559. In the next section it will be shown that in the fiducial volume of $0.3 < x < 1.0$, and $0.5 < abs(\cos\theta) < 0.97$ the asymmetry of the Michel distribution is -0.264. This allows us to estimate the muon polarization in data as -0.83, and the polarization in GEANT as -0.97. Since we know in GEANT that the polarization was at least -0.997, this gives a measure of how well we can trust this calculation.

Without fiducial volume cuts, the asymmetry from analysis of GEANT data is -0.189. With the same code run on real data, the asymmetry is approximately -0.15. To find the volume of integration of the Michel spectrum to roughly match the polarizations obtained from analysis of GEANT data with and without cuts (Refer to the next section for this calculation). The result is that without cuts on the fiducial volume, the region of integration to match polarizations is approximately $0 < x < 1$, and $0.1 < abs(\cos\theta) < 1$. With this region of integration, the asymmetry of the Michel spectrum is -0.183, which gives us an estimated GEANT polarization of $-0.189/0.183 \approx -0.99$, and real data polarization of $-0.15/0.183 \approx -0.82$.

From a cloud muon run, the asymmetry of the data with cuts on the fiducial volume is found to be +0.05. Using equation 9 in the next section, this gives an estimated Cloud Muon polarization of $.05/.264 \approx 0.19$.

3 Calculation of Expected Asymmetry

To calculate the expected Asymmetry, start with the expression for the differential decay rate in Equation 2[1].

$$\frac{d\Gamma}{dx d\cos\theta} \propto 3x^2 - 2x^3 + P_\mu \xi \cos\theta (2x^3 - x^2) \tag{2}$$

Now we integrate over momentum from lower limit p, to upper limit q:

$$\frac{d\Gamma}{d\cos\theta} \propto \int_p^q 3x^2 - 2x^3 + P_\mu \xi \cos\theta (2x^3 - x^2) dx \tag{3}$$

$$\frac{d\Gamma}{d\cos\theta} \propto q^3 - p^3 - \frac{1}{2}(q^4 - p^4) + P_\mu \xi \cos\theta (\frac{q^4 - p^4}{2} - \frac{q^3 - p^3}{3}) \tag{4}$$

Now set $r = q^3 - p^3$, and $s = q^4 - p^4$, then the number of forward counts (F) is given by integrating over $\cos\theta$ from a to b:

$$F \propto \int_a^b r - s/2 + P_\mu \xi \cos\theta (s/2 - r/3) d\cos\theta \tag{5}$$

$$F \propto (r - s/2)(b - a) + P_\mu \xi (s/2 - r/3)(\frac{b^2}{2} - \frac{a^2}{2}) \tag{6}$$

Similarily, the number of backward counts (B) is found by integrating over $\cos\theta$ from -b to -a:

$$B \propto \int_{-b}^{-a} r - s/2 + P_\mu \xi \cos\theta (s/2 - r/3) d\cos\theta \tag{7}$$

$$B \propto (r - s/2)(b - a) + P_\mu \xi (s/2 - r/3)(\frac{a^2}{2} - \frac{b^2}{2}) \tag{8}$$

The asymmetry is then given by:

$$G = \frac{F - B}{F + B} = \frac{P_\mu \xi (3(q^4 - p^4) - 2(q^3 - p^3))(b + a)}{6(2(q^3 - p^3) - q^4 + p^4)} \tag{9}$$

Using Equation 9, we can calculate the expected asymmetry for different cuts on reduced energy and momentum. The expected asymmetry of Michel muons with $P_\mu \xi = -1$, with different fiducial volume cuts is summarized in Table 1. Note that the analysis in this section has not accounted for radiative corrections which could effect the polarization obtained by a few percent.

Table 1: Asymmetry of the Michel spectrum for Surface Muon decays with $P_\mu \xi = -1$.

Desrciption	p	q	a	b	G
Complete Spectrum	0	1	0	1	$\frac{-1}{6} = -0.167$
Nominal Acceptance Cuts	0.3	1	0.5	0.97	-0.264
Approx. No Acceptance Cuts	0	1	0.1	1.0	-0.183

4 Conclusion

The method of calculating asymmetries is not perfect, but gives rough estimates of the polarization to about the three percent level for GEANT data. For real data there must still be biasses in the event classification which results fewer upstream events than expected. The average polarization of surface muons from real data in the Mylar target is estimated to be only -0.83. For real data Cloud muons were estimated to have polarization of +0.19.

References

[1] Review of Particle Physics, Phys. Rev. **D 66** (2002) 1.