1 Proposed Implementation of Energy Calibration

This is a short not to outline a proposed method of improving the energy calibration at momenta other than at the decay positron endpoint momentum (p_{edge}). Recall that in our first measurement of ρ and δ we used the corrected momentum p_{ec} given by:

$$p_{ec} = \frac{p_{rec}}{1 + \frac{\beta}{p_{edge}}} + \frac{\alpha}{|\cos\theta|} \tag{1}$$

where p_{rec} is the reconstructed momentum $\cos(\theta)$ is the reconstructed cosine of the decay positron angle, β is the shift in the spectrum from the theoretical endpoint momentum, and $\alpha = (\alpha_u, \alpha_d)$ depending on whether the track is upstream (u) or downstream (d) of the muon stop.

Rob's studies of michel spectrum momenta positron tracks that traverse the detector can be used to look at momenta that differ from the endpoint momenta. Specifically he has made plots of the difference between the upstream and downstream reconstructed momenta times $\cos\theta$ versus the momentum ($\Delta p \bar{\cos}\theta$ versus p). These plots show a clear almost linear dependence that can be written with the form:

$$\Delta p \bar{\cos}\theta = \alpha_0 + \alpha_1 p = \alpha_{tot}(p) \tag{2}$$

This momentum dependent $\alpha_{tot}(p)$ should be related by a constant scaling to our endpoint momentum calibration $\alpha_{sum} = \alpha_u + \alpha_d$ at the endpoint. We obtain momentum dependent $\alpha_u(p)$ and $\alpha_d(p)$:

$$\alpha_u(p) = \frac{\alpha_u(p_{edge})}{\alpha_{tot}(p_{edge})}(\alpha_0 + \alpha_1 p) \tag{3}$$

$$\alpha_d(p) = \frac{\alpha_d(p_{edge})}{\alpha_{tot}(p_{edge})}(\alpha_0 + \alpha_1 p) \tag{4}$$

If we use this formulation, then the energy correction on the reconstructed momentum will look like the following equation:

$$p_{ec} = \frac{p_{rec}}{1 + \frac{\beta}{p_{edge}}} + \frac{\frac{\alpha}{\alpha_{tot}(p_{edge})}(\alpha_0 + \alpha_1 p_{rec})}{|\cos\theta|} \tag{5}$$

Or equivalently,

$$p_{ec} = \frac{p_{rec}}{1 + \frac{\beta}{p_{edge}}} + \frac{\alpha}{|\cos\theta|} \frac{(\alpha_0 + \alpha_1 p_{rec})}{(\alpha_0 + \alpha_1 p_{edge})} \tag{6}$$

Just to repeat this, we then find α and β from the endpoint energy calibration. The α_0 and α_1 would come from a fit to Rob's "final" analysis of the upstream stops data (a straight line fit to $\Delta p \cos\theta$ versus p).