as shown in Fig. 8 for K^+, but this must be accompanied by increased care to eliminate high voltage noise.

Note added in proof. Since submitting this article for publication we have tried to operate eight ion detectors in a single vacuum chamber and have come to the following conclusions:

(a) A thin scintillator (0.023 cm thick in our case) with a light pipe works just as well as the thick scintillator originally described, and is much less sensitive to background radiation (e.g., from a cyclotron).

(b) In the test apparatus an ion beam of small cross section was used. However, ions can be accepted over the full entrance aperture of the detector and high voltage noise kept at acceptably low levels if the following precautions are taken:

(i) All insulators in the vicinity of the ion detector should be carefully shielded.

(ii) The detector should be kept as dust-free as possible.

(iii) Noise from field emission can be avoided by polishing or shielding all metal electrodes and by keeping interelectrode spacings large.

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Beam Steering with Quadrupole and with Rectangular Box Magnets*

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The exact field configuration in the aperture of an asymmetrically excited electric or magnetic quadrupole lens is derived. The resulting effectiveness as a combined focusing and steering device for particle beams is analyzed, and the limitations are explored. Simple expressions are obtained for the steering effect. These results are compared to the effectiveness of a rectangular array of current sheets and iron as a steering device. The latter is shown to be superior in many cases.

INTRODUCTION

IT is often necessary to impart a small steering deflection to a beam of charged particles from an accelerator as the beam passes through the beam transport system. Since quadrupole lenses are usually distributed throughout such a system it is of interest to determine to what extent they can be used for steering as well as focusing. Koltay and Szabo[1] have performed an approximate analysis, and they report that such steering is being put into use at the Tandem Laboratory of the UITF, Copenhagen, and at the Institute of Nuclear Research of the Hungarian Academy of Sciences, Debrecen, Hungary. On the other hand, a rectangular array of iron and currents has been used by Hand and Panofsky[2] to produce a quadrupole field for a lens. Since this gives identically a quadrupole focusing field it is of interest to determine whether asymmetric excitation of this structure gives the same fields as asymmetric excitation of the usual quadrupole magnet. Such a steering system (rectangular box) is in use at the University of Colorado.[3]

FIELD OF SYMMETRIC QUADRUPOLE

In the interior of the lens, the field is well-known as

$$B_x = gy \quad \text{and} \quad B_y = gx, \tag{1}$$

which evidently satisfies

$$\nabla \times \mathbf{B} = 0. \tag{2}$$

This is obtained from the complex potential

$$W = igz^2/2, \tag{3}$$

with

$$B^* = B_x - iB_y = -dW/dz. \tag{4}$$

The physical potential is

$$\mathrm{Re}(W) = -gxy, \tag{5}$$

which requires the hyperbolic pole pieces as equipotentials.

* Supported in part by the U. S. Atomic Energy Commission.

[1] E. Koltay and G. Y. Szabo, Nucl. Instr. Methods 35, 88 (1965).

[2] L. N. Hand and W. K. H. Panofsky, Bull. Am. Phys. Soc. 3, 421 (1958); and Rev. Sci. Instr. 30, 927 (1959).

[3] D. Lind (private communication).

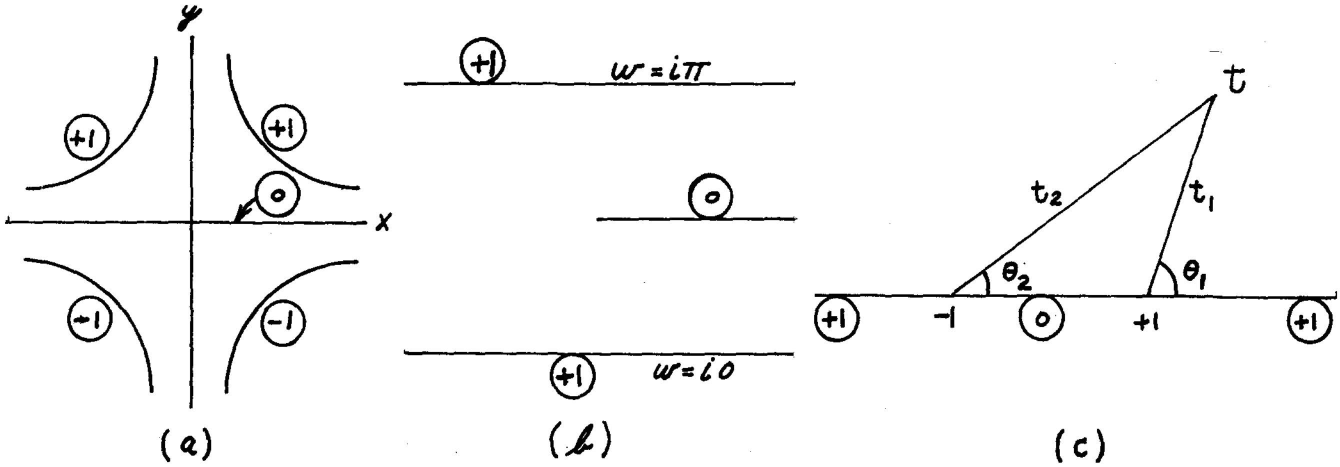

FIG. 1. (a) Pole pieces of quadrupole element with a steering excitation. The circled numbers ± 1 are the pole piece potentials. The circled zero denotes the zero potential along the x axis. (b) The transformation of Eq. (7) maps the upper half of (a) onto (b). (c) The transformation of Eq. (8) maps (b) onto (c).

These are excited S, N, S, N around the quadrants with equal magnitudes as required by Eq. (5).

FIELD OF ASYMMETRIC QUADRUPOLES

Consider hyperbolic pole pieces excited for steering as N, N, S, S, with equal magnitudes, around the quadrants, as shown in Fig. 1(a). The surfaces of the pole pieces are chosen as

$$|2xy| = \pi. \tag{6}$$

The fields for this configuration can be solved by conformal mapping, and the procedure is only outlined. We consider the pole pieces to be long enough so that this is a two dimensional problem in the x-y plane. Invoking symmetry, the fields are normal to the x axis. Thus, the problem concerns that part of the upper half plane bounded by the two upper hyperbolic pole pieces at potential $+1$ and the x axis at zero potential. This may be transformed to the w plane by

$$2w = z^2 + i\pi = (x^2 - y^2) + i(2xy + \pi), \tag{7}$$

as in Fig. 1(b). Churchill[4] gives the transformation from the w plane to the t plane of Fig. 1(c) as

$$w = i\pi - \tfrac{1}{2}\ln(t+1)(t-1), \tag{8}$$

where the equipotentials have now been mapped into the three parts of the axis $t = i0$.

The complex potential for Fig. 1(c) is

$$W = 1 + (i/\pi)\ln(t_1/t_2) = 1 + (i/\pi)\ln[(t-1)/(t+1)]. \tag{9}$$

Note that the real part is

$$\mathrm{Re}(W) = 1 + (\theta_2 - \theta_1)/\pi, \tag{10}$$

which evidently satisfies the boundary conditions everywhere along the pole pieces in the t plane.

[4] R. V. Churchill, *Complex Variables and Applications* (McGraw-Hill Book Company, Inc., New York, 1960).

Having obtained the complex potential [Eq. (9)] as a function of t we rewrite it to show its dependence on z as

$$W = 1 + (i/\pi)\ln\{[(1 - e^{-z^2})^{\frac{1}{2}} - 1]/[(1 - e^{-z^2})^{\frac{1}{2}} + 1]\}. \tag{11}$$

From Eq. (4) we obtain the field as

$$B^* = i(2z/\pi)(1 - e^{-z^2})^{-\frac{1}{2}}. \tag{12}$$

That this field satisfies the boundary conditions is proved by examining

$$\mathrm{Re}(B^* dz) = B_x dx + B_y dy. \tag{13}$$

On the x axis, we have $z = x$ so

$$B^* dz = i(2x dx/\pi)(1 - e^{-x^2})^{-\frac{1}{2}} = \text{pure imaginary}. \tag{14}$$

Thus the field is normal to the x axis. On the poles, $2xy = \pm\pi$ and $dz = (z^*/x)dx$. Then

$$B^* dz = i(2/\pi)\{zz^* dx/x[1 - e^{-(x^2 - y^2)} e^{\pm i\pi}]^{\frac{1}{2}}\}$$
$$= \text{pure imaginary}, \tag{15}$$

so the field is normal to the pole piece surfaces.

The magnetic field of Eq. (12) has the desired shape for steering near the origin since

$$\lim_{z \to 0} B^* = i(2/\pi), \tag{16}$$

so

$$B_x \to 0 \quad \text{and} \quad B_y \to -2/\pi. \tag{17}$$

However, when the particle beam appreciably fills the quadrupole aperture it is clear that this uniform field of constant magnitude and direction is not a good approximation.

Evidently, by unequal excitation of the poles the field at the center may be given any direction and the field is obtained from superpositions of rotated fields like that of Eq. (12). If we specify the excitations by quadrants in sequence, Fig. 1(a) would be $+1$, $+1$, -1, -1, producing a central field downward. The excitation -1, $+1$, $+1$, -1

TABLE I. The numbers are $(-\pi/2)B_y$ and $(-\pi/2)B_x$, respectively, from Eqs. (12) and (4) for the quadrupole. The evaluation below for the first quadrant gives the values in the other quadrants by symmetry. These numbers are also the deflections Δx and Δy, respectively, as discussed in the text. The dashed lines denote the approximate boundaries of a circular pipe filling the aperture.

	y: 0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
x: 0.0	(1.00, 0.00)	(0.99, 0.00)	(0.96, 0.00)	(0.91, 0.00)	(0.84, 0.00)	(0.76, 0.00)	(0.67, 0.00)	(0.57, 0.00)	(0.46, 0.00)	(0.36, 0.00)
0.2	(1.01, 0.00)	(1.00, 0.02)	(0.97, 0.04)	(0.92, 0.06)	(0.85, 0.08)	(0.77, 0.09)	(0.67, 0.10)	(0.57, 0.10)	(0.46, 0.10)	(0.26, 0.10)
0.4	(1.04, 0.00)	(1.03, 0.04)	(1.00, 0.08)	(0.95, 0.12)	(0.88, 0.15)	(0.79, 0.18)	(0.68, 0.20)	(0.57, 0.22)	(0.45, 0.21)	
0.6	(1.09, 0.00)	(1.08, 0.06)	(1.05, 0.12)	(0.99, 0.18)	(0.92, 0.24)	(0.82, 0.28)	(0.71, 0.32)	(0.58, 0.34)	(0.44, 0.34)	
0.8	(1.16, 0.00)	(1.15, 0.08)	(1.12, 0.17)	(1.06, 0.25)	(0.98, 0.33)	(0.88, 0.40)	(0.74, 0.45)	(0.59, 0.49)	(0.43, 0.49)	
1.0	(1.26, 0.00)	(1.25, 0.11)	(1.21, 0.21)	(1.16, 0.32)	(1.07, 0.42)	(0.96, 0.52)	(0.81, 0.61)	(0.62, 0.67)	(0.40, 0.68)	
1.2	(1.37, 0.00)	(1.36, 0.13)	(1.33, 0.25)	(1.28, 0.39)	(1.19, 0.52)	(1.07, 0.66)	(0.91, 0.79)	(0.67, 0.91)		
1.4	(1.51, 0.00)	(1.50, 0.15)	(1.47, 0.30)	(1.42, 0.45)	(1.35, 0.62)	(1.24, 0.80)	(1.07, 1.00)	(0.81, 1.21)		
1.6	(1.67, 0.00)	(1.66, 0.16)	(1.63, 0.33)	(1.59, 0.51)	(1.53, 0.70)	(1.45, 0.92)	(1.33, 1.18)			
1.8	(1.84, 0.00)	(1.83, 0.18)	(1.81, 0.36)	(1.78, 0.55)	(1.74, 0.76)	(1.70, 0.99)				

would produce a central field to the right. Superposition of the solutions like Eq. (12) for excitations $+1, +1, -1, -1$ and $-1, +1, +1, -1$ would correspond to excitation $0, +2, 0, -2$, giving a central field downward to the right at 45°.

The magnetic field of Eq. (12) was evaluated by computer for the first quadrant of Fig. 1(a). The results are listed in Table I. The other quadrants can be obtained by symmetry considerations. The more extensive numerical results actually computed with a grid of intervals of 0.1 have been condensed in Table I to a grid of intervals of 0.2 to save space. Note that the radius of the aperture is $\pi^{\frac{1}{2}}=1.77$ which sets the distance scale.

FIELD OF MAGNETIC QUADRUPOLE WITH RECTANGULAR APERTURE FOCUSING

It has been shown by Hand and Panofsky[2] that a rectangular array of current sheets backed by iron of infinite permeability [Fig. 2(a)] produces a quadrupole field. The use of such a magnet has been reported by them.[2] In Fig. 2(a) the four sides of the heavy black rectangle represent the four uniform current sheets with currents in and out of the paper as shown. The total current in each leg is of the same magnitude for any rectangle. Here the boundary condition at the surface is obtained by the line integral of Maxwell's equation

$$\nabla\times\mathbf{H}=4\pi(\mathbf{j}/10) \tag{18}$$

around the contour shown in Fig. 2(c). The current density $\mathbf{j}$ is in amperes. Thus H_t the field tangential to the boundary is given as

$$H_t=4\pi(j/10), \tag{19}$$

since $H_t=0$ in the infinitely permeable iron. This constitutes a boundary condition specifying one component of $\mathbf{B}$ everywhere on a closed boundary of a space in which $\nabla\times\mathbf{B}=0$. Thus the field is determined. Furthermore, the boundary condition of Eq. (19) is of the same form as Eq. (1) evaluated on the same rectangle. This proves that the array of Fig. 2(a) produces the same quadrupole focusing field in the interior of the space as do the hyperbolic pole pieces in the conventional quadrupole. This, of course, was the conclusion of Hand and Panofsky[2] but a derivation was not given there, and the method is needed for the following considerations.

FIELD OF MAGNETIC QUADRUPOLE WITH RECTANGULAR APERTURE DEFLECTING

Consider the same rectangular array excited now as in Fig. 2(b). Since the rectangular array of Fig. 2(a) and the hyperbolic poles of the conventional quadrupole produce identical fields it might be supposed that when the rectangular array is excited in the deflecting mode [Fig. 2(b)] it would give the same field as in Fig. 1(a) and Eq. (12). This is not the case as is demonstrated.

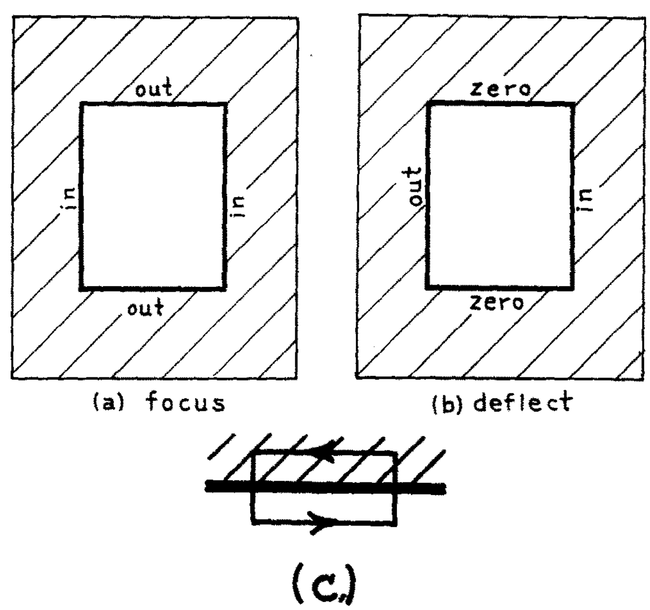

FIG. 2. Rectangular box magnet. The heavy lines are uniform current sheets and the shaded region is iron of large permeability. The directions of the uniform current sheets are shown in (a) for a quadrupole focusing field and in (b) for a deflecting field. The contour of integration is indicated in (c) for obtaining Eq. (19).

Again Eq. (19) applies, so $H_t=0$ everywhere on the top and bottom surfaces and $H_t=(+\text{const})$ everywhere on the side surfaces. Consequently the field in the interior is everywhere constant and directed upward for Fig. 2(b). This is the ideal deflecting field unlike Eq. (12), and is the system used at Colorado.[3]

Evidently, by supplying current to the top and bottom sheet with no current in the side sheets the field may be turned to the horizontal. By having the equal and opposite side currents different from the equal and opposite top-bottom currents an identically constant field in any direction in the plane can be obtained as the vector sum of the constant vectors for each separate excitation.

BEAM STEERING WITH RECTANGULAR BOX

The ideal steering device appears to be the rectangular box of Fig. 2(b). Since the field is everywhere constant, all particles experience the same deflection. The beam optical images from the other lenses in the system are not distorted. This conclusion neglects end effects.

DISPLACEMENT OF FIELD ZERO FOR ASYMMETRIC EXCITATION OF QUADRUPOLE

There are cases where a quadrupole magnet is already in the system, and it is not desirable to use the rectangular box for steering. We first formulate the location of the zero field in the aperture when the quadrupole is asymmetrically excited as discussed earlier.

Consider the focusing excitations of strengths $+f$, $-f$, $+f$, $-f$ for poles 1 to 4, respectively, of Fig. 1(a). For a positive particle moving toward $+z$ this is vertically focusing and horizontally defocusing. Consider superposed excitations of strengths $+s$, $+s$, $-s$, $-s$ for steering. This would deflect the particle toward $+x$. The combined excitation is then $+(f+s)$, $-(f-s)$, $+(f-s)$, $-(f+s)$.

Let us first analyze the problem with the approximation that the part of the field caused by the steering excitation is everywhere

$$B^*=i2s/\pi, \tag{20}$$

which is the limit of Eq. (12) for small z. This is the approximation used throughout by Koltay and Szabo.[1] Then the uniform field, along $-y$ in this example, is added to the linearly varying B_y [Eq. (1)] to give an exact quadrupole focusing field centered at that point where the net field is zero.

We determine the location of this zero field. From Eqs. (1) and (3), if the magnitude of the potential of the pole piece for pure focusing is f then

$$W_{\text{pole-piece}}=f=g(\pi/2), \tag{21}$$

so

$$g=f(2/\pi). \tag{22}$$

For a purely steering pole potential of s then Eq. (20) and Eq. (1) show that the field is zero for $x=x_0$, where

$$f(2/\pi)x_0+s(2/\pi)=0, \tag{23}$$

or

$$x_0=-s/f. \tag{24}$$

In order to compare this result with the estimate given by Koltay and Szabo[1] we must superpose two steering fields from excitations $+\frac{1}{2}s$, $+\frac{1}{2}s$, $-\frac{1}{2}s$, $-\frac{1}{2}s$ and excitations $-\frac{1}{2}s$, $+\frac{1}{2}s$, $+\frac{1}{2}s$, $-\frac{1}{2}s$ equivalent to 0, s, 0, $-s$. These give two steering fields at right angles to each other, so Eq. (24) becomes

$$\text{zero displacement}=\sqrt{2}\,(\tfrac{1}{2}s)/f=0.707\,(s/f). \tag{25}$$

The factor 0.707 is to be compared with their[1] multiplying constant $\frac{1}{2}\pi^{\frac{1}{2}}=0.89$ in their Eq. (2b) estimates. Note that that their Fig. 1 is rotated 45° from our Fig. 1(a). When the approximation of constant steering field is inadequate, Eq. (12) or Table I gives the exact result.

STEERING WITH ASYMMETRICALLY EXCITED QUADRUPOLE

A. Approximation of Uniform Steering Field

First, we consider the approximation used throughout the analysis of Koltay and Szabo[1] that the effect of asymmetric excitation of the quadrupole is to shift the field zero to a new location x_0, y_0, Eq. (24). The result is a pure quadrupole field about a new center as noted in the last section. Thus the optic axis for this lens is no longer at (0,0) but at (x_0,y_0). This is simply taken into account as in Fig. 3. The image is shifted (steered) by a distance mx_0, where m is the magnification in this plane.[5] The matrix statement of this is given by Koltay and Szabo,[1] but it is

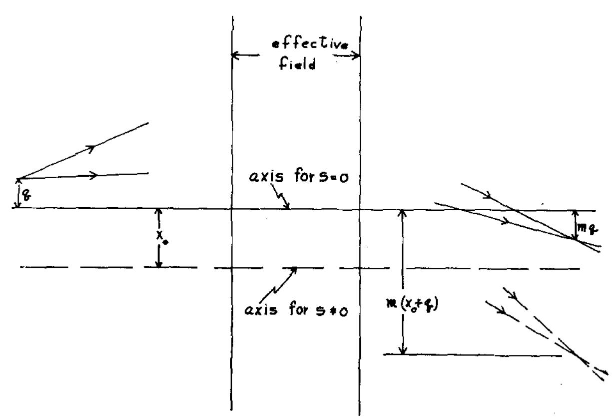

FIG. 3. Schematic showing how the steering deflection for an asymmetrically excited quadrupole is mx_0 in the approximation of uniform steering field. Here q is the transverse distance of the object from the optic axis with no steering excitation, m is the magnification, and x_0 is the displacement of the field zero when the steering excitation is applied. The solid rays are for no steering, the dashed rays are for steering.

[5] The application of this result to steering by physically moving the quadrupole is evident.

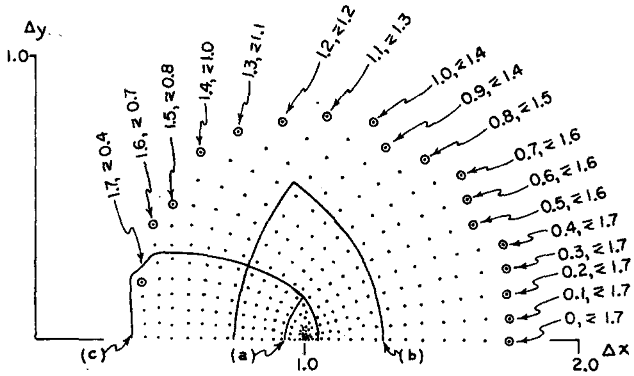

FIG. 4. Plot of the portion of Table I within the dashed boundaries (i.e., approximately within a circular beam pipe filling the aperture). However the grid for x and y here is finer, being at intervals of 0.1, rather than the intervals of 0.2 used in Table I. Each curve of dots is labeled with the y coordinate in the quadrupole aperture as the first number, and the maximum x coordinate as the second number. For example, the label 1.3, $\lesssim$1.1 denotes $y=1.3$ and $x=0$, 0.1, 0.2, . . ., 1.1 (or twelve points in all). The circled dots have x, y at the extreme edges of the beam pipe. As discussed in the text, this plot may be regarded as the field values from Eq. 12 or as individual contributions to a steered image which would be a single point image at $\Delta x=1$, $\Delta y=0$ if Eq. (12) represented a uniform steering field. The curves (a), (b), and (c) are discussed in the text.

complicated and requires an augmented 3×3 matrix. In the exact semigraphical method previously given,[6] the distance D in Fig. 2 therein is simply measured from the new optic axis.

B. Exact Steering Field

Unfortunately, the actual steering field is not uniform but is given by Eq. (12), the limit of which, for small z, was used above. As each particle moves along the beam pipe through the quadrupole aperture we could evaluate $\bar{x}$, the average value of the x coordinate, and $\bar{y}$, the average value of the y coordinate, as the particle traverses the effective field region including the effect of the steering field. The approximation is made that each particle experiences a constant steering field given by Eq. (12) evaluated at $z=\bar{x}+i\bar{y}$. Of course $\bar{x}$ and $\bar{y}$ are different for each particle.

Since B_y produces the x component of force and B_x produces the y component of force, the x and y components of Eq. (12) are proportional to the y and x components, respectively, of the steering deflection. If the particle traversing the quadrupole with $\bar{x}=0$, $\bar{y}=0$ has a steering deflection defined as $\Delta x=1$, $\Delta y=0$ then it is evident that the numbers in Table I represent the coordinates Δx, Δy of beam spots in the steered image.

This is shown in Fig. 4 where the values of Δx and Δy are plotted for the range of x and y enclosed by dotted lines in Table I. These would be contained in a circular beam pipe filling the quadrupole aperture. To be specific, consider the case where a quadrupole doublet is excited so that a point object gives a point image with no steering excitation. Then consider that the steering excitation $+s$, $+s$, $-s$, $-s$ is added to one of the members of the doublet to produce steering in the $+x$ direction. Particles traversing with $\bar{x}=0$, $\bar{y}=0$ experience unit steering, by definition, to a point $\Delta x=1$, $\Delta y=0$ on the image plane. Particles traversing at other values of $\bar{x}$, $\bar{y}$ go to other points Δx, Δy on the image plane. If the steering field were uniform all dots in Fig. 4 would coalesce to $\Delta x=1$, $\Delta y=0$.

Figure 4 represents $\bar{x}$, $\bar{y}$ in the first quadrant of the aperture. The pattern for the second quadrant is the mirror image of Fig. 4 about the Δx axis. The pattern for quadrants 3 and 4 reproduces that for the first two quadrants so the dots from the former and the latter are identical.

If the beam uniformly filled a circular beam pipe (radius $=\pi^{\frac{1}{2}}=1.77$) filling the quadrupole aperture the image would be smeared out over all the dots shown (and their plot mirrored about the Δx axis). The local density of dots is proportional to the local intensity of the image. Other examples shown in Fig. 4 are:

(a) $|\bar{x}|\lesssim 0.5$, $|\bar{y}|\lesssim 0.5$

(b) $|\bar{x}|\lesssim 1$, $|\bar{y}|\lesssim 1$

(c) $|\bar{x}|\lesssim 0.5$, $|\bar{y}|=$any value within the circular pipe.

From Fig. 4 it can be seen that for $|\bar{x}|\lesssim 0.5$, $|\bar{y}|\lesssim 0.5$ [domain (a)] the deflected image of a point object is entirely contained within an area of dimensions ~0.1 of the deflection. This area contains 12% of the beam if the beam were uniformly spread over the circular aperture. The image intensity (dot density) is an order of magnitude larger in this area than that for beam near the edges of the aperture.

Figure 4 is completely general since it can be used for any distribution of beam within the quadrupole aperture. Then each dot should be weighted according to the relative intensity of beam at that $\bar{x}$, $\bar{y}$ in the quadrupole element used for steering. If the unsteered image is not a point then the steered image can be synthesized from Fig. 4.

ACKNOWLEDGMENTS

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[6] J. E. Draper, Rev. Sci. Instr. 34, 679 (1963).