First Feasibility Studies for Negative Muon Decay with $\mathcal{TWIST}$

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Abstract

Typically, $\mathcal{TWIST}$ uses a highly polarised beam of positive muons to measure the free muon decay spectrum and test the results against the standard model prediction. When a negative muon beam is used, however, the $\mathcal{TWIST}$ experimental apparatus can also be used to make measurements of the negative muons as they decay from atomic bound states of a target material.

This report documents the first feasibility studies of doing negative muon decay measurements using the $\mathcal{TWIST}$ experimental setup. I give a thorough discussion of the conditions under which preliminary data from negative muon decays was taken and the efforts that were made to analyse this data. The results of this preliminary analysis are also shown. This study revealed that $\mathcal{TWIST}$ is capable of making a measurement of the bound muon decay spectrum. This report concludes with a discussion of what the future possibilities are for negative muon decay studies with $\mathcal{TWIST}$.

Chapter 1

Introduction

The TRIUMF Weak Interaction Symetry Test ($\mathcal{TWIST}$) uses free muon decay to test the standard model of particle physics. In order to measure the free decay spectrum, a beam of positively charged muons is stopped inside a detector using a stopping target. It is a simple matter, however, to use the existing $\mathcal{TWIST}$ experimental setup to generate a beam of negatively charged muons (μ^-s) and have them stop in the target. When the negative muon beam is used, the physics which affects the muons in the stopping material changes. This report documents the first studies into the feasibility of $\mathcal{TWIST}$ making use of a negative muon beam to make physics measurements.

1.1 Introduction to the $\mathcal{TWIST}$ experiment

$\mathcal{TWIST}$ is an experiment currently being conducted at the Tri-University Meson Facility (TRIUMF) in Vancouver, BC, Canada. The goal of this experiment is to make a precision measurement of the parameters that characterise the decay of free muons via normal muon decay ($\mu^+ \rightarrow e^+\nu_e\bar{\nu_\mu}$). The results can then be compared with predictions made by the standard model. Because the normal decay of a muon is independent of any strong force interactions, the muon is a convenient laboratory for studying weak interaction symmetry.

The measurement of the muon decay parameters is done by tracking the path of decay positrons through a strong, uniform magnetic field. $\mathcal{TWIST}$ has designed a spectrometer to make the required measurements to high precision (see section 1.4). The momentum and angular distributions can then be reconstructed and the parameters can be fit from the resulting spectrum.

The $\mathcal{TWIST}$ experiment is not discussed in detail in this report, and the following discussion of the experiment is intended to be an overview. I have focused primarily on the aspects which are important to understanding the negative muon decay studies that are the topic of this report.

1.2 The Physics of Muon Decay

There are two known decay modes for the muon: normal muon decay ($\mu^+ \rightarrow e^+\nu_e\bar{\nu_\mu}$), and radiative muon decay ($\mu^+ \rightarrow e^+\nu_e\bar{\nu_\mu}\gamma$), which has a branching ratio of about 10^{-4} as compared to the normal decay mode. Ignoring the corrections due to the radiative decay mode, the probability for a muon decay resulting in an electron with an energy E_e into an angle $d\cos(\theta)$ is given in terms of four constants: ρ, η, ξ, and δ [1] by

$$\frac{d^2\Gamma}{dx d\cos(\theta)} = \frac{m_\mu}{4\pi^3} W_{\epsilon\mu}^4 G_F^2 \sqrt{x^2 - x_0^2}[F_{IS}(x,\rho,\eta) + \cos(\theta) P_\mu F_{AS}(x,\delta,\xi)]. \tag{1.1}$$

Here,

- m_e and m_μ are the electron and muon masses, respectively
- G_F is the Fermi coupling constant
- $W_{\epsilon\mu} \equiv \frac{m_e^2+m_\mu^2}{2m_\mu}$ is the maximum positron energy
- $x_0 \equiv \frac{m_e}{W_{\epsilon\mu}}$ is the minimum positron energy
- $x \equiv \frac{E_e}{W_{\epsilon\mu}}$ is the reduced positron energy
- $\cos(\theta) \equiv \frac{\vec{P_\mu}\cdot\vec{p_e}}{|\vec{P_\mu}||\vec{p_e}|}$ is the cosine of the angle between the muon spin axis and the momentum of the decay positron
- P_μ is the polarisation of the muon beam ($-1 \leq P_\mu \leq 1$) [2]

In equation (1.1), the parameters ρ, η, ξ, and δ are known as the Michel parameters. It is these four parameters which characterise the decay spectrum (see figure 1.1). F_{IS} and F_{AS} are, respectively, the isotropic and anisotropic terms in the decay. Explicitly written, they are:

$$F_{IS} = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x) \tag{1.2}$$

$$P_\mu F_{AS} = \frac{1}{3}\sqrt{x^2 - x_0^2}\left[P_\mu\xi(1-x) + \frac{2}{3}P_\mu\xi\delta(4x - 4 + \sqrt{1-x_0^2})\right]. \tag{1.3}$$

In the standard model, we assume that the only non-zero coupling for weak interactions is the vector to axial vector coupling (V-A). Under this assumption, the Michel parameters assume the following values:

$$\rho = \frac{3}{4} \qquad \eta = 0 \qquad \xi = 1 \qquad \delta = \frac{3}{4}. \tag{1.4}$$

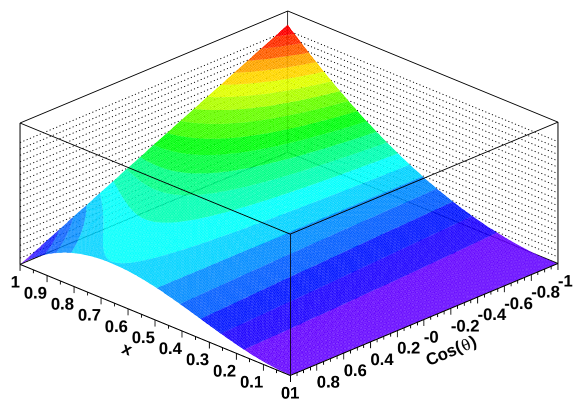

Figure 1.1: Momentum of a decay positron vs. $\cos(\theta)$

Ultimately, the $\mathcal{TWIST}$ experiment hopes to produce measurements of ρ, δ, and $P_\mu\xi$ to a few parts in 10^{-4}. To date, two physics publications have come from $\mathcal{TWIST}$ documenting the measurements of $\delta = 0.74964 \pm 0.00066(stat.) \pm 0.00112(syst.)$ [3] and $\rho = 0.75080 \pm 0.00032(stat) \pm 0.00097(syst) \pm 0.00023$ [4]. The final uncertainty in the ρ measurement is due to the dependence on ρ of η. These measurements are both consistent with the standard model predictions.

1.3 $\mathcal{TWIST}$ Muon Beam Production

$\mathcal{TWIST}$ uses a proton beam provided by the 500 MeV cyclotron at TRIUMF to produce the muon beam that is sent into the detector. The proton beam is incident upon a beryllium target. Among the particles produced from the production target are pions. The primary decay mode for these pions is $\pi^+ \rightarrow \mu^+\nu_\mu$. Because this is a two body decay, the muon has a well defined momentum of 29.79 MeV/c when the pion decays from rest.

The primary operational mode of our beam is designed to select muons with a momentum of 29.6 MeV/c that result from the decay of pions that came to rest on the surface of our production target. These muons are referred to as 'surface muons'. The acceptance of our muon beam-line (figure 1.2) is $\sim 1\%$, so this momentum setting allows us to select the surface muons while maintaining a high muon rate. These muons have a high negative polarisation, and so, they make an ideal beam for $\mathcal{TWIST}$.

A secondary operational mode for beam production produces what is referred to as 'cloud muons'. The magnetic dipoles, which control which particle momentum is selected for the beam, are tuned to select particles of momentum 32.8 MeV/c. With this setting, we select muons from pions that decay in flight outside of the production target. These muons have a net positive polarisation of only $\sim 1/4$ and are used for systematics studies.

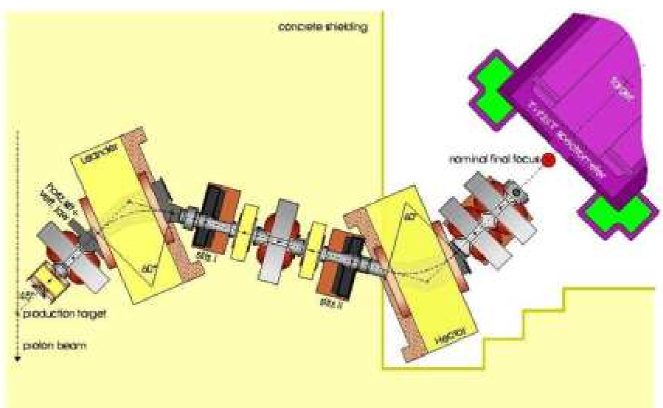

Figure 1.2: A schematic diagram of the $\mathcal{TWIST}$ beam-line

1.4 The $\mathcal{TWIST}$ Spectrometer

The $\mathcal{TWIST}$ spectrometer is designed to make precision measurements of the momentum and decay angle of the positrons that are emitted from muons as they decay from rest in the centre of the detector. A schematic diagram of the spectrometer is shown in figure 1.3.

The detector stack inside the spectrometer is composed of 44 drift chambers (DCs) and

12 proportional chambers (PCs). These wire chambers are the main source of information from the spectrometer. The PCs use a CF_4/isobutane gas mixture which produces a fast signal response. Thus, the PCs are ideal for resolving which tracks are left by different particles by using the time separation of their signals. The DCs use dimethylether (DME) gas to provide the high spatial resolution required to track the particles' paths through the detector. The detector stack is symmetric between the upstream and downstream halves of the detector.

A superconducting MRI solenoid magnet is used to create a very high (2 Tesla), nearly uniform magnetic field inside the detector. This magnetic field is oriented in the direction of the beam. The component of the magnetic field along the beam direction was mapped, and a simulation of the magnet was done using OPERA. This provides us with a map of the magnetic field throughout the tracking region that is used by our simulation software to determine the paths of the particles. The magnetic field causes particles travelling through the detector to trace a helical path. The geometry of the helix is then used by the analysis software to reconstruct the momentum and angle of the decay positrons.

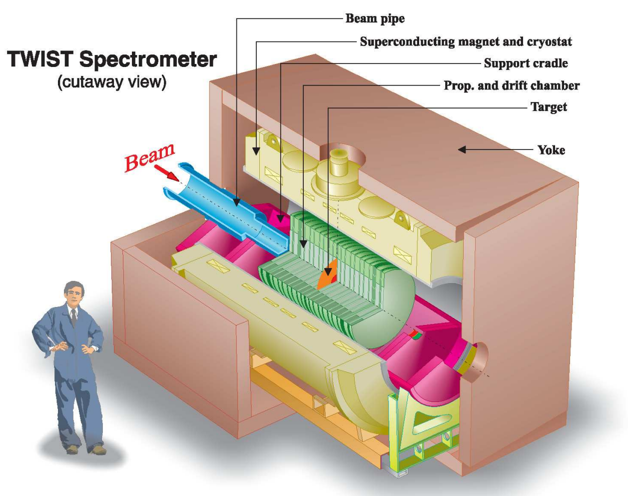

Figure 1.3: A schematic diagram of the *TWIST* spectrometer.

A main feature of the *TWIST* spectrometer is the small amount of material in the tracking region. As the muons travel through the detector material, they undergo multiple scattering as well as other interactions. This scattering can cause a depolarisation of the muon beam. Also, particles travelling through the detector lose energy as they traverse material. This can lead to distortions of the track, which complicates the analysis. Therefore, by minimising the material in the detector, we can preserve the properties of the particle's track through the detector and improve the accuracy of our analysis.

1.5 $\mathcal{TWIST}$ Monte Carlo

The $\mathcal{TWIST}$ analysis relies heavily on the results of Monte Carlo simulated events. Thus, a very detailed simulation program was written using GEANT3. The detector geometry that is used in the simulation is virtually identical to the actual detector geometry and includes all the materials in the detector that a muon or positron could interact with. This includes, for example, each of the individual wires from the wire chambers.

The Monte Carlo simulation is responsible for generating a simulated muon beam according to input taken from real experimental data. Both the position and the spin of each muon is then tracked through the mapped solenoid field. Tracking the muon spin through the magnetic field is a feature that was added to the $\mathcal{TWIST}$ simulation software and it is not standard to GEANT3. The software also simulates energy loss as well as other physical processes as the particles travel through the detector material. Finally, a muon decay subroutine samples the Michel spectrum and a decay positron is generated with a momentum and angular distribution which reproduces the Michel spectrum.

1.6 $\mathcal{TWIST}$ Analysis Software

The primary goal of the analysis software is to reconstruct a two-dimensional histogram of the muon decay spectrum in momentum and angle ($\cos(\theta)$). There are a number of steps that the software must go through to accomplish this goal. The first step is to look at the timing information provided by the PCs in order to separate particles in time and determine the event type and which tracks belong to which particle. This process is called windowing and event classification. Once the tracks from each particle have been separated from each other, a pattern recognition is attempted on each positron track to make an initial identification of the helix parameters for the track. This pattern recognition is then used as the first guess input to a more complex helix fitting program.

The track fitter is capable of fitting the helical path that particles trace inside the detector with high efficiency and high accuracy. Energy loss corrections to the particle path are made at each DC plane of the detector. The geometry of the track is then used to infer the properties of the deay positrons. For example, the radius of the helix gives the transverse momentum of the tracked particle while how tightly wound the helix is gives the longitudinal component of the momentum. After each event has been analysed, an entry can be made into a two-dimensional histogram to store the momentum and $\cos(\theta)$ of the particle. Once a data set has been completely analysed, this histogram is our decay spectrum. We can then fit the decay spectrum to the Michel spectrum to measure the Michel parameters.

Because $\mathcal{TWIST}$'s analysis strategy depends on a very detailed simulation analysis, it is important that the same analysis software be used for both GEANT Monte Carlo and data. The only difference is that we can output GEANT specific information to confirm the proper operation of our analysis software.

1.7 My Contribution

A majority of my time working with $\mathcal{TWIST}$ was spent working on a study exploring the feasibility of $\mathcal{TWIST}$ making a measurement of the decay spectrum of μ^-s decaying from orbit within muonic atoms. The effect of the muonic binding on the decay spectrum makes μ^-s a poor choice for studying the Michel spectrum, however the decay of bound muons provides a laboratory for studying the properties of muonic atoms (see section 2.1). Because μ^-s are not useful for measuring the Michel spectrum, the μ^- decay studies are outside of the mainstream of $\mathcal{TWIST}$'s research. For this reason, I was able to make it my own small project and was able to do most of the work on it myself.

Aside from working on the μ^- decay studies, I was also involved in studies to better understand the muon beam polarisation, P_μ, for the upcoming measurement of the $P_\mu\xi$ Michel parameter. I contributed to understanding P_μ through three projects.

The first of these projects was a systematic search through changes in the Monte Carlo beam tune to see if we could match the beam properties that we observe in the data. Specifically, the goal was to resolve a discrepance between the beam position and size between Monte Carlo and data. The beam spot is measured at a certain position in our detector, however, the input parameters to the simulation are for a beam position outside the detector where we have no experimental data. In between the input position and the position where the beam is measured, there is a phase-space rotation of the beam due to the magnetic field outside the detector. There are also four parameters which can be adjusted to change the beam position: The x and y coordinates of the beam at the input plane as well as the angle the beam momentum makes from the z-axis. This makes matching the beam tune between data and Monte Carlo a difficult task. By systematically searching through a variety of possible Monte Carlo beam tunes, I was able to find a combination which matched the data better than the existing Monte Carlo beam tune.

Another obstacle in understanding P_μ was a discrepancy between the muon stopping distributions observed in real data and in Monte Carlo simulations. I was able to model the stopping distribution in data by introducing pions into the simulated muon beam with the same momentum as the surface muons. The pions then decayed into muons inside the simulated detector and the result was a simulation which better recreated the muon stopping distribution of the data. From this model, we were able to make an estimate of the contribution to the depolarisation caused by pion contamination in the beam.

The final contribution I made to the $P_\mu\xi$ studies was in improving the efficiency of the helix fitter for low helix-radius particle tracks. The incoming muons have a low transverse component to their momentum and so the radius of the helix inside the detector is very small. The pattern recognition and fitting software, which was designed to fit decay positron tracks with a larger radius, could not efficiently track low radius particles. I was able to make changes to the pattern recognition software so that the helix fitter was able to fit the muons more often (see figure 1.4). This in turn helps us to understand effect that the transverse momentum of the muons has on the polarisation of the beam.

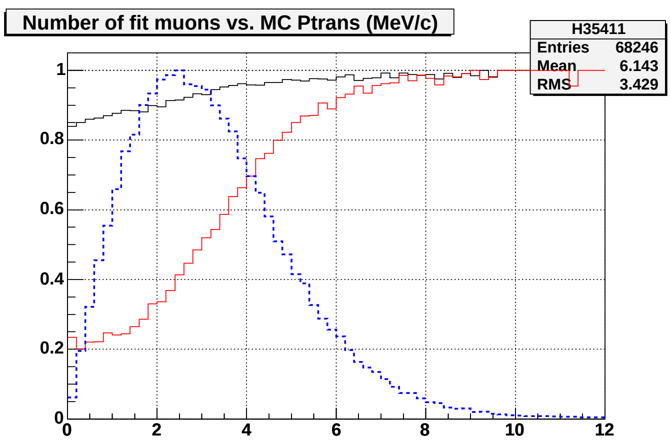

Figure 1.4: A comparison between the tracking efficiency vs. muon transverse momentum (MeV/c) before (red) and after (black) my changes to the pattern recognition software. The muon transverse momentum distribution is displayed in blue.

Chapter 2

Motivations for Negative Muon Decay Studies

When negatively charged muons are stopped in a target, they become bound in orbits around the atoms of the target material. This situation is undesirable for studying the Michel spectrum because the decay spectrum is influenced by the Coulomb potential of the nucleus and the muon's momentum in the bound state, among other factors. The motivations for studying negative muon decay are, then, separate from $\mathcal{TWIST}$'s primary research goals. Nevertheless, a measurement of the bound muon decay spectrum provides us with an opportunity to make contributions to other areas of physics.

The primary physics motivation of these studies is to make a precise measurement of the momentum distribution of electrons from muons that decay in orbit about Aluminium nuclei. This measurement allows us to compare to several theoretical predictions which have been made and resolve discrepancies between them. With such a measurement, we can also make a contribution to muon-to-electron conversion ($\mu \to e$) experiments by aiding in the understanding of the background. Finally, using negative muons in the $\mathcal{TWIST}$ apparatus is a good way to test systematics and the analysis software. These motivating factors are discussed in greater detail below.

2.1 Bound Muon Decay

The decay spectrum of μ^- decaying from a stopping target varies considerably from that of the free muon Michel spectrum. This is due to the fact that the negatively charged muons become bound in atomic orbits. Because the muon mass ($m_\mu \approx 106$ MeV/c^2) is roughly 207 times the mass of an electron, a negatively charged muon in the vicinity of a nucleus sees a stronger binding potential than the atomic electrons. Thus, muons in a target material quickly become bound in atomic orbits and form muonic atoms. While the dominant decay mode for μ^- decay, $\mu^- \rightarrow e^-\nu_\mu\bar{\nu}_e$, is simply the charge conjugation of positive muon decay, the nature of the decay spectrum is influenced by the properties of the muonic bound state [5].

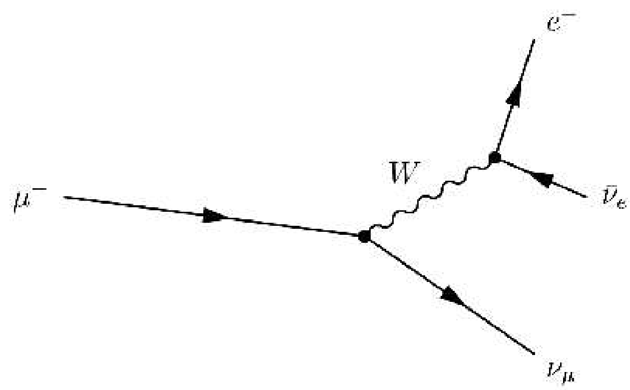

Figure 2.1: The Feynman Diagram for negative Muon Decay

In the free muon momentum spectrum, there is a peak at the maximum $e^{+/-}$ momentum (52.8 MeV/c). This corresponds to the case where the two decay neutrinos are emitted in the same direction and the $e^{+/-}$ is given the maximum kinetic energy in the opposite direction. Due to the Coulomb potential of the nucleus, however, this peak position is shifted to lower momenta for bound muon decay. The sharp cutoff at 52.8 MeV/c that we observe in free muon decay is Doppler blurred in the case of bound muon decay due to the momentum distribution of the muon in orbit. Thus, the bound muon decay momentum spectrum does not cut off at 52.8 MeV/c but instead displays a high energy tail which extends out to a maximum decay energy of the μ energy minus the nuclear recoil energy. This high energy tail can extend to energies higher than 100 MeV for Aluminium.

A muon typically becomes bound in an atom and descends to the ground state in a time less than 10^{-10} s [6]. Because this process happens in such a short time compared to the muon lifetime (2.2 μs), nearly 100% of the muons will decay from the ground state. The decay probability of a bound muon decaying from the $1s_{1/2}$ ground state of an atom into an electron of energy E into a solid angle $d\Omega_e$ is given by

$$W(E,\theta)\frac{dEd\Omega_e}{4\pi} = [N(E) + A(E)P_\mu \cos(\theta)]\,\frac{dEd\Omega_e}{4\pi} \tag{2.1}$$

where $N(E)$ is the energy spectrum and $A(E)$ is the asymmetry coefficient [7]. $N(E)$ and $A(E)$ are plotted in figure 2.2 for several different nuclei as well as for the free decay case.

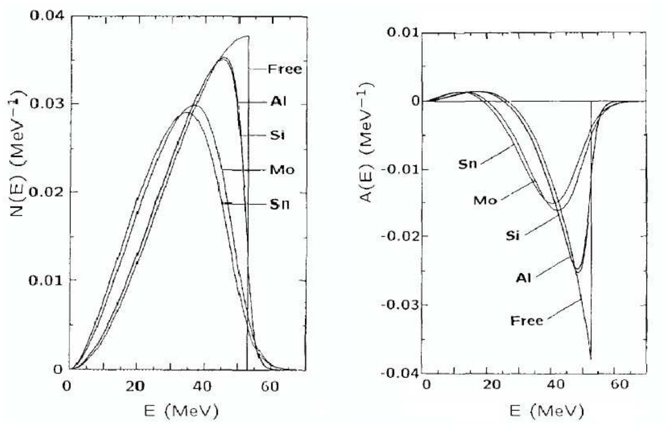

Figure 2.2: The momentum spectrum ($N(E)$) and asymmetry coefficient ($A(E)$) for ground state muon decays from several different nuclei [7]

Because TWIST was designed to make precision measurements of the free muon decay spectrum, a measurement of the bound muon decay spectrum is within the capabilities of both the detector and the analysis software. Moreover, TWIST can make a measurement of the momentum distribution of electrons from bound muon decay more precisely than any previously published results. Thus, the primary physics motivation of doing μ^- decay studies with TWIST is to make a precision measurement of the decay electron momentum distribution.

2.2 MECO experiment

MECO is an experiment being performed at Brookhaven National Lab looking for the rare electron and muon number violating process $\mu^- N \rightarrow e^- N$. That is, a muon converting directly to an electron in the field of a nucleus. MECO can discover this process for a branching ratio as low as 10^{-16}. Using a pulsed muon beam incident on an aluminium stopping target, MECO will look for decay electrons in an energy range between 103.9 MeV and 105.4 MeV. Electrons from muon conversion have a well defined energy of the μ mass minus the binding energy ($m_\mu c^2 - B.E. \approx 105$ MeV but the observed energy is shifted below 105 MeV because of energy loss in the target). So, the muon conversion electrons will be observed in this energy range [8].

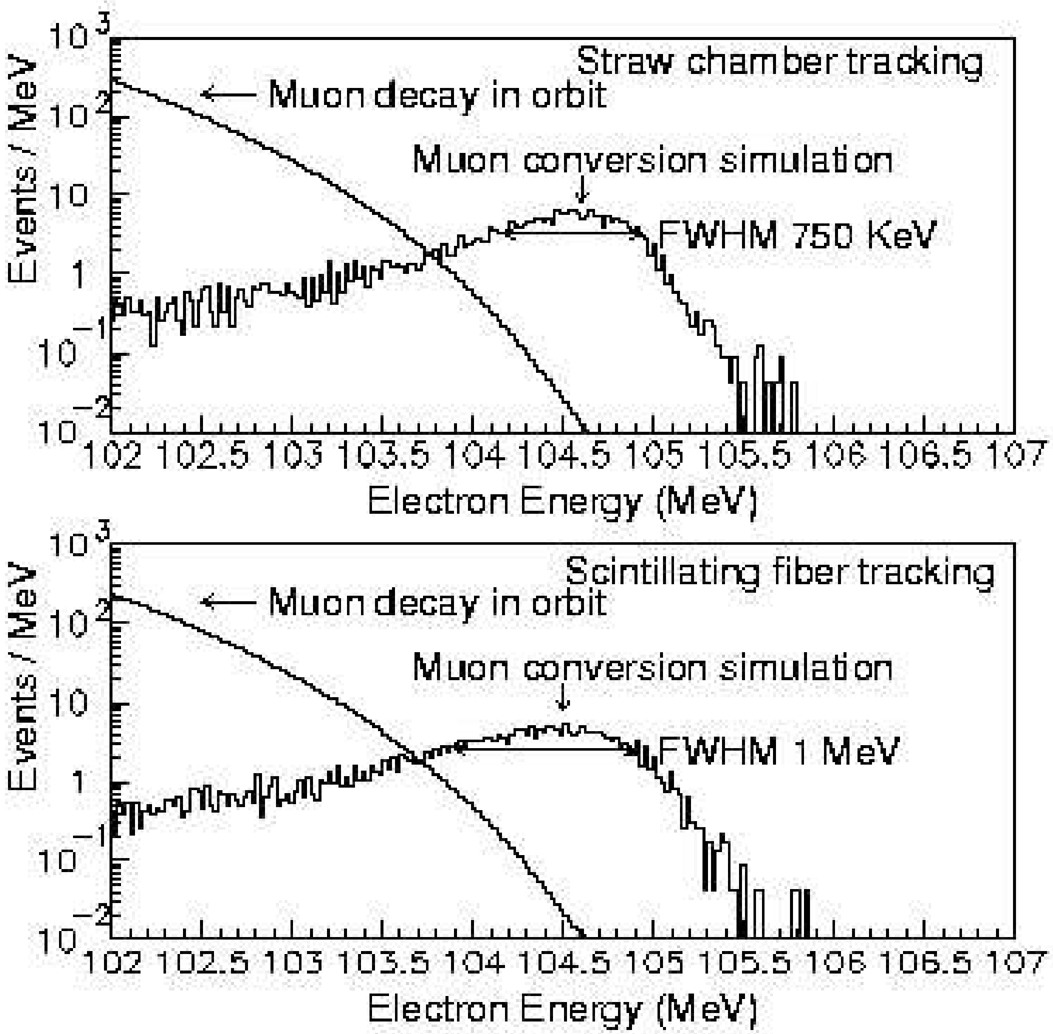

Figure 2.3: The expected background for MECO from muon decays in orbit for two proposed detectors. The above graphs are from a simulation which assumes a branching ratio of 10^{-16} [8]

Because of the Doppler blurring of the decay electron energy from bound muon decay, however, there will be a certain number of weak interaction decay electrons in this energy range (see figure 2.3). Thus, bound muon decay is an important source of background for MECO. The precision measurement of the bound muon decay momentum spectrum that

$\mathcal{TWIST}$ is capable of making would aid MECO in understanding this background source. It is especially important to have a precise measurement considering the large discrepancies in the theoretical predictions at the required energy range. The fact that $\mathcal{TWIST}$ can make this contribution to MECO through μ^- decay studies is an important motivating factor for the studies[1].

2.3 Test of Analysis and Systematics

As is the case with any experiment, doing something new with one's experimental setup and analysis software provides a good test of how well all the aspects of the experiment are understood. In this sense, it is hoped that doing μ^- decay studies with $\mathcal{TWIST}$ will be a good test of our analysis software as well as systematic uncertainties involved with the experiment.

One example of such a test comes from the fact that the negatively charged electrons spiral in the opposite direction than do the positrons which we use to measure the Michel spectrum. This then allows us to test how well our analysis software can track particles which spiral in the wrong direction. This helps us to track not only particles that have negative charge, but particles which are travelling toward the target as opposed to away from it.

It is difficult to predict all the ways that a μ^- study could help us to understand $\mathcal{TWIST}$ analysis and systematics. The fact that this could likely point to weaknesses in the $\mathcal{TWIST}$ analysis coupled with the physics motivations of doing the study make it more than worthwhile to pursue.

[1] On Aug. 11, 2005, while this report was being composed, the National Science Foundation voted to cancel the Rare Symmetry Violating Processes physics project which included the MECO experiment. However, we believe that other muon-to-electron conversion experiments will be taken up in the future, and they remain a motivating factor for continuing our research into bound muon decay.

Chapter 3

Negative Muon Decay First Feasibility Studies

Due to the motivating factors listed in chapter 2, interest was generated into investigating whether or not $\mathcal{TWIST}$ is capable of making interesting physics measurements using a μ^- beam. After ensuring that there were no unacceptable safety concerns due to increased neutron production when switcing the polarities of the selection magnets, some beam time was dedicated to taking data with a μ^- beam for the purposes of a feasibility study.

3.1 Experimental Setup and Conditions

The μ^- data was taken in a single afternoon in November of 2002. A total of 4 runs were taken and roughly 75,000 events were observed during the runs. In contrast, a standard $\mathcal{TWIST}$ data set is composed of 100 runs, and contains 3×10^8 events.

The conditions were standard for $\mathcal{TWIST}$'s positive muon decay runs (see chapter 1 for a discussion of $\mathcal{TWIST}$'s experimental setup) except that the polarities of the beam magnets were reversed to select 29.6 MeV/c negatively charged muons instead of 29.6 MeV/c positively charged surface muons. The μ^- beam was not, however, composed of surface muons. Pions that stop inside the production target, or on its surface become bound in atomic orbits and the decay muon will have less than 29.6 MeV/c of momentum because of the binding potential. The muons selected were from the cloud muons surrounding the production target that exist at a continuum of momenta. This, unfortunately, meant that the event rate inside the detector was greatly reduced. The beam rate for the μ^- data was only about 250 muons per second. To contrast, the beam rate for positive cloud muons at 32.8 MeV/c is about 600 muons per second while the beam rate for surface muons is 2.5×10^3 muons per second. Using cloud muons also meant that the muons had a low positive polarisation of only $\sim +0.25$ (compared to -1 for surface muons).

The stopping target that was in use during the μ^- runs was a mylar target with a graphite coating. This target is not ideal for a precise measurement of the decay spectrum because it is composed of a variety of elements, predominantly carbon and hydrogen. Thus, there are a variety of nuclei for the muons to become bound in and it is not possible to disentangle which muons decayed from orbit about which type of nucleus. Because we were collecting data for the purposes of a first feasibility study, however, it was decided that the target was suitable enough for our purposes.

3.2 Analysis

Initially, I ran the μ^- data through the standard $\mathcal{TWIST}$ analysis software. The helix fitter code was intentionally designed to be able to fit to particles with either positive or negative charge so it was expected that the analysis code would need very little modification, if any at all, before it would analyse μ^- events. An investigation of the χ^2 distribution for the helix fits done during the first analysis attempt, however, revealed that the helix fitter was failing to properly fit the decay electrons to a helix.

In order to test the analysis software, I wrote a Monte Carlo simulation in which positive muons were stopped in the detector and then decayed into negatively charged electrons. The reason positive muons were used for this situation is because the Monte Carlo code for the positive muon's spin and decay parameters was much more mature than for μ^-s. Also, the analysis software is well tested with incoming positive muons so we could ensure that the charge of the incoming muon would not affect the analysis. This test lead to a bug in $\mathcal{TWIST}$'s analysis software being discovered. While the bug did not affect the positive muon data, it resulted in strange results when μ^- data was analysed (see figure 3.1).

The bug, which prevented the analysis software from correctly tracking decay electrons, was quickly identified and fixed. After this bug fix, the $\mathcal{TWIST}$ analysis software correctly tracked decay electrons from the μ^- data. An example of a correctly tracked event from this data set is shown in figure 3.2.

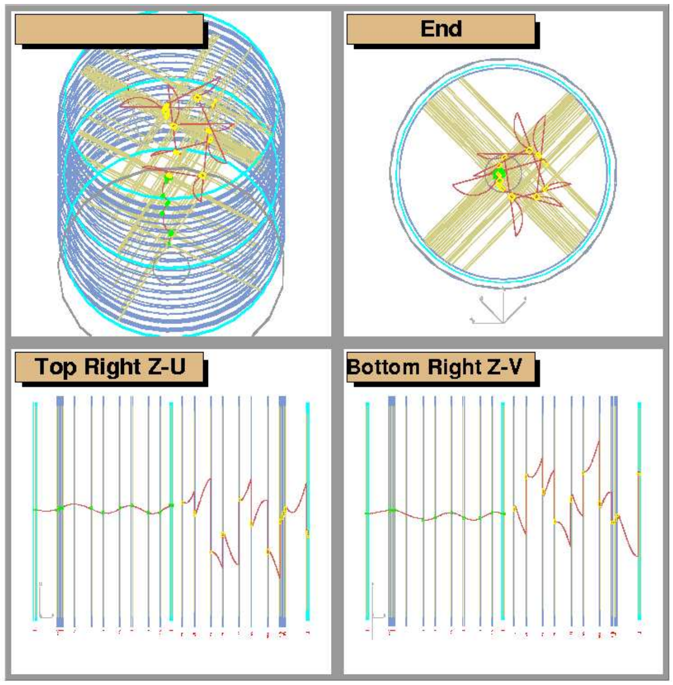

Figure 3.1: The above is a visualisation of a simulated $\mu^+ \rightarrow e^-$ decay event in which the e^- was not tracked properly by the existing TWIST software. The failure of the analysis was obvious because the red analysis tracks are discontinuous at each of the DC planes

3.3 GEANT Monte Carlo Simulation

Unlike standard GEANT, in TWIST simulations the muon spin is tracked through the magnetic field. Thus, to simulate the μ^- data, a new particle had to be added to the TWIST simulation representing a negatively charged muon containing the relevant spin information. This allows the TWIST analysis to track the depolarising effects on the beam as it travels through the detector.

A negative muon decay subroutine was written which samples a decay electron momentum and decay angle according to theoretical values for Aluminium 27. This theoretical decay spectrum was obtained from [7]. Because the theoretical decay spectrum exists as a data table and because calculating it analytically for each event is a computationally intense task, I used an interpolation process using Newton's divided difference method to sample values from the spectrum. While, for the most part, the interpolation was a reasonable curve through the points on the data table, the high momentum tail showed negative values and unusual features between the points. For this reason, an exponential fit was done to

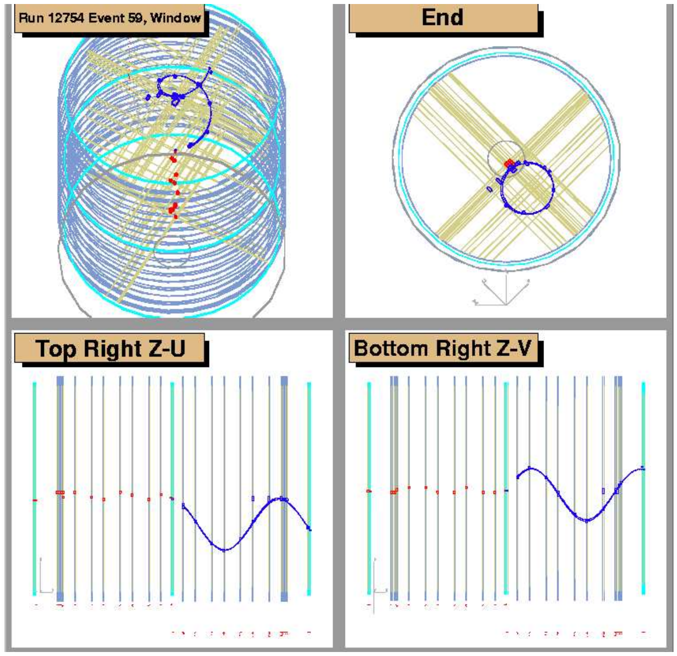

Figure 3.2: A visualisation of a typical negative muon decay event generated by the TWIST analysis software. The red tracks are the hits from the incoming muon. The decay electron hits are tracked to a helix, which is displayed in blue

the theoretical calculations for momenta greater than 60 MeV/c. This exponential was then used in the Monte Carlo generator to provide a more reasonable decay spectrum than the interpolation alone produced.

While the data used for the first feasibility studies were not from an aluminium target, this simulation will become useful when the analysis of the aluminium target data begins to compare the observed experimental decay spectrum with the theoretical decay spectrum. Simulating the decay spectrum of the small data set obtained for these studies is difficult due to the fact that the muon stopping target is composed of several different types of atoms (carbon, and hydrogen for example). For our purposes, it was only necessary to determine if the analysis software could correctly analyse μ^- simulation data. The details of the decay were not important for this purpose. Therefore, we chose aluminium as our stopping target in the simulation because the theoretical spectrum was easily available and because the simulation will become useful when we begin to analyse μ^- data using an aluminium target.

The momentum spectrum generated by the Monte Carlo bound muon simulation is shown in figure 3.3. The same figure shows a comparison with the negative muon data that was obtained for these studies. While the comparison is not fair since the muons are becoming bound in different atoms in each case, the figure shows what one would expect to see. In the case of the data, the muons are becoming bound primarily in carbon and hydrogen. Both of these elements are lighter than aluminium. As expected, we see that the data spectrum is closer to that of free muons than the aluminium spectrum is. The peak position is not shifted as far from 52.8 MeV/c as is the simulated aluminium peak and the high energy tail is not as pronounced in the data.

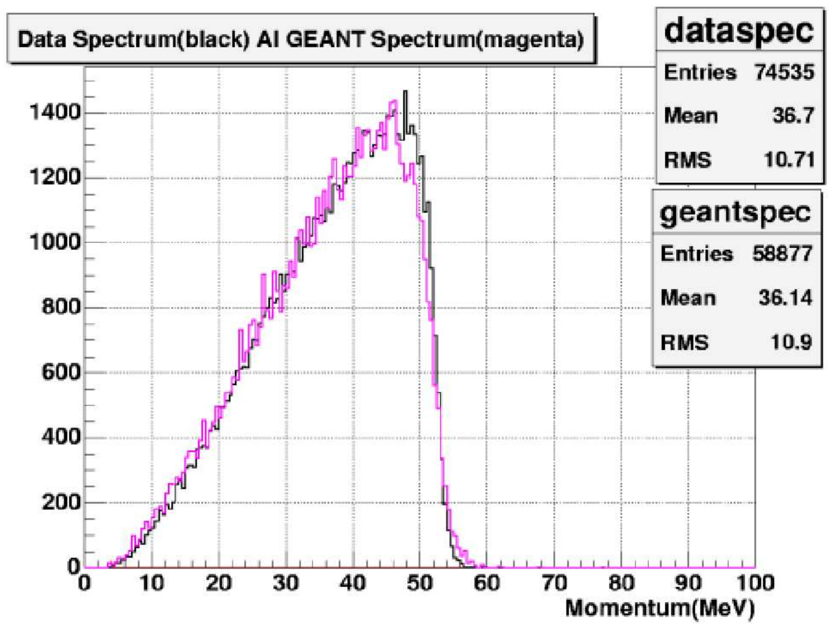

Figure 3.3: The Monte Carlo spectrum of muon decays from ^{27} Al compared to the μ^- data

3.4 Results

While these first feasibility studies were not intended to result in any physics measurements, it is apparent from the data that we are beginning to see the first hints at the physics which will come from the future of these studies. Figure 3.4 shows a comparison between the μ^- decay data used in this study (in black) and data from a regular positive muon $\mathcal{TWIST}$ data set (in magenta). The magenta line in the plot is very precise, and so, the deviations that the black histogram makes from it are significant. Already, we can see that the peak position is shifted to lower momenta. We can also see the effects of the doppler blurring in the form of extra events for the μ^- data beyond 52.8 MeV/c. Figure 3.5 highlights the deviations between the two spectra by showing their comparison on a log plot, and by showing the difference between the plots.

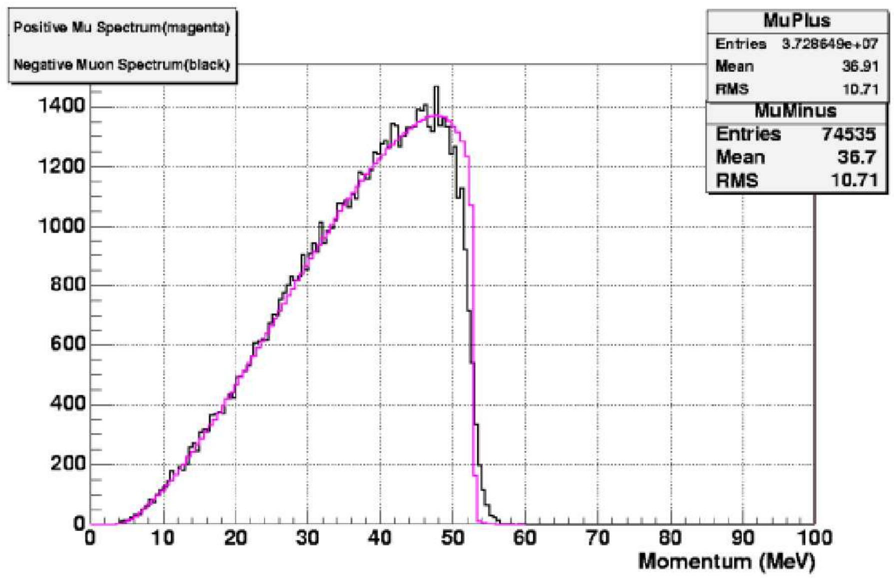

Figure 3.4: A comparison between the positive muon decay data (magenta) and the μ^- decay data (black)

These results lend confidence to the idea that $\mathcal{TWIST}$ is capable of doing interesting physics using a μ^- beam and their existing experimental aparatus. From these first feasibility studies, a number of the tools that will be required for the physics measurements have already been developed. For example, the GEAT simulation has been modified to allow for a μ^- beam simulation and the analysis software has been modified so that it can correctly track decay electrons through the detector and reconstruct the decay spectrum. Moreover, we now have some experience working with a μ^- beam both in terms of taking data and in analysing it.

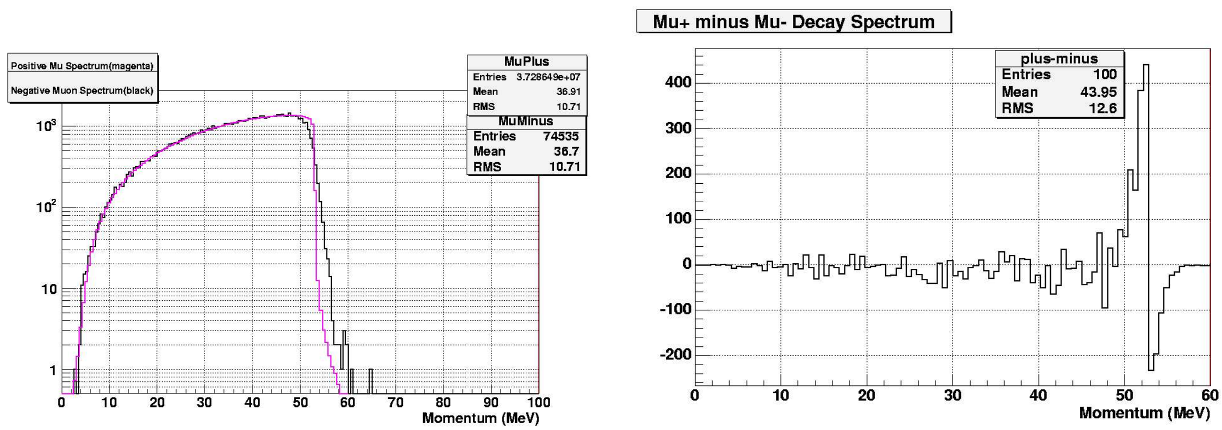

Figure 3.5: left: The positive muon spectrum (magenta) and the negative muon spectrum (black) on a log scale plot. right:The difference $(\mu^+ - \mu^-)$ between the positive and negative muon spectra.

Chapter 4

Future plans for $\mathcal{TWIST}$'s μ^- studies

A good deal of confidence has been generated in $\mathcal{TWIST}$'s ability to make a measurement of the decay spectrum of μ^-s due to these first feasibility studies. Thus, there are already plans to continue with μ^- decay studies. As soon as the experimental apparatus can be prepared to take more data, we will dedicate some of our run time to collect more μ^- data. This year, we are using an aluminium stopping target so we will be able to properly compare our data to the theoretical prediction given by our GEANT simulation. Moreover, we will collect more data so that we have enough statistics to make a precise measurement of the momentum spectrum.

Once this data is in hand, we are confident of being able to measure the μ^- decay spectrum. From the feasibility studies, it appears that our analysis software is now capable of tracking decay electrons in our detector. Thus, we should be able to reconstruct the decay electron momentum distribution and compare it to theoretical predictions. It is also possible for $\mathcal{TWIST}$ to measure the asymmetry of the decay. However, the cloud muon beam is not well polarised, so it is likely difficult to extract meaningful information from this aspect of the spectrum.

There are a number of other measurements that $\mathcal{TWIST}$ can make from a good μ^- data set. For example, the muon is initially captured in high energy states of the atomic orbit. Because the muon's mass is roughly 207 times that of the mass of the electrons in the atom, however, the muon rapidly cascades through the energy levels until it reaches the atomic ground state. There is a depolarisation that occurs during this cascade [9]. By looking at the asymmetry of the decay, and by considering the polarisation of the muon beam as it entered the target, it is possible to make a measurement of the depolarisation that occurs during the cascade.

$\mathcal{TWIST}$ can also measure the lifetime of the bound muons by looking at the timing between the muon stopping in the target and the first observation of the decay electron. The reason this lifetime is physically significant is because the effective lifetime of bound muons is reduced from that of the free muon. This is due to a probability of the muon being captured by the nucleus. This process competes with decay and thus reduces the effective lifetime of the bound muon.

Another measurement that $\mathcal{TWIST}$ can make from the μ^- data is a limit on the branching ratio of $\mu^- N \rightarrow e^- N$.

A lot more work has yet to be done towards producing a useful physics measurement from this project. The bulk of this work will be in understanding what the systematic sources of error are for doing μ^- decay studies and how they affect any of the measurements that we attempt to make. It is also our intention to produce a Monte Carlo decay spectrum directly from an analytical calculation. Using interpolation introduces some uncertainty in between data points which could be eliminated by doing an analytical calculation for each randomly thrown energy and decay angle.

While it is possible for $\mathcal{TWIST}$ to make the above mentioned measurements, it is not yet clear what kind of precision can be achieved. All of these measurements are outside the scope of what the $\mathcal{TWIST}$ spectrometer was designed to measure. The precision to which we can test the theoretical predictions depends on both the statistics that we can collect and the systematics. Both of these factors remain unknown until they can be studied in more detail. While we are confident that we can beat the existing precision of μ^- momentum spectrum measurements, the chances of the other measurements that we can make being more precise than existing measurements is far smaller.

Chapter 5

Conclusions

Through these first feasibility studies, I have clearly demonstrated that $\mathcal{TWIST}$ is capable of measuring the decay spectrum of bound μ^-s using the existing experimental apparatus. We have already developed a number of the tools that are needed to analyse μ^- data. The relevant changes to the analysis software have been made and implemented in the standard versions of the $\mathcal{TWIST}$ software. The $\mathcal{TWIST}$ softwa re is now capable of analysing μ^- data and reconstructing the μ^- decay spectrum. Also, the Monte Carlo has been modified to allow tracking the spin of μ^-s through the simulated detector solenoid field and a subroutine has been written to generate a theoretical decay spectrum of μ^-s decaying from aluminium.

Now that we are confident that our detector and analysis software can make the required measurement, plans are underway to collect more data. Later this summer, a number of μ^- data runs will be performed and recorded for later analysis. There is, however, still a considerable amount of work that must go into the project. For example, the systematic uncertainties which affect the measurement need to be considered in order to understand the level of precision that we can measure the decay parameters to. Also, we have yet to determine the expected precision to which we can make the other measurements listed in chapter 4. Ultimately, after much more work in studying the systematics and performing a thorough analysis, we hope to produce a physics publication for a measurement of the μ^- decay spectrum from Aluminium.

Appendix A

The purpose of this appendix is to share more specific information about what was done on this project, and to point out the relevant files that were used. This section will be useful for anyone who may be working on this project in the future. All the refered-to directories are on the $\mathcal{TWIST}$ filesystem.

A.1 GEANT

A.1.1 $\mu^+ \rightarrow e^-$

The change which needs to be made to the GEANT code to simulate the reaction $\mu^+ \rightarrow e^-$ is simple. In mudcay.F, there is a line of code which reads

```
GKIN(5,NGKINE) = 2.
```

This line defines which particle the muon is to decay to. If this number is changed from 2 to 3, it will produce an electron instead of a positron.

Two GEANT runs were simulated using the reaction $\mu^+ \rightarrow e^-$. The .dat files for these runs are /twist/data23/mazur/geant_runs/run7.dat and /twist/.../run8.dat. The results of the analysis for these files is stored in
/home/mazur/e614soft/triumf/geant/devel/run/results/muplustoeminus.

A.1.2 ^{27}Al GEANT

The spectrum that GEANT samples for the muon decay spectrum is calculated in a function called sample_michel(). In order to facilitate negative muon simulation, I wrote a new function called sample_michel_minus() which is called from sample_michelb.F using the following structure:

```
if(ipart.eq.65) then
   call sample_michel(e, ctheta)
else if(ipart.eq.66) then
   call sample_michel_minus(e,ctheta).
```

Here, particle number 66 is a negatively charged muon with spin information which can be tracked through the magnetic field and particle number 65 is the positive muon equivalent. Then, the decay particle can be defined as an electron in mudcay.F by changing the line listed in A.1.1 to

```
if(ipart.eq.65) then
   GKIN(5,NGKINE) = 2
else if(ipart.eq.66) then
   GKIN(5,NGKINE) = 3
endif
```

The code for the sample_michel_minus() function is defined in /home/mazur/e614soft/-triumf/geant/devel/source/sample_michel_minus.F.

The function returns an electron energy, e, and the cosine of the decay angle, ctheta. This is calculated based off an interpolation of a data table which appeared in a paper by Watanabe [7]. A random energy and $\cos(\theta)$ are generated as well as a third random number representing the probability of that decay event occuring. If the spectrum falls beneath the randomly generated point in 3-D, the event is rejected and a new attempt is made.

Three .dat files were generated using the sample_michel_minus spectrum generator. They can all be located in the directory /twist/data23/mazur/geant_runs/ with file names run11.dat, run12.dat, and run13.dat.

For these runs, I used a rough estimate of the polarization of the muons as they decay for the initial polarization of the simulated muon beam. This polarization was -0.04. The other notable change to the .ffcard file was to change the PGUN settings to produce a beam of negative muons with spin (particle number 66). The FFCARDS file used for run12, for example, can be found at /home/mazur/e614soft/triumf/geant/devel/run/run12.ffcards.

The results of the analysis of these files as well as the tree sum of all three runs can be found in the folder /twist/data23/mazur/muminus/geant. /twist/data23/mazur/muminus/-production_raw2.kcm is an example of a typical .kcm that I've used for GEANT analysis.

A.2 Analysis

Now that the bug in the analysis software has been corrected, the standard CVS version of MOFIA can analyse negative muon data. There were four negative muon data runs available at the time of this report (12753-12756), which were all analysed with the photo executable /twist/data23/mazur/muminus/photo_narrowwindowfix using the default .kcm file /twist/data23/mazur/muminus/production_raw.kcm. The results of the analysis and the subsequent tree summing can be found in the directory /twist/data23/mazur/muminus/-NWFanal/.

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