

Theoretical Implications of the TWIST Experiment Results

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SSSP2009, National Taiwan University, May 2009



- 1 Muon decay formalism
- 2 TWIST experiment
- 3 Theoretical implications

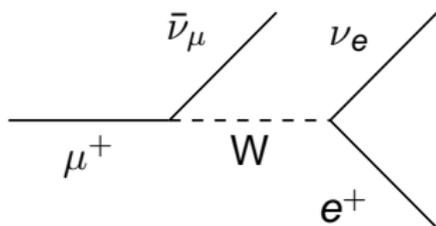
Muon decay to probe the weak interaction

Muon decay is ideal to study the weak nuclear interaction at low energy.

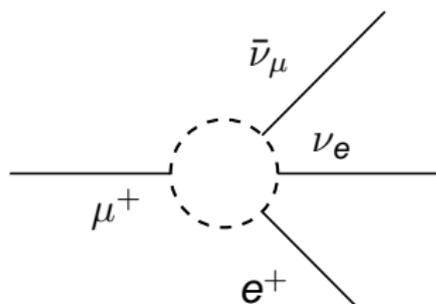
- Only weak interaction involved
- Muons are easy to produce
- One decay mode dominates ($\approx 100\%$)

One can study muon decay at low energy in a model independent way.

Standard Model



4-fermion interaction



The interaction can be described as a derivative-free, Lorentz-invariant and lepton-number conserving matrix¹:

$$M = 4 \frac{G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \epsilon,\mu=R,L}} g_{\epsilon\mu}^{\gamma} \langle \bar{e}_{\epsilon} | \Gamma^{\gamma} | \nu_e \rangle \langle \bar{\nu}_{\mu} | \Gamma_{\gamma} | \mu_{\mu} \rangle$$

$\gamma =$ S(calar), V(ector), T(ensor)

$\epsilon, \mu =$ R(ight-handed), L(eft-handed)

¹W. Fetscher, H. J. Gerber, and K.F. Johnson, *Phys. Lett.* **B173** (1986) 102 

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- $g_{RR}^T \equiv g_{LL}^T \equiv 0$
- A common phase doesn't matter

Standard Model, V-A interaction

$$g_{LL}^V = 1$$

⇒ 19 real and independent parameters

¹W. Fetscher, H. J. Gerber, and K.F. Johnson, *Phys. Lett.* **B173** (1986) 102

The muon decay parametrization

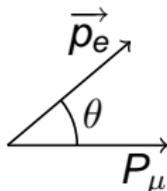
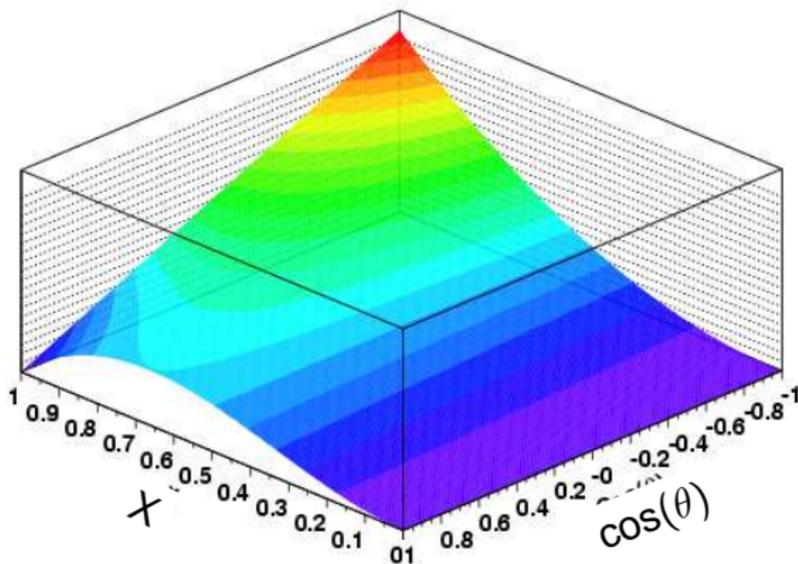
The differential decay rate can be written:

$$\frac{d^2\Gamma}{dx d\cos\theta} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} (F_{IS}(x) + P_\mu \cos\theta F_{AS}(x)) + \text{R.C.}$$

$$W_{e\mu} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$$

$$x = \frac{E_e}{W_{e\mu}}$$

$$x_0 = \frac{m_e}{W_{e\mu}}$$



The muon decay parametrization

The isotropic and anisotropic parts are:

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$F_{AS}(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2} \left[1 - x + \frac{2}{3}\delta(4x - 3 + (\sqrt{1 - x_0^2} - 1)) \right]$$

Standard Model predictions

$$\rho = \frac{3}{4}, \quad \eta = 0, \quad P_\mu \xi = 1, \quad \delta = \frac{3}{4}$$

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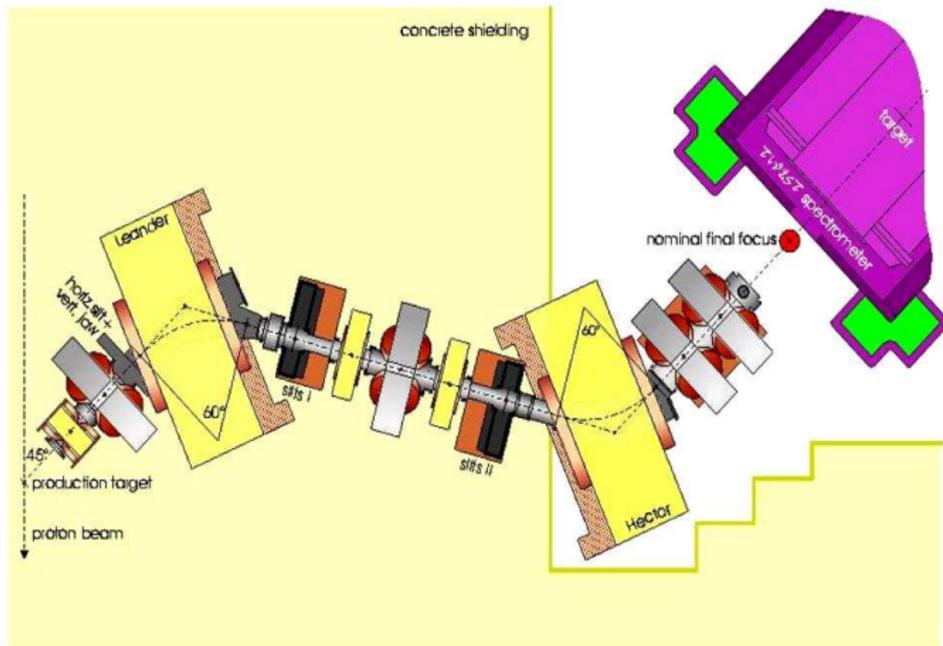
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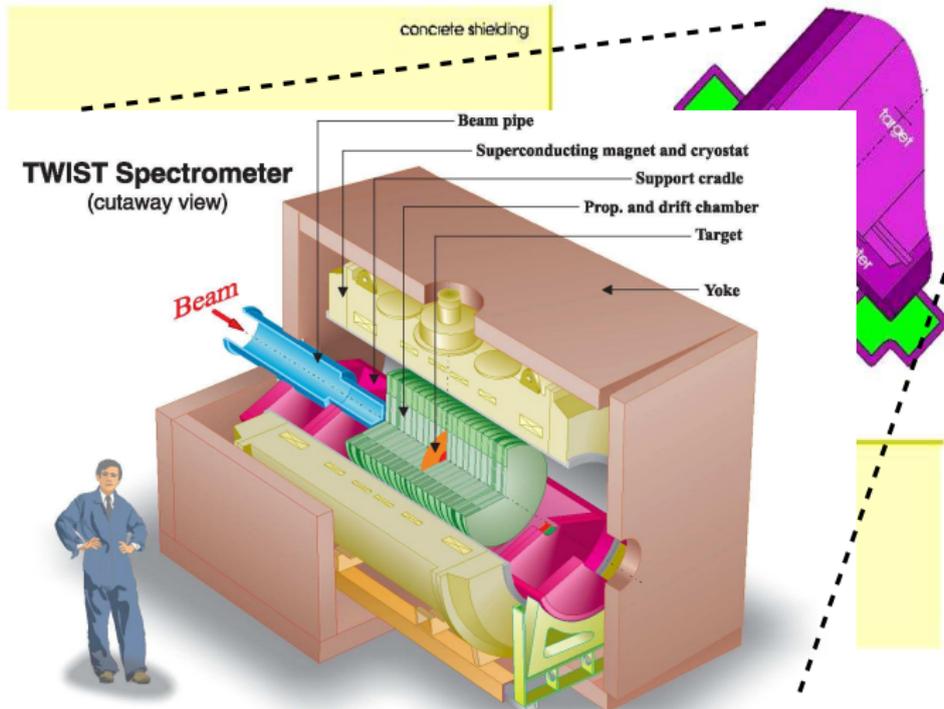
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The TWIST experiment

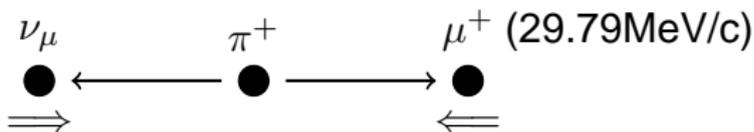
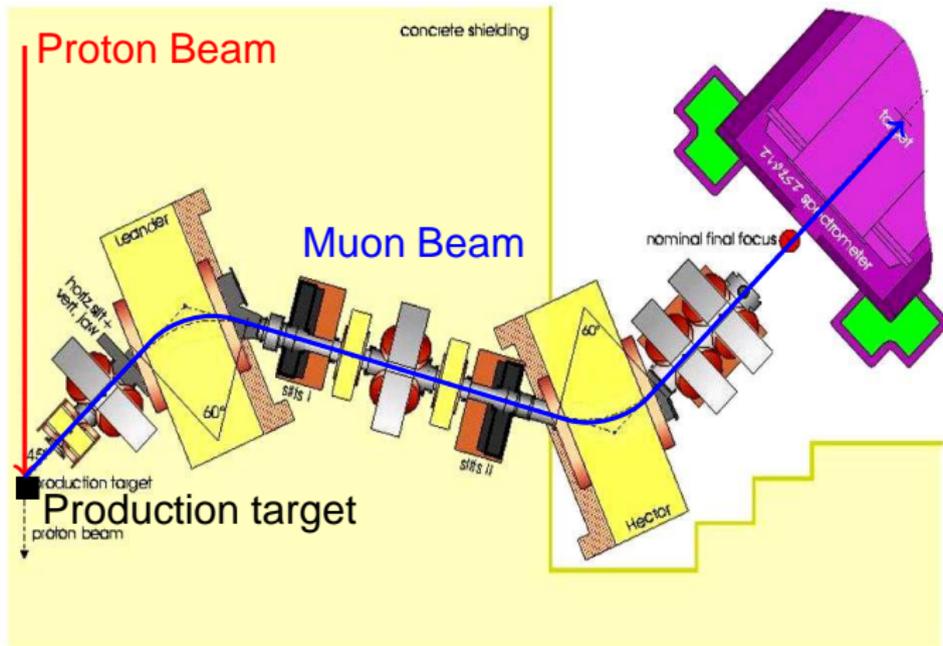


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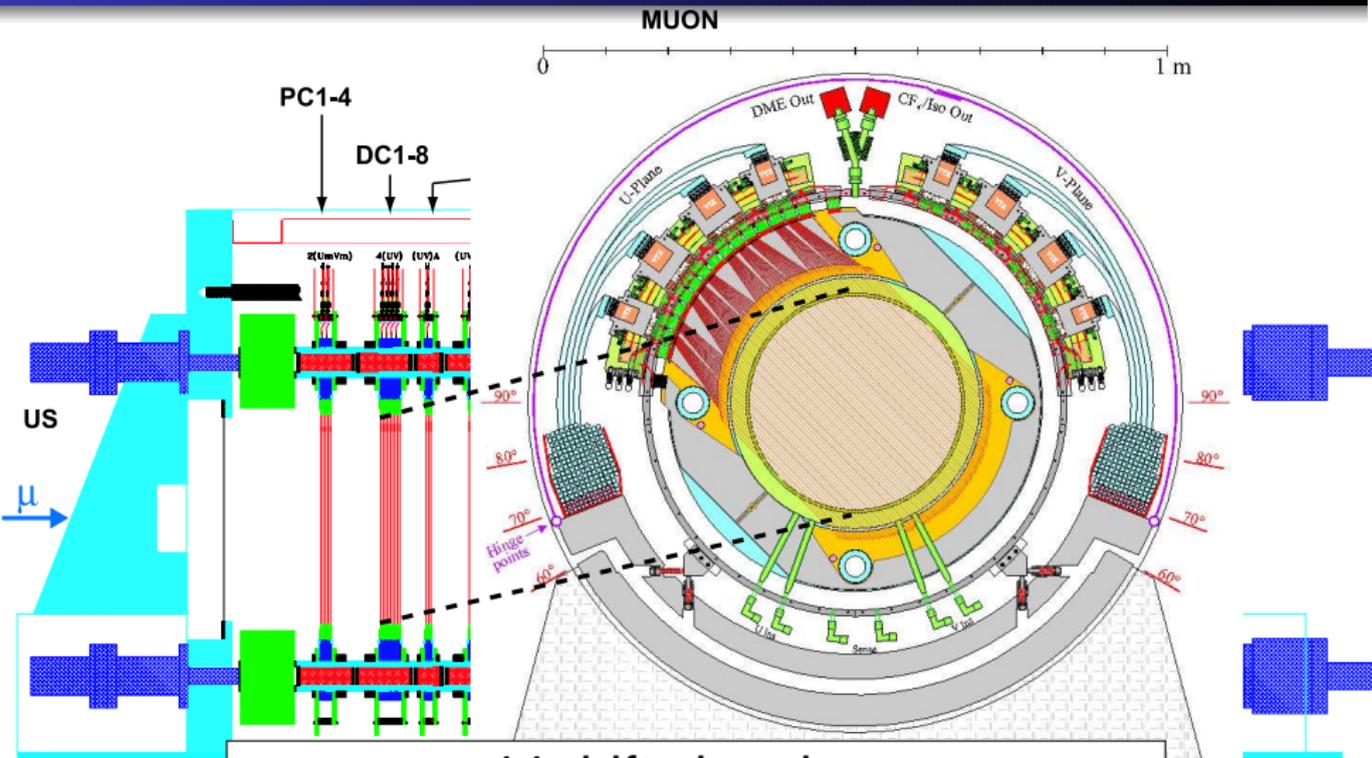


TWIST = TRIUMF Weak Interaction Symmetry Test

The TWIST experiment

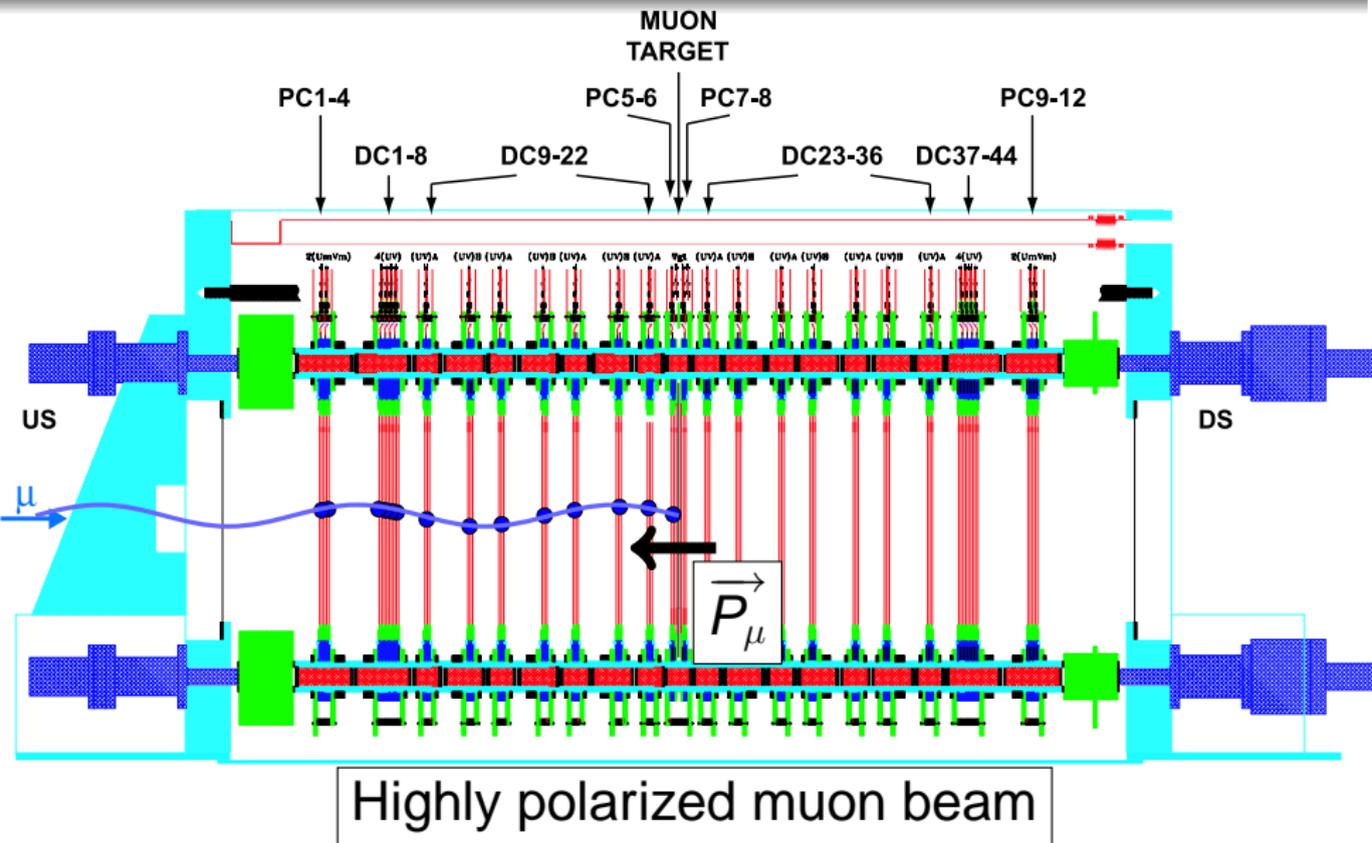


Typical event

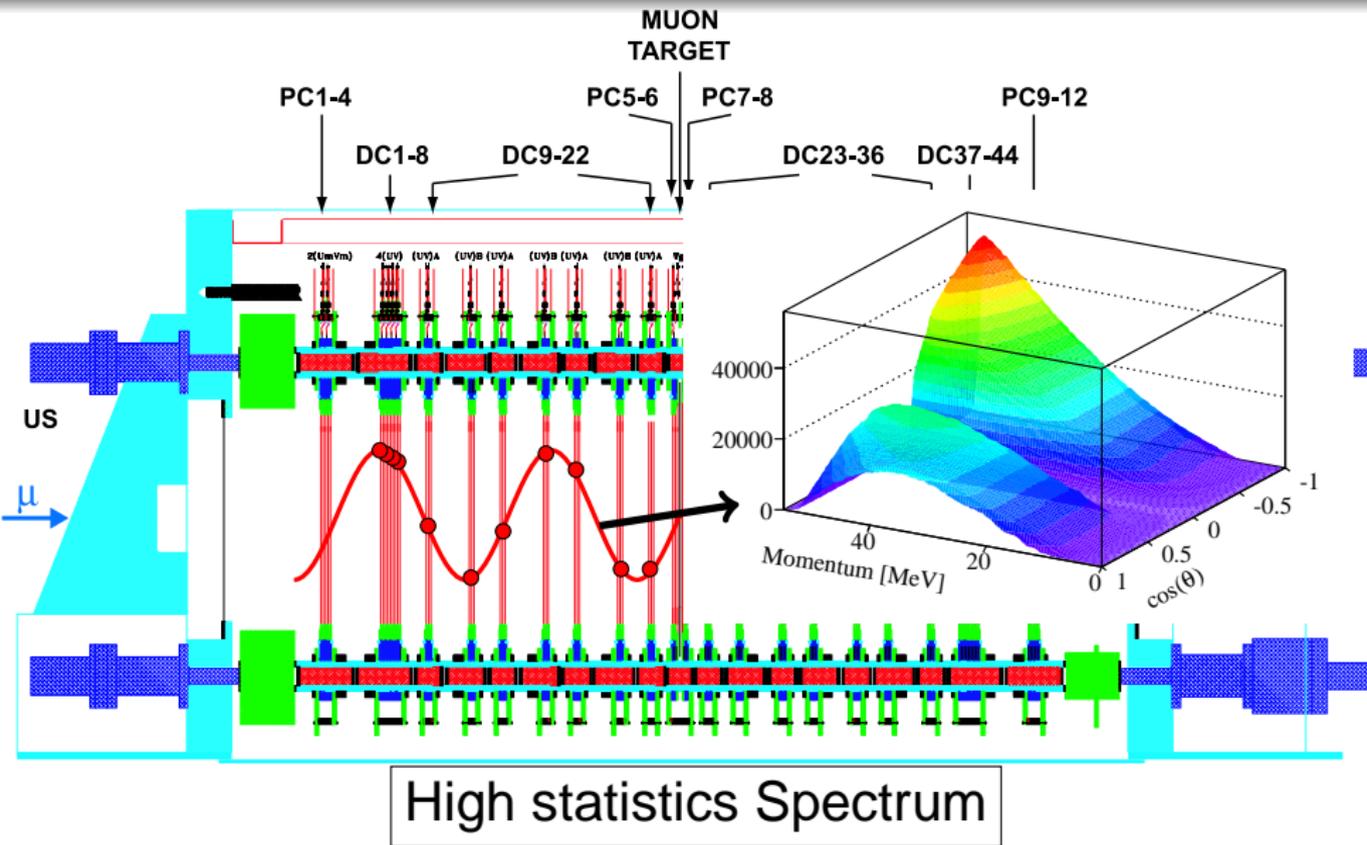


44 drift chambers
Wire position known to 5 parts in 10^5

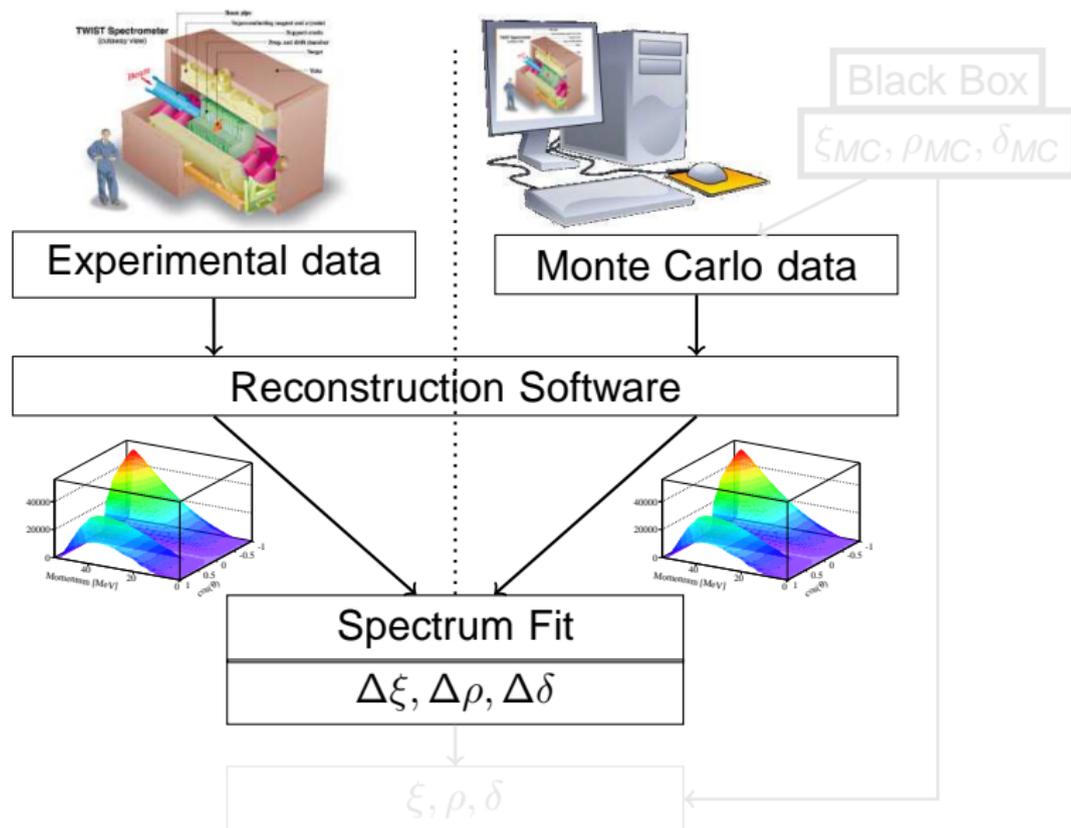
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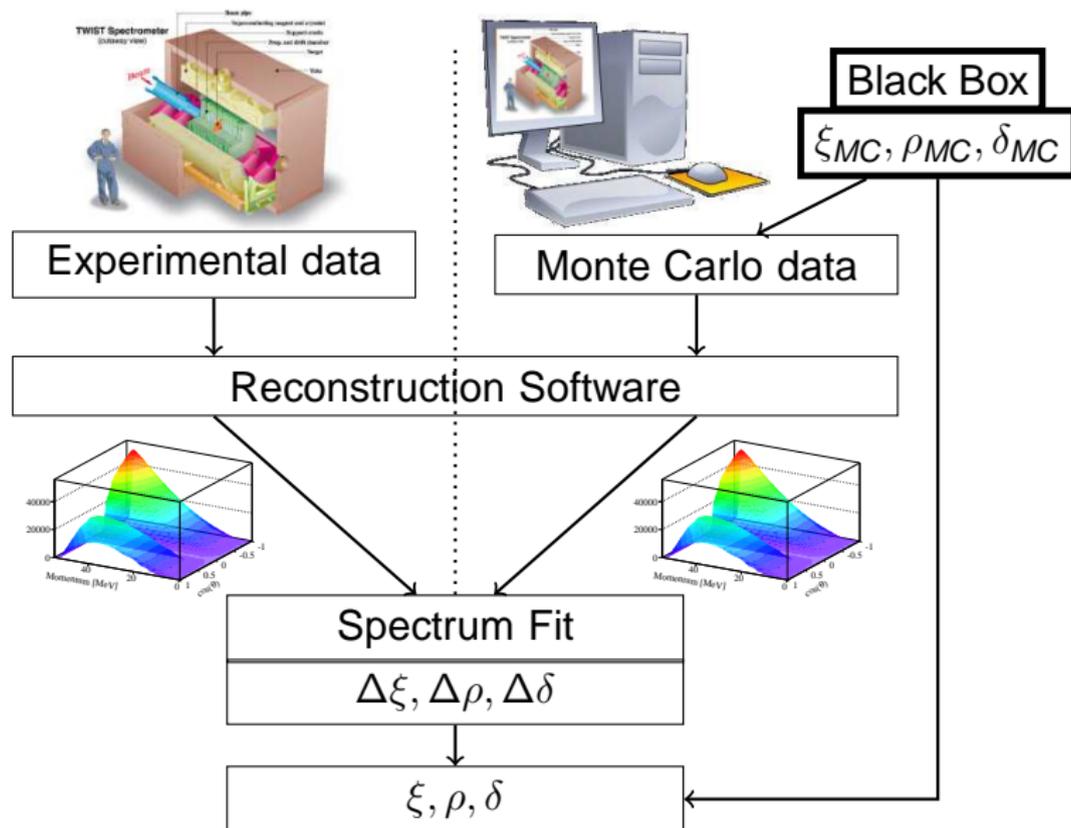
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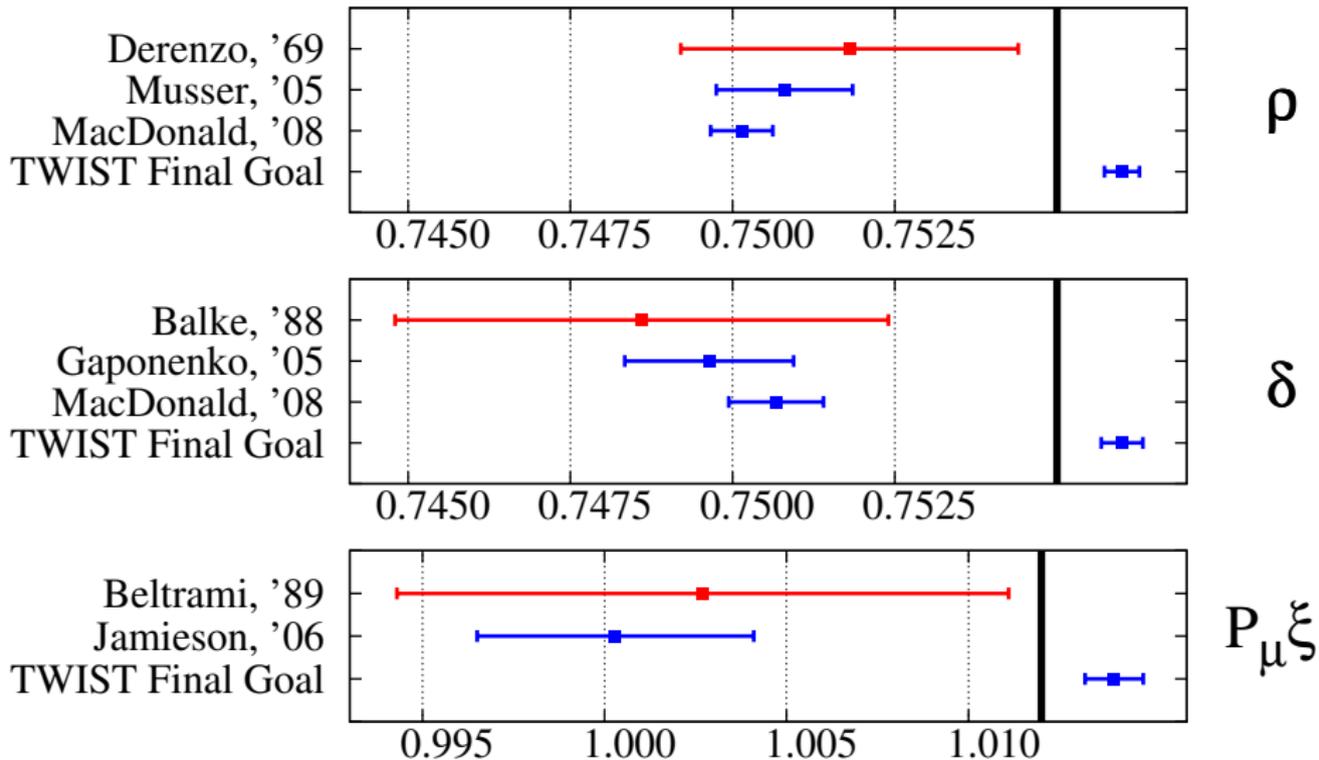
The software analysis



The software analysis



The TWIST Results



Pre-TWIST

TWIST

Towards the Final Measurement

A lot of improvements were implemented in order to achieve our goals for the final measurement:

- The final data represents more than three times the amount of data taken for the previous analysis.
- Hardware improvement
 - Drift chambers rearrange
 - Beam steering installed on the beamline
 - ...
- Calibrations improvement
 - Space time relations in the drift chambers measured instead of simulated
 - Better alignment of the experiment components
 - ...

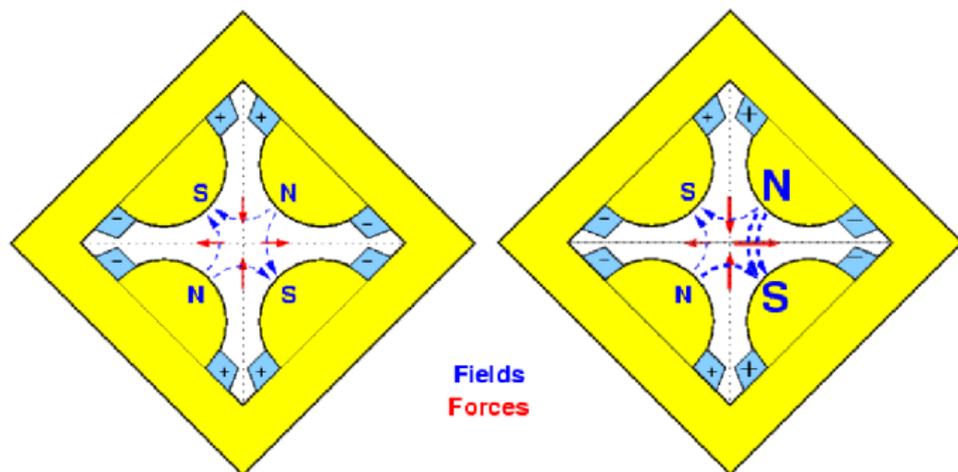
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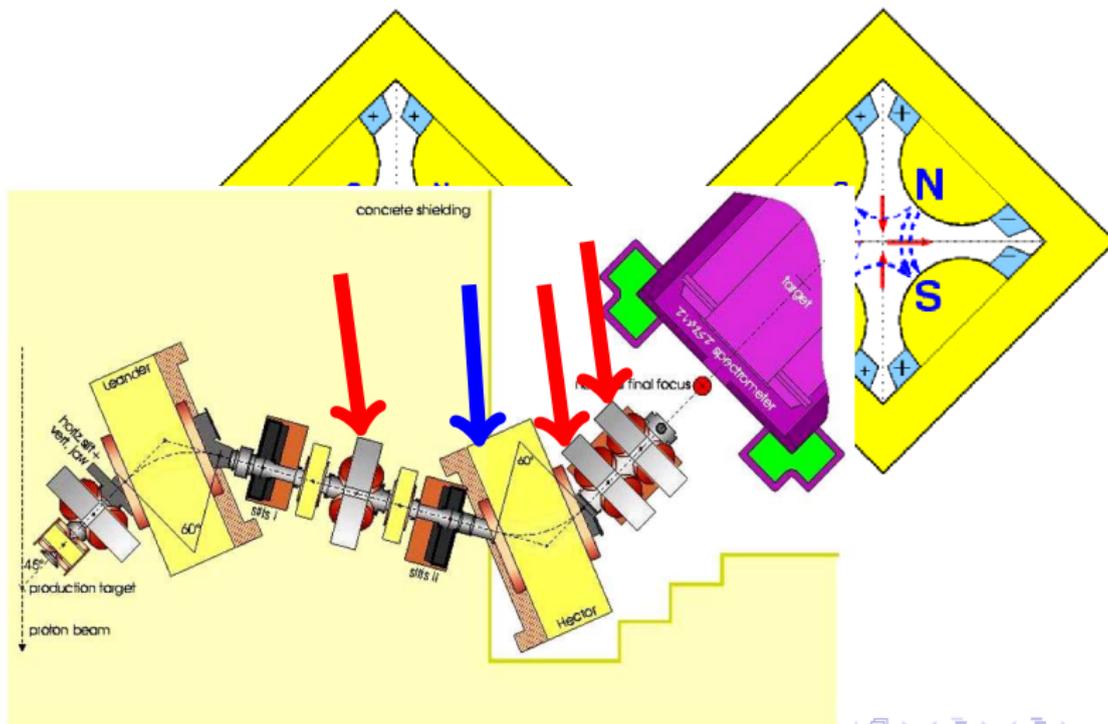
Better control on the muon beam

Installation of extra power supply on the beam line to steer the beam.



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New muon beam monitoring

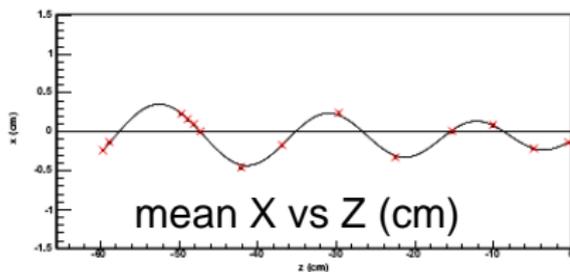
- The individual muon tracks cannot be reconstructed
- The muon beam spots at each plane are used instead

Improvements

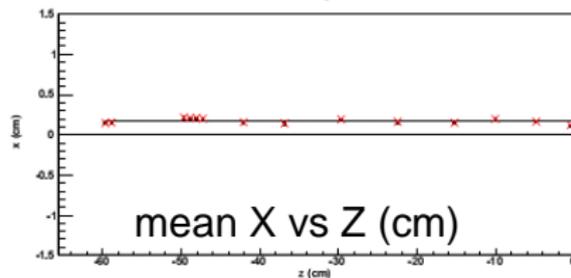
- Muon beam stability monitored
- Reduced $p_T \rightarrow$ lower depolarisation

Position of the muon beam spot center in the spectrometer.

High p_T



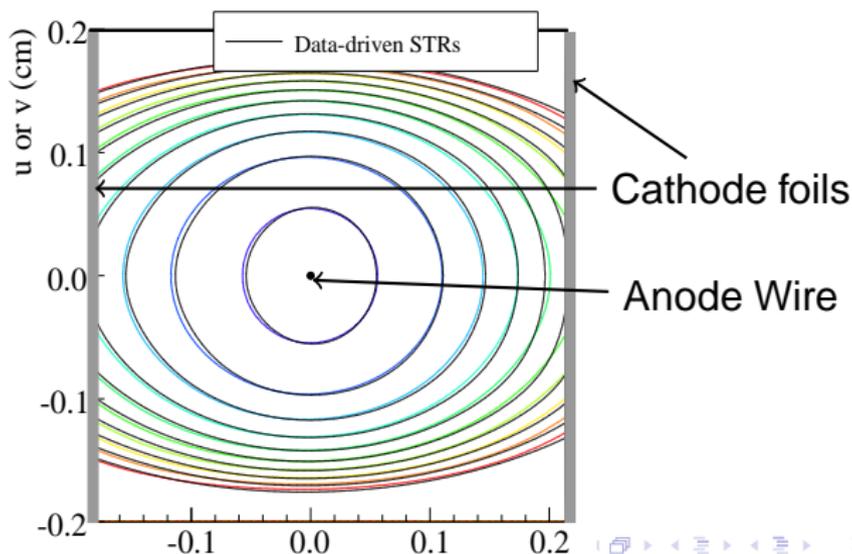
Small p_T



Better Space Time Relations (STRs) for the drift chambers

- Previous analyzes used simulated STRs (from Garfield)
- Now the STRs are extracted from the decay positron tracks

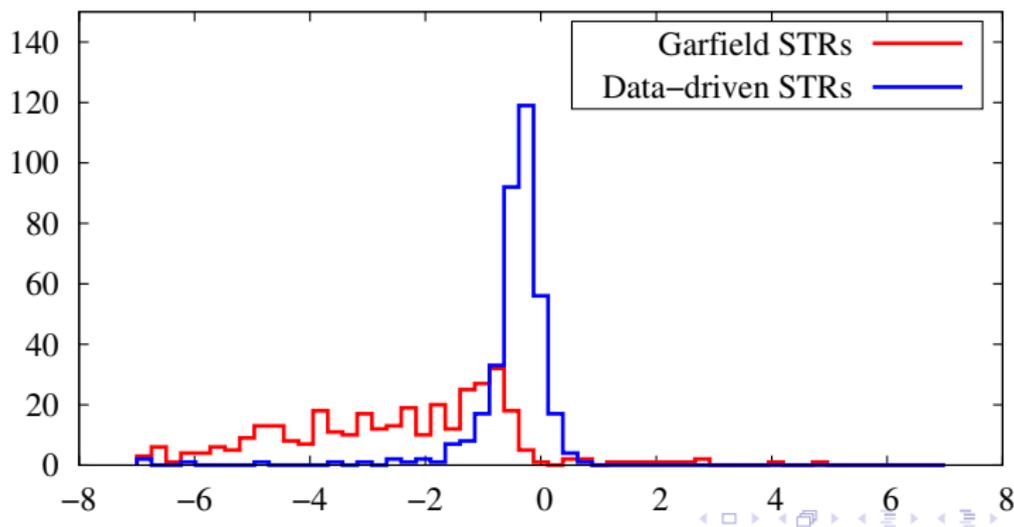
⇒ **Track reconstruction greatly improved and the corresponding systematic uncertainty is now below 10^{-4} .**



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Measurement of the muon decay parameters with a precision of an order of magnitude better than the experiments prior to TWIST.

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Measurement of the muon decay parameters with a precision of an order of magnitude better than the experiments prior to TWIST.

Publication of the results planned for early 2010.



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Left Right Symmetry Test

In left-right symmetric models the (V+A) current is suppressed, but not exactly zero².

The left- and right-handed gauge boson fields are given by:

$$W_L = W_1 \cos \zeta + W_2 \sin \zeta$$

$$W_R = e^{i\omega}(-W_1 \sin \zeta + W_2 \cos \zeta)$$

The following notations assume possible differences in left and right coupling and CKM character:

$$t = \frac{g_R^2 m_1^2}{g_L^2 m_2^2}, \quad t_\theta = t \frac{|V_{ud}^R|}{|V_{ud}^L|}, \quad \zeta_g^2 = \frac{g_R^2}{g_L^2} \zeta^2$$

²Herczeg, P., Phys. Rev. D, 34, 3449–3456, 1986

Left Right Symmetry Test

$$\rho = \frac{3}{4}(1 - 2\zeta_g^2), \quad \xi = 1 - 2(t^2 + \zeta_g^2)$$

$$P_\mu = 1 - 2t_\theta^2 - 2\zeta_g^2 - 4t_\theta\zeta_g^2 \cos(\alpha + \omega)$$

90% confidence level limits can be deduced from TWIST results with a minimal set of assumptions about the left-right symmetry model:

$$\text{Pre-TWIST: } |\zeta_g| < 0.066, \quad \left(\frac{g_L}{g_R}\right) m_2 > 294 \text{ GeV}/c^2$$

$$\text{Current}^3: |\zeta_g| < 0.022, \quad \left(\frac{g_L}{g_R}\right) m_2 > 364 \text{ GeV}/c^2$$

³R.P.MacDonald et al., Phys. Rev. D 78, 032010 (2008) 

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To extract the couplings $g_{\epsilon\mu}^\gamma$ from muon decay, one needs 11 (not all independent) parameters:

- the four muon decay parameters ρ , η , $P_\mu\xi$ and δ
- the measurement of $P_\mu\xi\delta/\rho$
- the parameters ξ' and ξ'' from the longitudinal polarisation of the outgoing electrons
- the parameters η'' , α , β , α' and β' from the transverse polarisation of the outgoing electrons
- the parameter $\bar{\eta}$ from the radiative muon decay

Gagliardi and al. (Phys. Rev. D 72, 073002) performed a global fit analysis extracting the coupling constants from the most recent results.

The following parametrization is used:

$$Q_{RR} = \frac{1}{4}|g_{RR}^S|^2 + |g_{RR}^V|^2 \quad Q_{LL} = \frac{1}{4}|g_{LL}^S|^2 + |g_{LL}^V|^2$$

$$Q_{RL} = \frac{1}{4}|g_{RL}^S|^2 + |g_{RL}^V|^2 + 3|g_{RL}^T|^2$$

$$Q_{LR} = \frac{1}{4}|g_{LR}^S|^2 + |g_{LR}^V|^2 + 3|g_{LR}^T|^2$$

$$B_{LR} = \frac{1}{16}|g_{LR}^S + g_{LR}^T|^2 + |g_{LR}^V|^2$$

$$B_{RL} = \frac{1}{16}|g_{RL}^S + g_{RL}^T|^2 + |g_{RL}^V|^2$$

$$I_\alpha = \frac{1}{4}[g_{LR}^V(g_{RL}^S + 6g_{RL}^T)^* + (g_{RL}^V)^*(g_{LR}^S + 6g_{LR}^T)]$$

$$I_\beta = \frac{1}{2}[g_{LL}^V(g_{RR}^S)^* + (g_{RR}^V)^*g_{LL}^S]$$

In this parametrization:

- The $Q_{\epsilon\mu}$ are total probabilities of a μ -handed muon decays into a ϵ -handed electron. Example:

$$Q_{RR} = \frac{1}{4}|g_{RR}^S|^2 + |g_{RR}^V|^2$$

The corresponding normalization condition is used to eliminate Q_{LL} from the analysis:

$$Q_{RR} + Q_{LR} + Q_{RL} + Q_{LL} = 1$$

- There are useful constraints:

$$0 \leq Q_{\epsilon\mu} \leq 1,$$

where $\epsilon, \mu = R, L$

$$0 \leq B_{\epsilon\mu} \leq Q_{\epsilon\mu},$$

where $\epsilon\mu = RL, LR$

$$|I_\alpha|^2 \leq B_{LR}B_{RL},$$

$$|I_\beta|^2 \leq Q_{LL}Q_{RR}$$

The global analysis uses a Monte Carlo integration techniques.

Extraction of the joint probability distributions of Q_{RL}, B_{RL}, \dots

11 decay parameters (ρ, δ, \dots) \implies 9 fit parameters (Q_{RL}, B_{RL}, \dots)

Model-Independent search for right-handed interactions

Model-independent measure of the right-handed muon decay probability:

$$Q_R^\mu = Q_{RR} + Q_{LR}$$

$$Q_R^\mu = \frac{1}{4}|g_{LR}^S|^2 + \frac{1}{4}|g_{RR}^S|^2 + |g_{LR}^V|^2 + |g_{RR}^V|^2 + 3|g_{LR}^T|^2$$

Results from the global analysis at a 90% confidence level:

- **Pre-TWIST:** $Q_R^\mu < 0.0051$
- **Gagliardi:** $Q_R^\mu < 0.0031$
- **Current⁴:** $Q_R^\mu < 0.0024$

⁴R.P.MacDonald et al., Phys. Rev. D 78, 032010 (2008)

Limits on the coupling constants

Weak Coupling	pre-TWIST	Gagliardi	Current ⁵
$ g_{RR}^S $	< 0.066	< 0.067	< 0.063
$ g_{LR}^S $	< 0.125	< 0.088	< 0.076
$ g_{RL}^S $	< 0.424	< 0.417	< 0.415
$ g_{LL}^S $	< 0.550	< 0.550	< 0.550
$ g_{RR}^V $	< 0.033	< 0.034	< 0.032
$ g_{LR}^V $	< 0.066	< 0.036	< 0.027
$ g_{RL}^V $	< 0.110	< 0.104	< 0.105
$ g_{LL}^V $	> 0.960	> 0.960	> 0.960
$ g_{RR}^T $	$\equiv 0$	$\equiv 0$	$\equiv 0$
$ g_{LR}^T $	< 0.036	< 0.025	< 0.022
$ g_{RL}^T $	< 0.112	< 0.104	< 0.104
$ g_{LL}^T $	$\equiv 0$	$\equiv 0$	$\equiv 0$

Couplings most sensitive to
TWIST results

90% confidence level

⁵R.P.MacDonald et al., Phys. Rev. D 78, 032010 (2008)

The TWIST Collaboration

TRIUMF

Alberta

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		Texas A&M
		Carl Gagliardi Bob Tribble
Regina	Montréal	
Ted Mathie Roman Tacik	Pierre Depommier	Valparaiso
	Kurchatov Institute Vladimir Selivanov	Don Koetke Shirvel Stanislaus

◇ Graduated

† Graduate student

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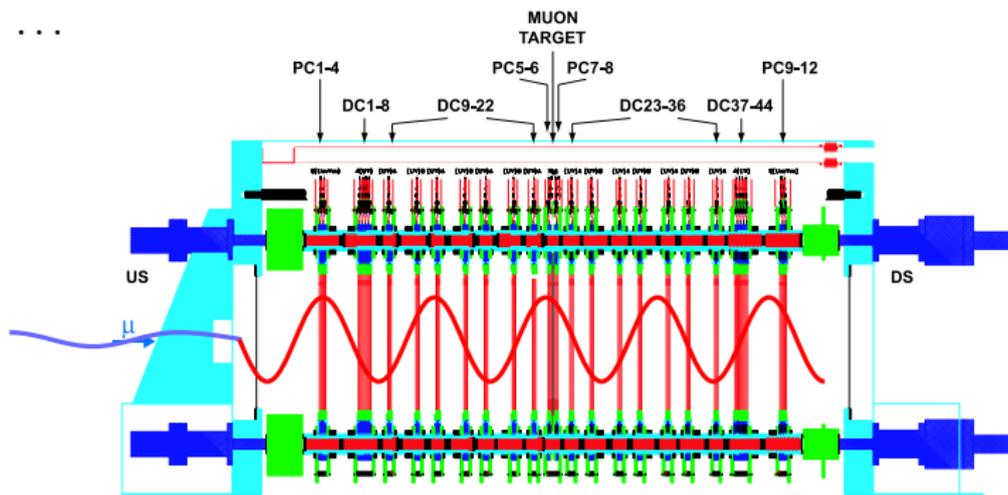
EXTRA SLIDES

Upstream stops data

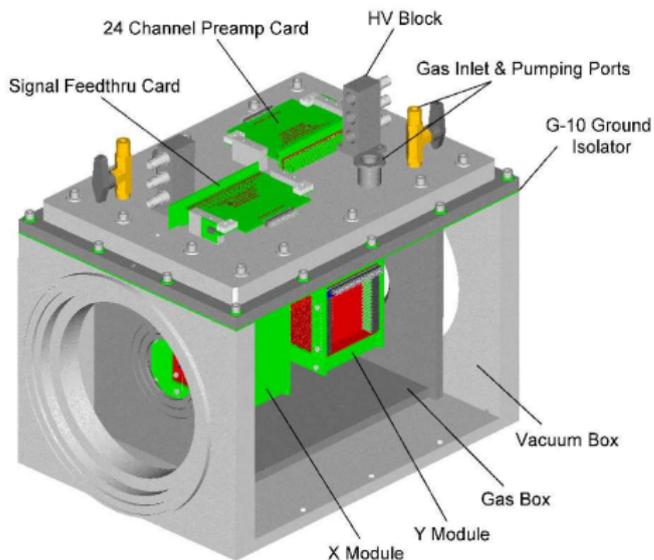
A special set of data was taken with the muons stopping at the far upstream end of the detector.

This data gives us information on the physics and the response of the detector:

- Test of the detector asymmetry
- Measure of the positron interaction with the target
- ...



The Time Expansion Chamber (TEC)

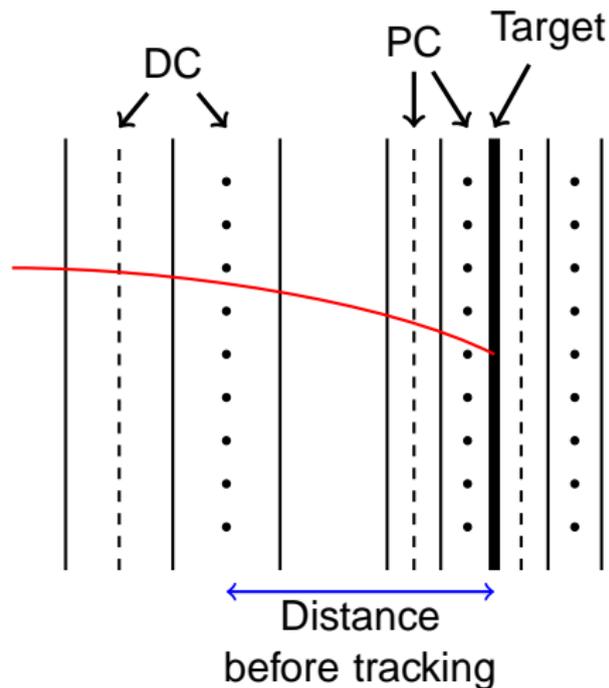
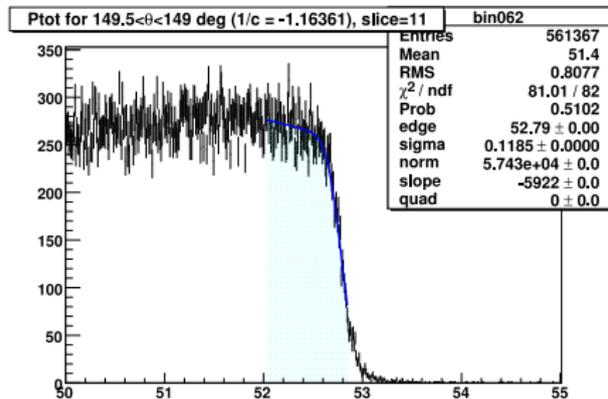


The TEC:

- is low mass
- operates in the beamline vacuum
- is removed during data taking

Energy calibration using the kinematic endpoint

The kinematic endpoint of the positron energy spectrum is used to correct a difference between the data and the MC simulation.



Muon decay parameters derivatives

$$\underbrace{\left. \frac{d^2\Gamma}{dx d(\cos\theta)} \right|_{\rho_{MC}, \delta_{MC}, \xi_{MC}}}_{\text{MC spectrum}} + \underbrace{\sum_{\alpha=\rho, \xi, \xi\delta} \frac{\partial}{\partial\alpha} \left[\frac{d^2\Gamma}{dx d(\cos\theta)} \right]}_{\text{Derivatives fitted}} \Delta\alpha$$

