

Final Results on Muon Decay from **TWIST**

Carl A. Gagliardi

Texas A&M University

for the **TWIST Collaboration**

Outline

- Introduction to muon decay
- **TWIST** experiment
- Previous **TWIST** results
- Final **TWIST** results

Muon decay spectrum

- The energy and angle distributions of positrons following polarized muon decay obey the Michel spectrum:

$$\frac{d^2\Gamma}{x^2 dx d(\cos\theta)} \propto (3 - 3x) + \frac{2}{3}\rho(4x - 3) + 3\eta\frac{x_0}{x}(1 - x) + P_\mu\xi\cos\theta\left[(1 - x) + \frac{2}{3}\delta(4x - 3)\right] \quad (+ \text{rad. corr.})$$

where $x = \frac{E_e}{E_{e,\max}}$

- Pre-**TWIST** accepted values for the muon decay (Michel) parameters:

	SM
$\rho = 0.7518 \pm 0.0026$	3/4
$\eta = -0.007 \pm 0.013$	0
$P_\mu\xi = 1.0027 \pm 0.0079 \pm 0.0030$	1
$\delta = 0.7486 \pm 0.0026 \pm 0.0028$	3/4
$P_\mu(\xi\delta/\rho) > 0.99682$ (90% c.l.)	1

Muon decay matrix element

- Most general Lorentz-invariant, local, lepton-number conserving muon decay matrix element:

$$M = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \varepsilon,\mu=R,L}} g_{\varepsilon\mu}^{\gamma} \langle \bar{e}_{\varepsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle (\bar{\nu}_{\mu})_m | \Gamma_{\gamma} | \mu_{\mu} \rangle$$

- The muon decay parameters are bi-linear combinations of the $g_{\varepsilon\mu}^{\gamma}$
- In the Standard Model, $g_{LL}^V = 1$, all others are zero
- Pre-**TWIST** global fit results (all 90% c.l.):

$ g_{RR}^S < 0.066$	$ g_{RR}^V < 0.033$	$ g_{RR}^T \equiv 0$
$ g_{LR}^S < 0.125$	$ g_{LR}^V < 0.060$	$ g_{LR}^T < 0.036$
$ g_{RL}^S < 0.424$	$ g_{RL}^V < 0.110$	$ g_{RL}^T < 0.122$
$ g_{LL}^S < 0.550$	$ g_{LL}^V > 0.960$	$ g_{LL}^T \equiv 0$

Goal of TWIST

- Search for new physics that can be revealed by **order-of-magnitude improvements** in our knowledge of ρ , δ , and $P_\mu \xi$

Two examples

- Model-independent limit on muon handedness

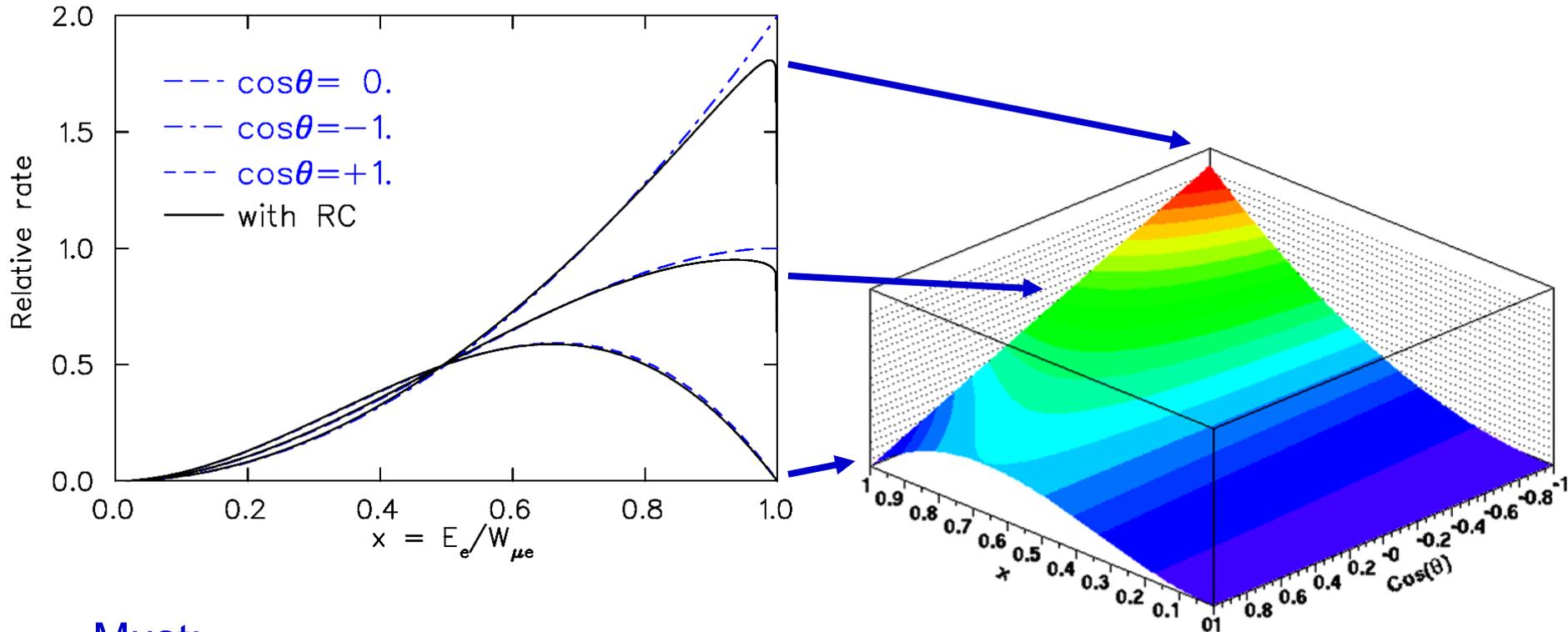
$$Q_R^\mu = \frac{1}{2} \left[1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right]$$

- Left-right symmetric model: $SU(2)_L \times SU(2)_R \times U(1)$

$$\begin{aligned} W_L &= W_1 \cos \zeta + W_2 \sin \zeta \\ W_R &= e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \frac{3}{4} - \rho &= \frac{3}{2} \zeta^2 \\ 1 - P_\mu \xi &= 4 \left(\zeta^2 + \zeta \left(\frac{M_1}{M_2} \right)^2 + \left(\frac{M_1}{M_2} \right)^4 \right) \end{aligned}$$

-

What is required?



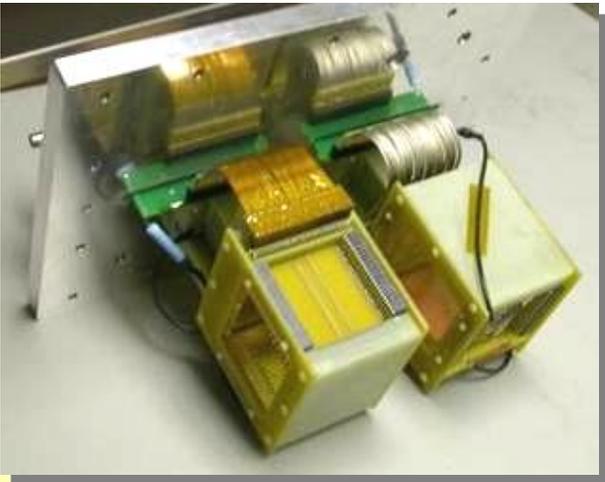
Must:

- Understand sources of **muon depolarization**
 - P_{μ} and ξ come as a product
- Determine positron **yield vs. momentum and angle**
 - All three parameters

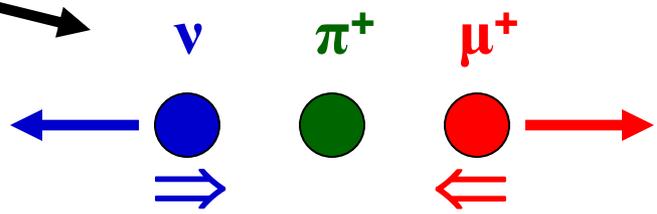
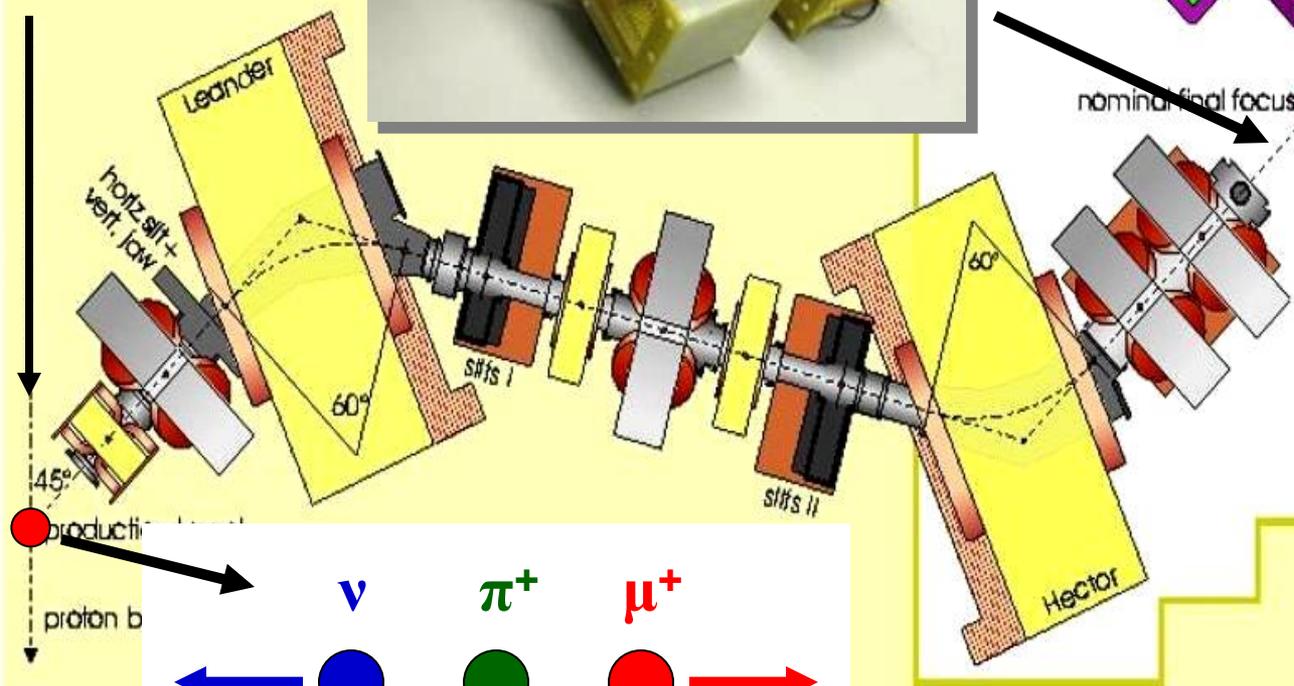
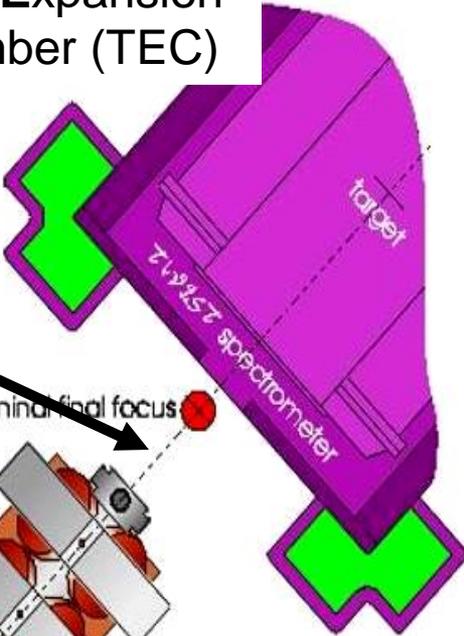
to within a **few parts in 10^4**

Surface muon beam

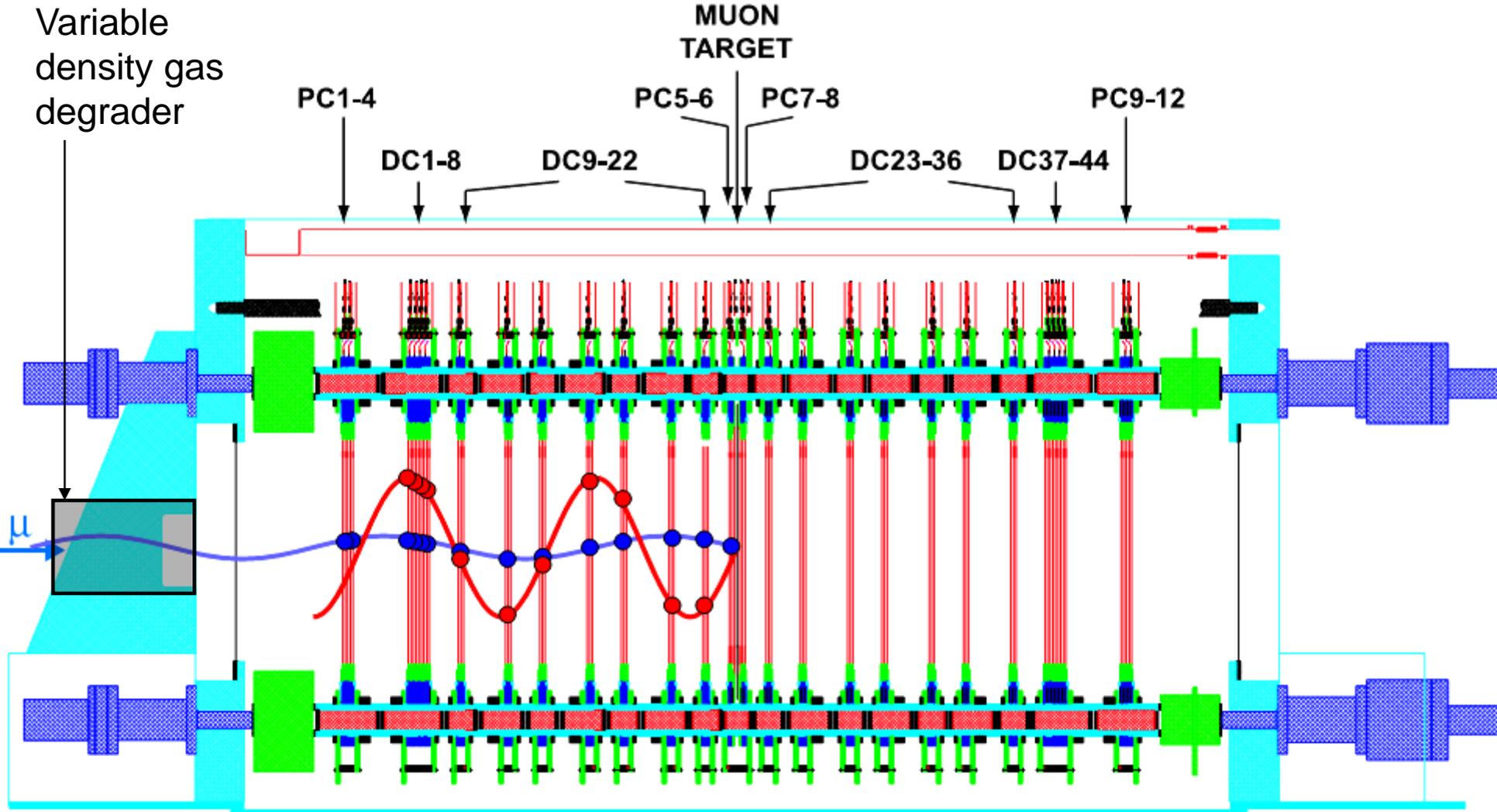
500 MeV
proton beam



Time Expansion
Chamber (TEC)

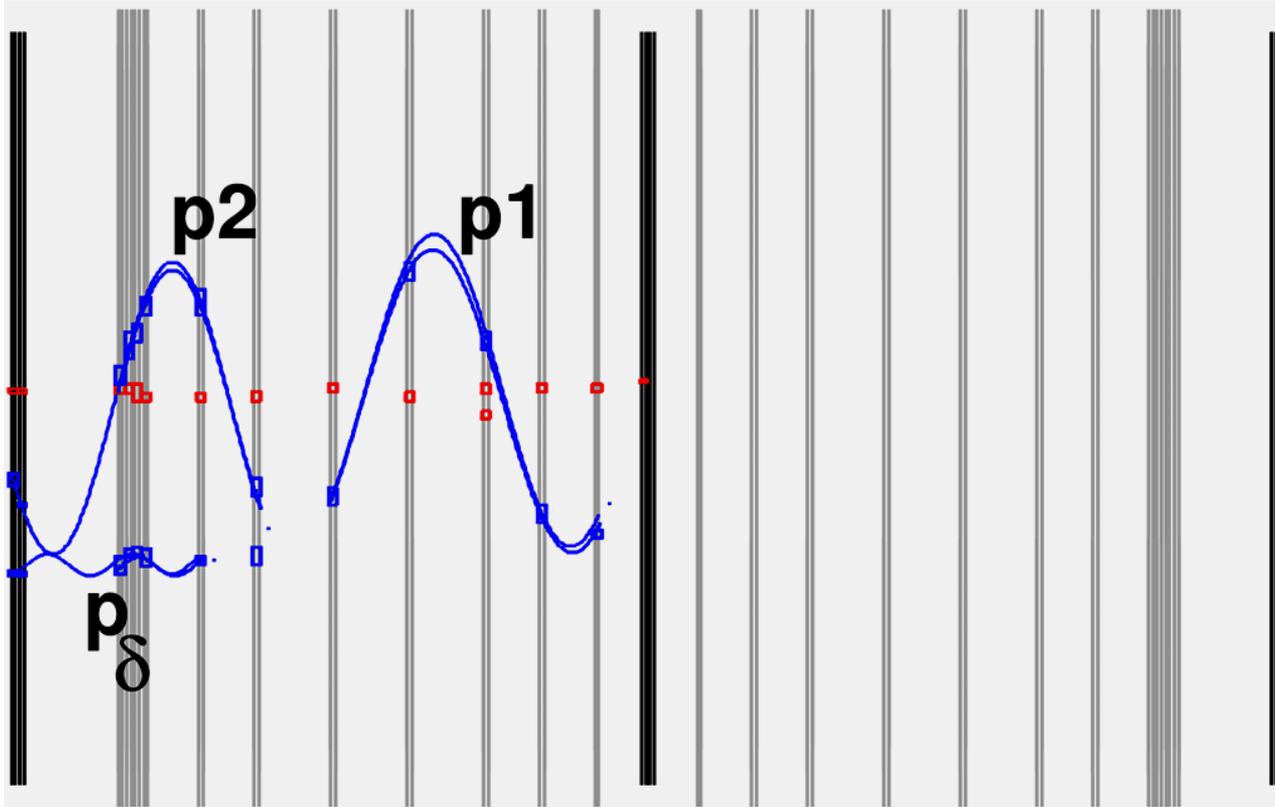


Detector array



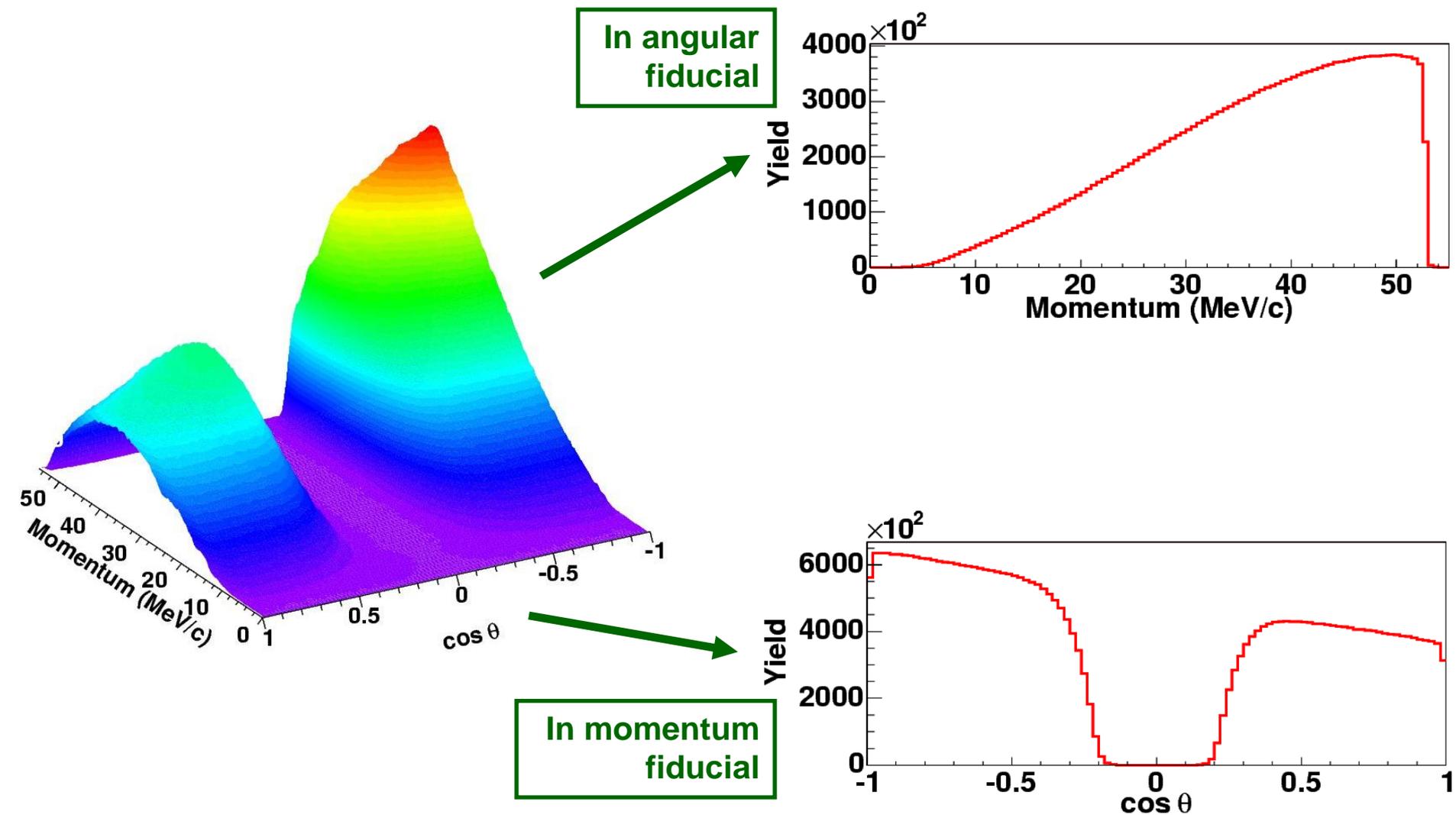
Event topologies

- Many events are simple
 - A muon enters and stops
 - The decay positron leaves
- Other events **aren't simple**



- We must be able to handle all kinds

2-d momentum-angle spectrum



Acceptance of the **TWIST** spectrometer

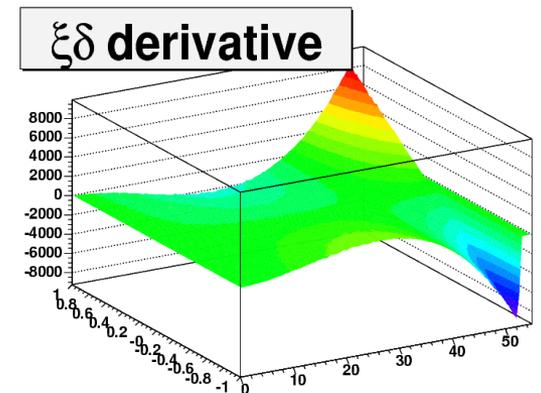
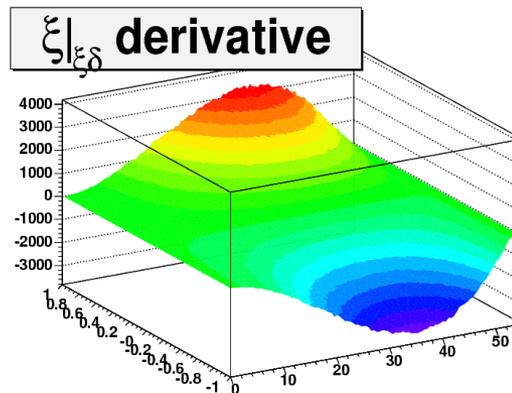
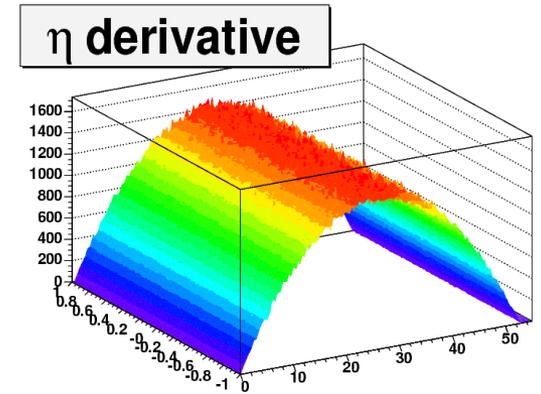
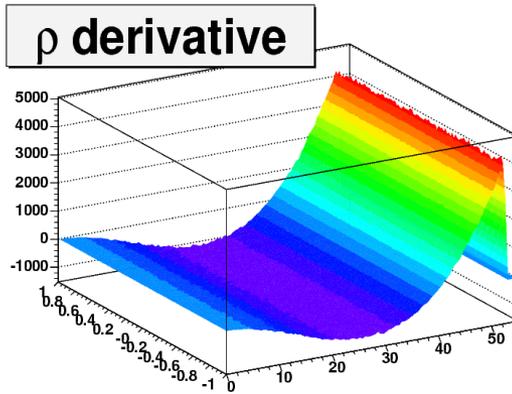
Fitting the data distributions

α_{MC} hidden
 → blind analysis

$$n_i(\alpha_{\text{data}}) = n_i(\alpha_{MC}) + \frac{\partial n_i}{\partial \alpha} \Delta \alpha,$$

$$\alpha = [\rho, \eta, \xi, \xi\delta]$$

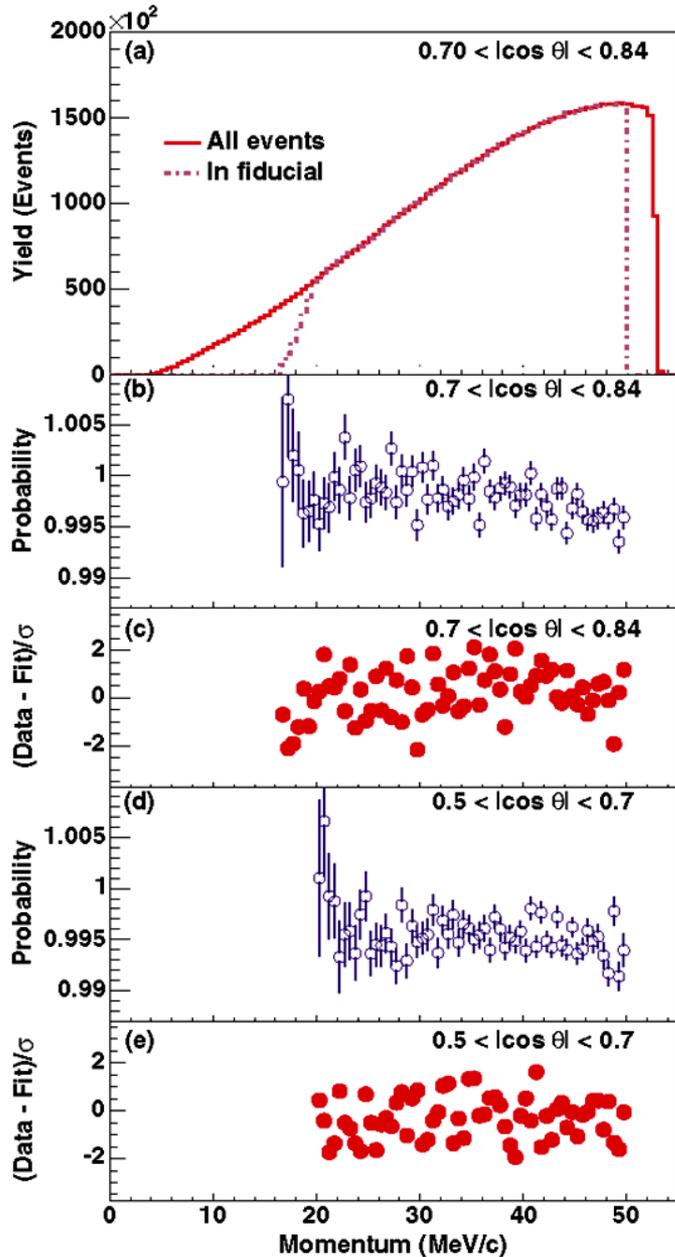
- Fit data to sum of a MC base spectrum plus MC-generated derivative distributions.
- Decay distribution is linear in the muon decay parameters, so this is exact, no matter what values (α_{MC}) are used in the MC base spectrum.



Physics data sets

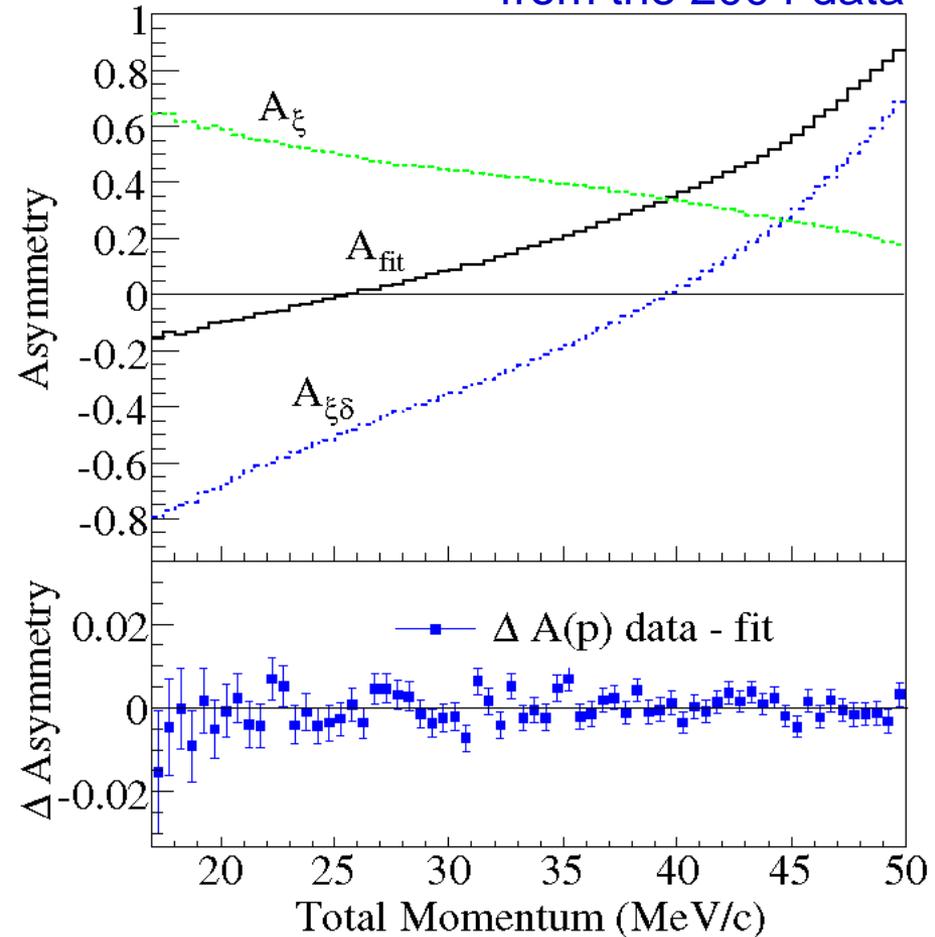
- Fall 2002
 - Test data-taking procedures and develop analysis techniques
 - First physics results – ρ and δ
 - Graphite-coated Mylar target not suitable for $P_{\mu}\xi$
- Fall 2004
 - Aluminum target and Time Expansion Chamber enabled first $P_{\mu}\xi$ measurement
 - Improved determinations of ρ and δ
- 2006-07
 - Both silver (2006) and aluminum (2007) targets
 - Ultimate **TWIST** precision for ρ , δ , and $P_{\mu}\xi$
 - Also measured negative muon decay-in-orbit when bound to Al

Data distributions from 2002 and 2004

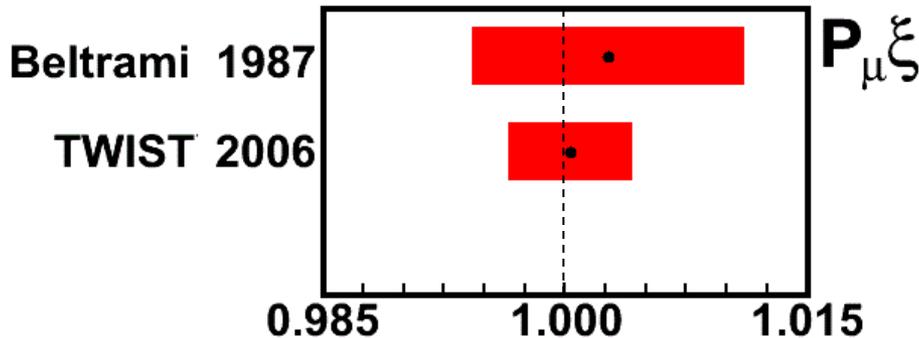
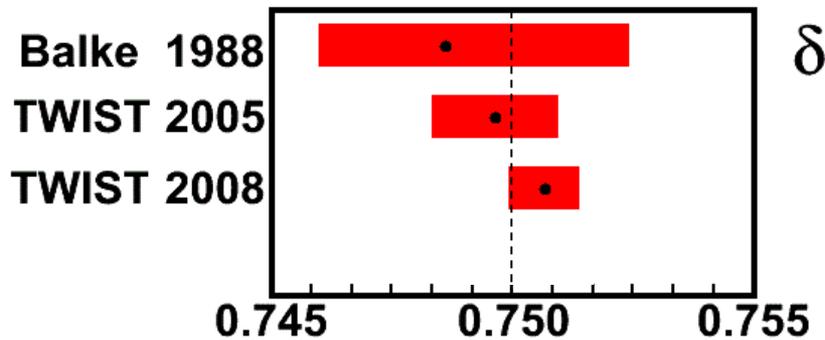
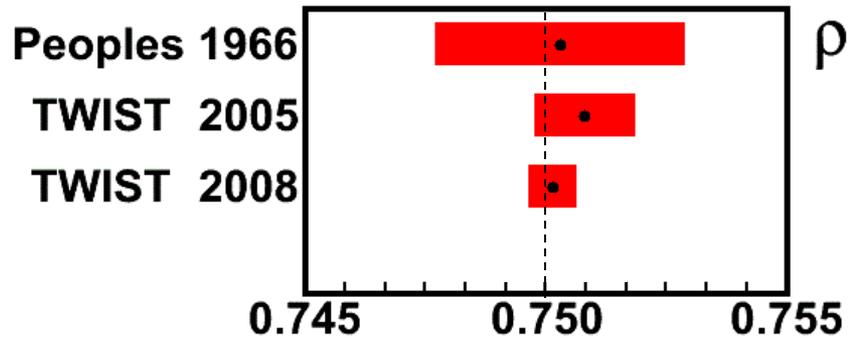


Angle-integrated spectrum from the 2002 data

Asymmetry vs momentum from the 2004 data



TWIST results before now



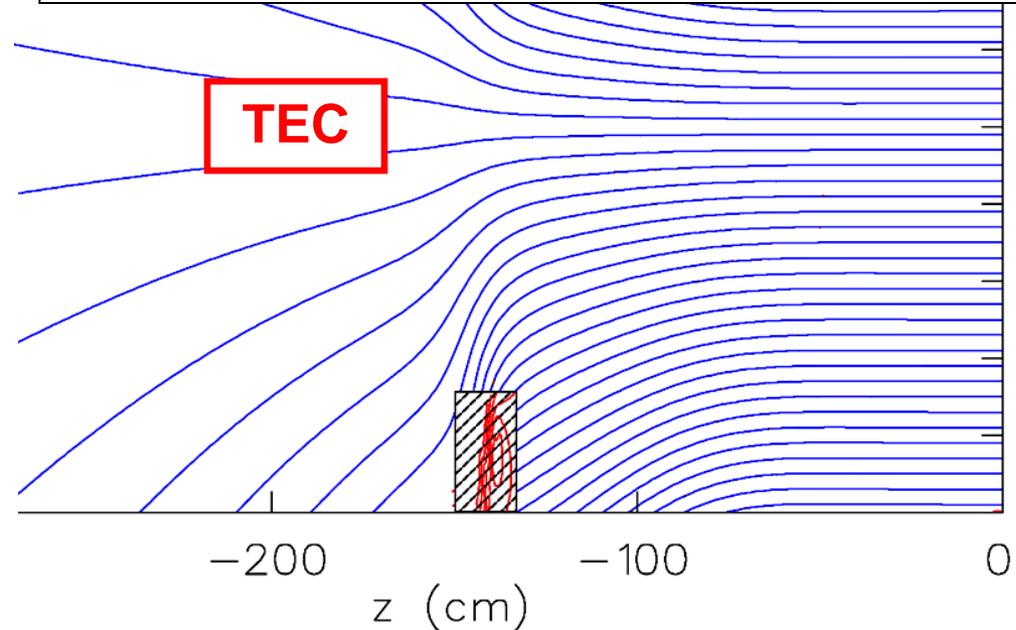
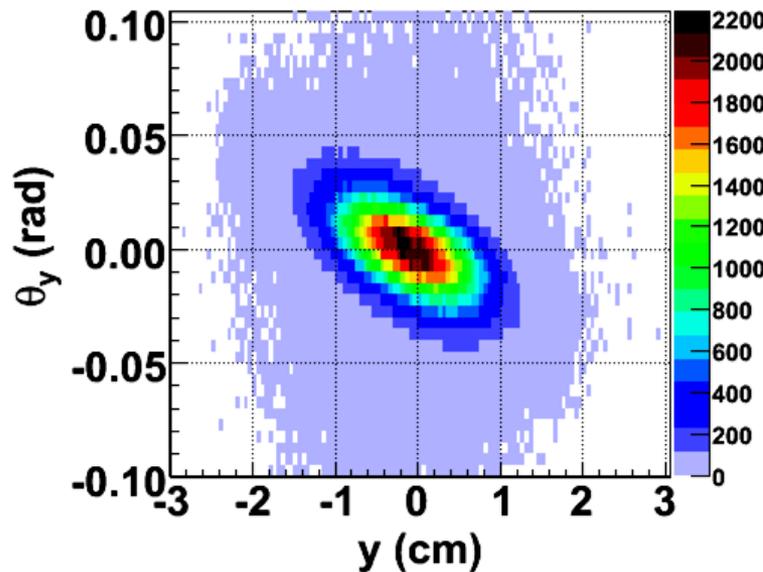
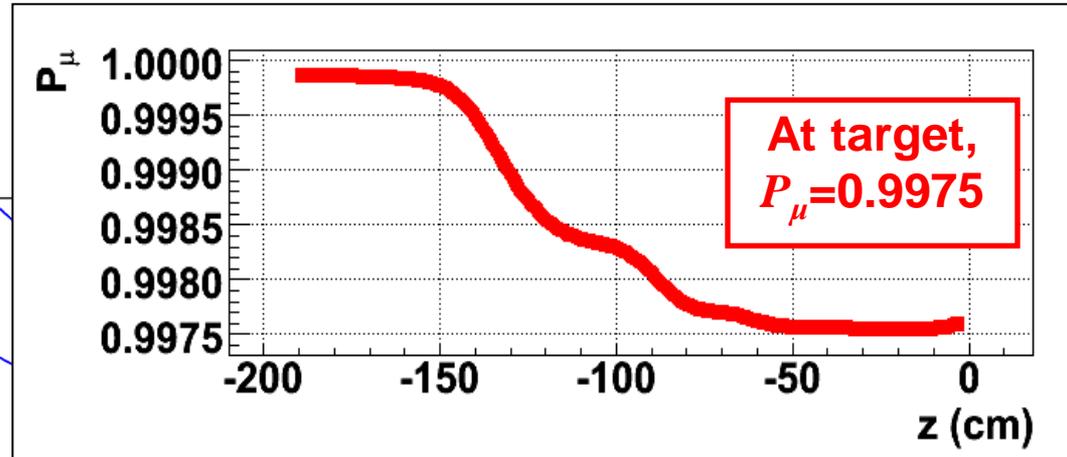
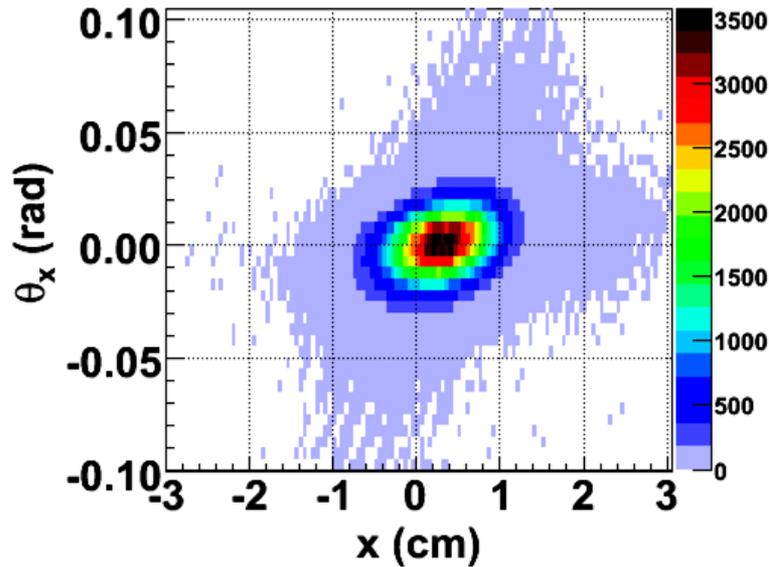
A global analysis that combined the first TWIST ρ and δ results and a concurrent measurement from PSI of the e^+ transverse polarization (PRL 94, 021802) together with all pre-TWIST muon decay parameter measurements found:

$$\eta = -0.0036 \pm 0.0069$$

How to do better?

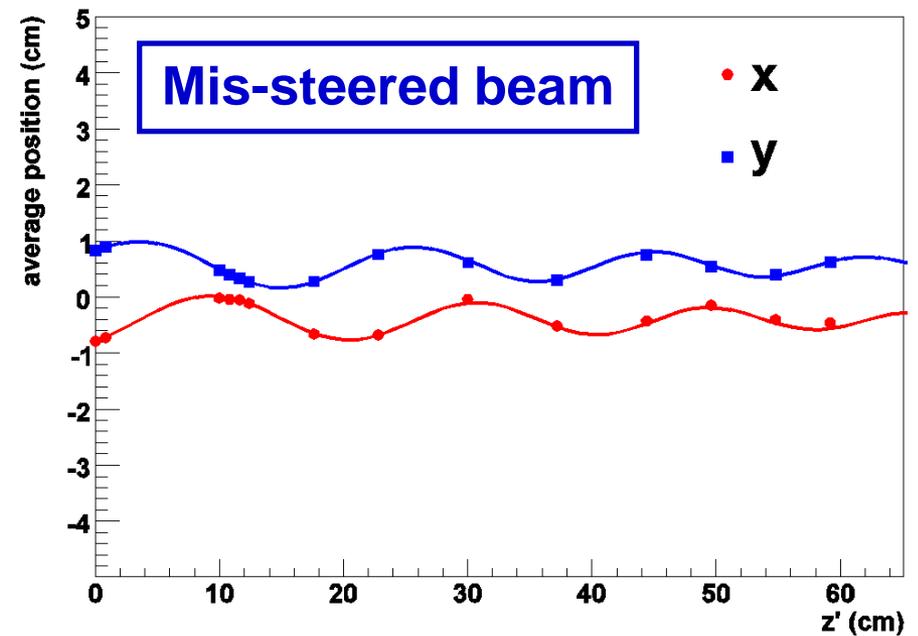
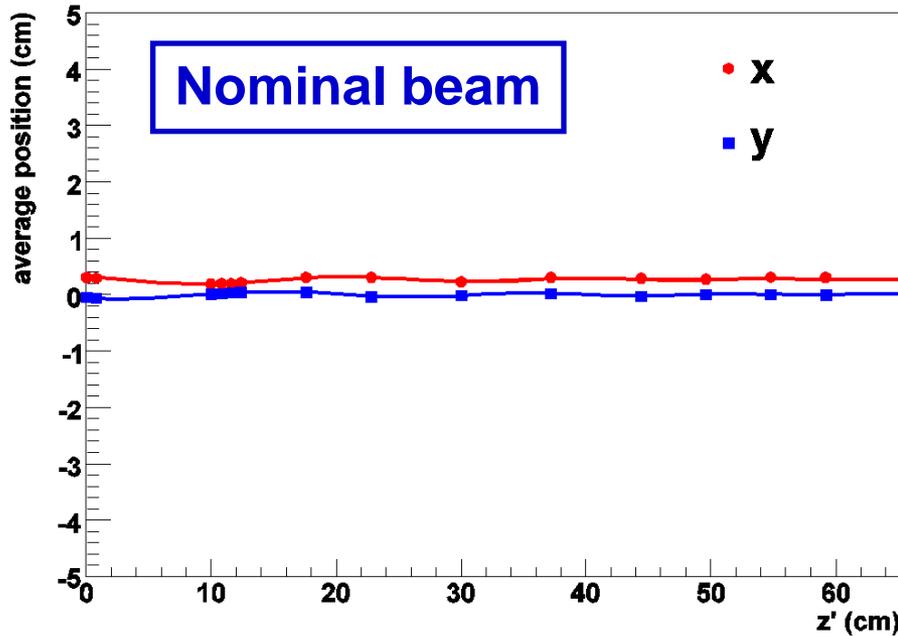
- **TWIST** is a **systematics-dominated experiment**
- Must have:
 - Improved data-taking procedures
 - Better understanding of the detector
 - Improved analysis techniques
- Leading systematics in our previous P_{μ}^{ξ} measurement
 - Muon depolarization while entering the solenoid
 - Time-dependent muon depolarization in the stopping target
- Leading systematics in our previous ρ and δ measurements
 - Chamber response
 - Momentum calibration
 - Positron interactions

Entering the solenoid



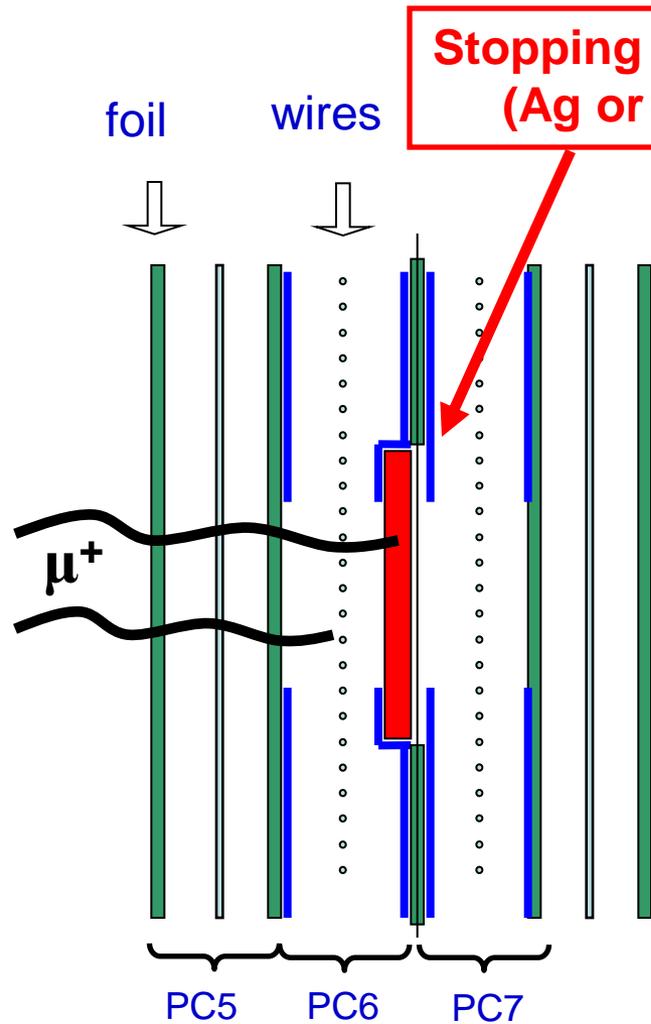
Muons can be depolarized as they cross flux lines when entering the solenoid

Validating the fringe field effects

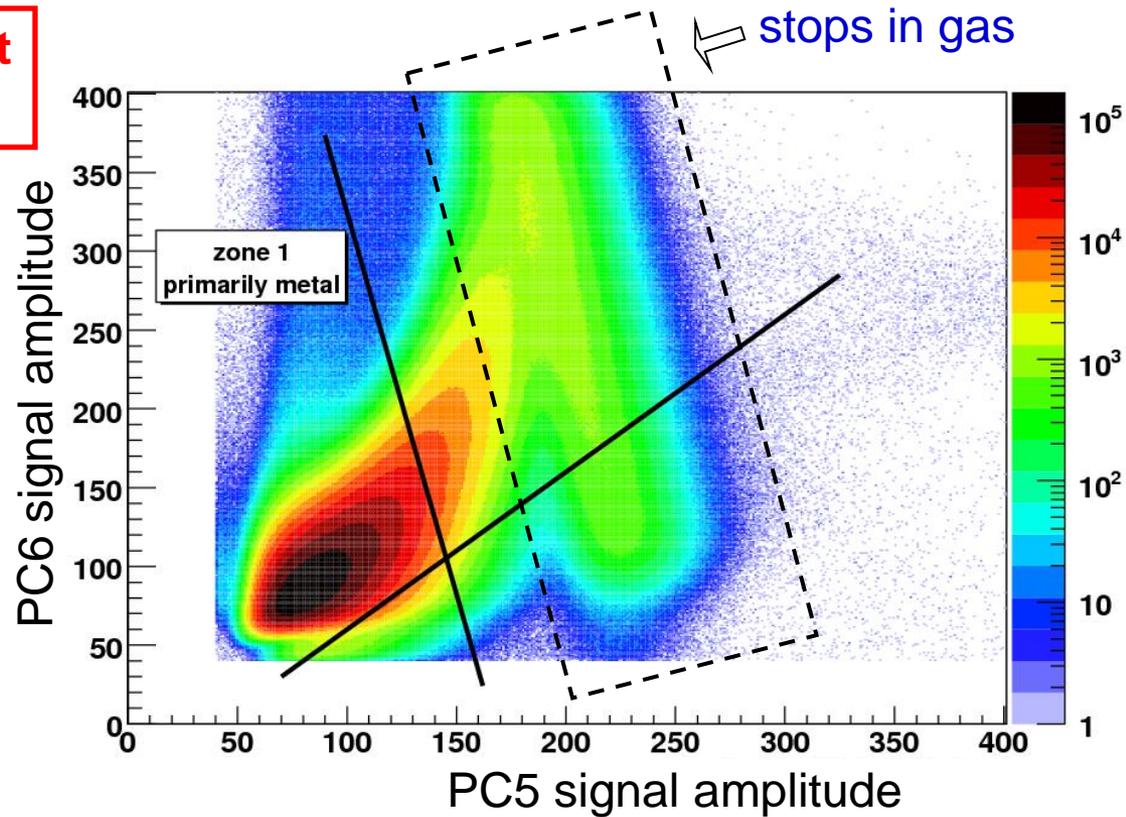


- The average muon beam trajectory inside the detector is **sensitive to the muon transverse momentum**
- Identify changes in muon beam properties between TEC measurements
- Comparisons between nominal and mis-steered beams
 - Observed muon beam trajectories within the detector
 - Difference in the decay asymmetry in data vs that predicted by the Monte Carlo

Ensuring muons stop in the metal target



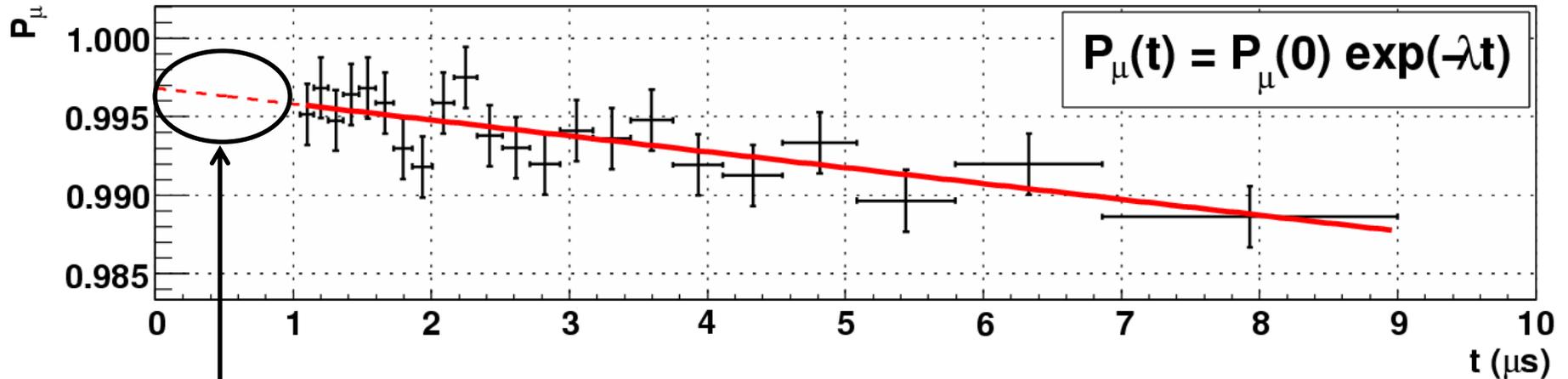
Stopping target
(Ag or Al)



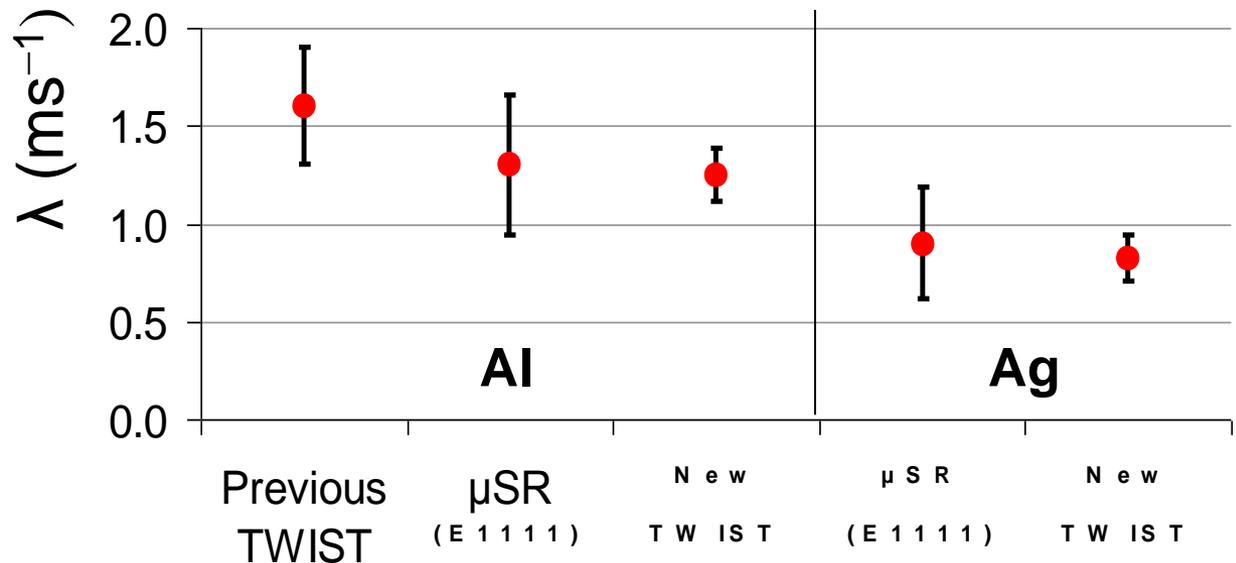
- Muons that stop in gas can depolarize through muonium formation
- Use muon energy depositions near the stopping target to reject those that stop in gas

Measuring depolarization after stopping

High purity (>99.999% purity) Al and Ag targets

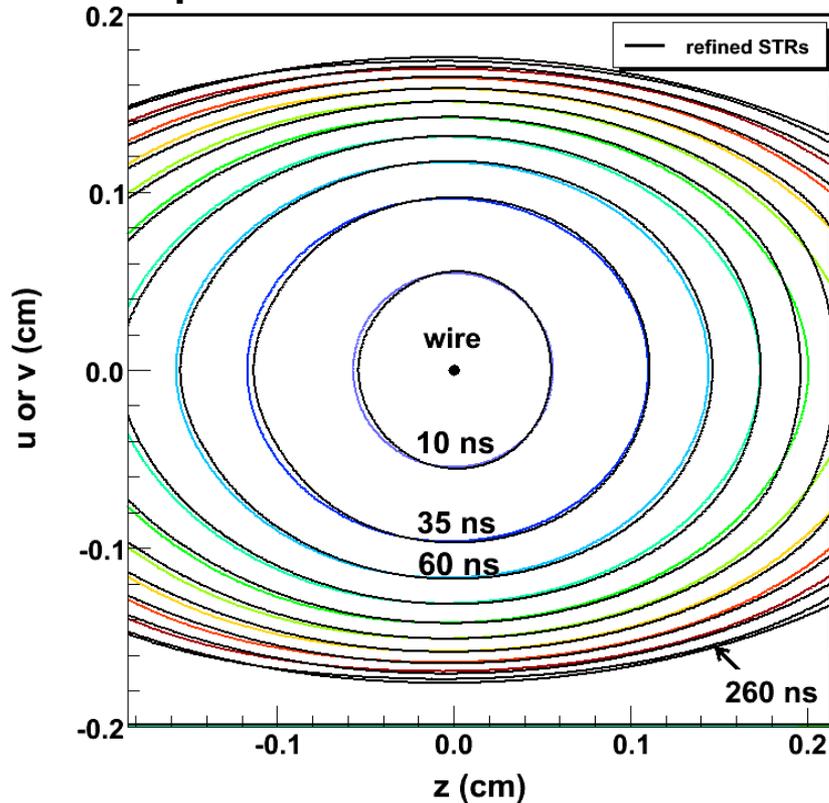


Subsidiary μ^+ SR study:
no “fast depolarisation”
down to 5 ns

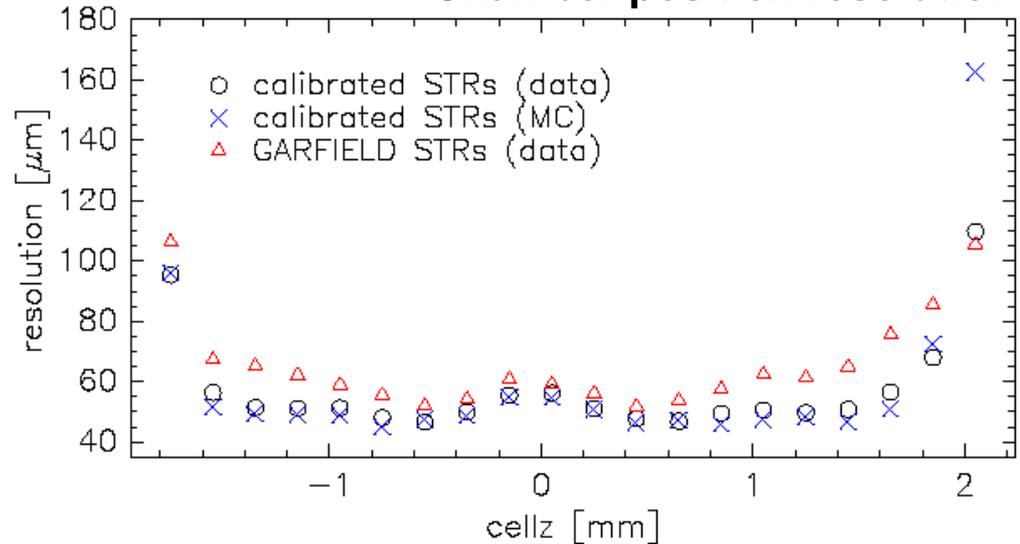


Improved drift chamber calibration

Equal-time contours

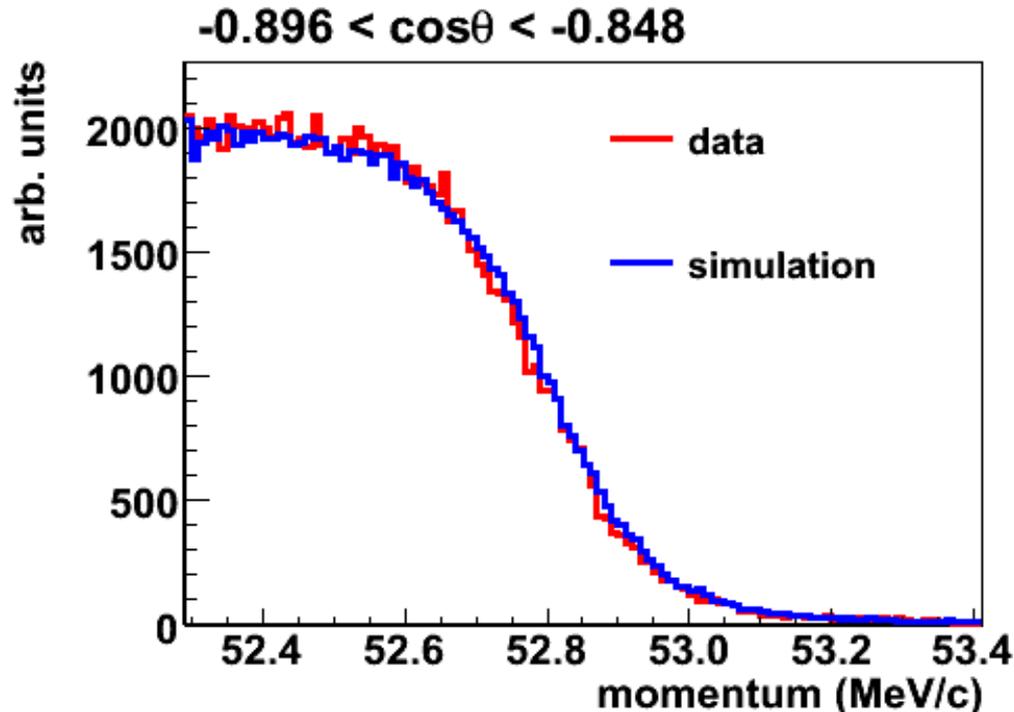


Chamber position resolution



- Direct determination of the effective distance vs time relation
- Accounts for small plane-to-plane fabrication differences
- Compensates for small (few keV) biases in the helix fitter
- Improved momentum resolution (near the endpoint)
 - Was $\sim 69 \text{ keV}/\sin(\theta)$ in Monte Carlo and $\sim 74 \text{ keV}/\sin(\theta)$ in data
 - Now $\sim 58 \text{ keV}/\sin(\theta)$ in both

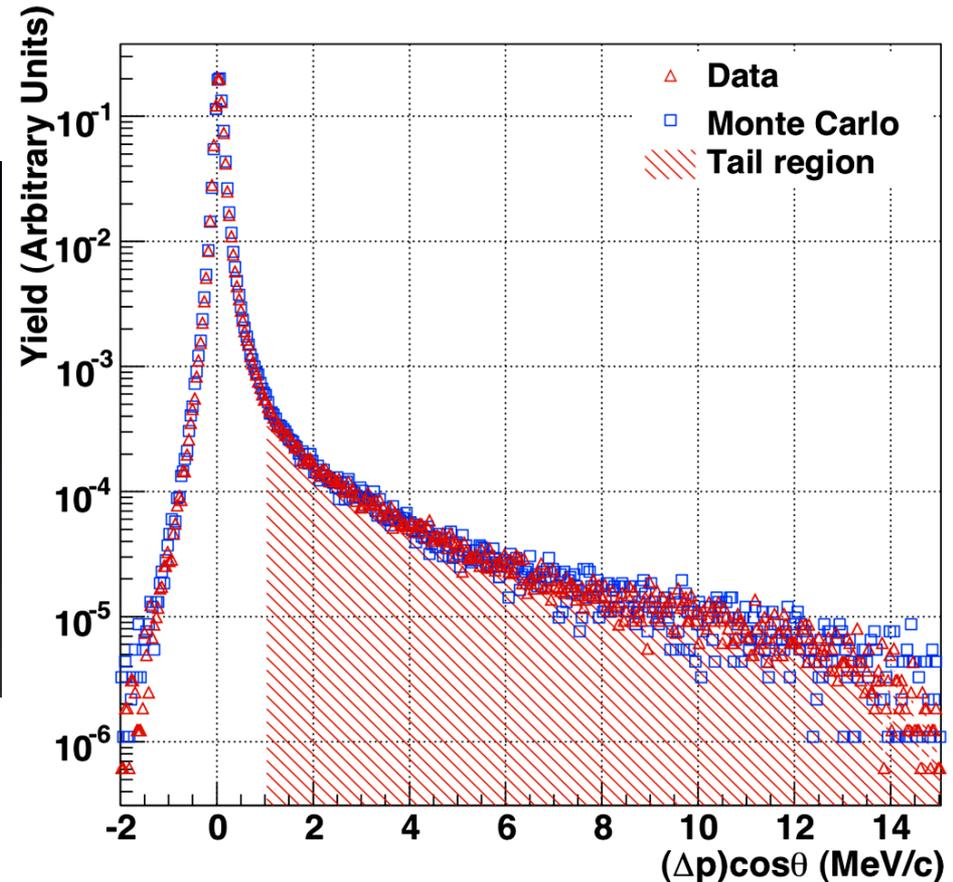
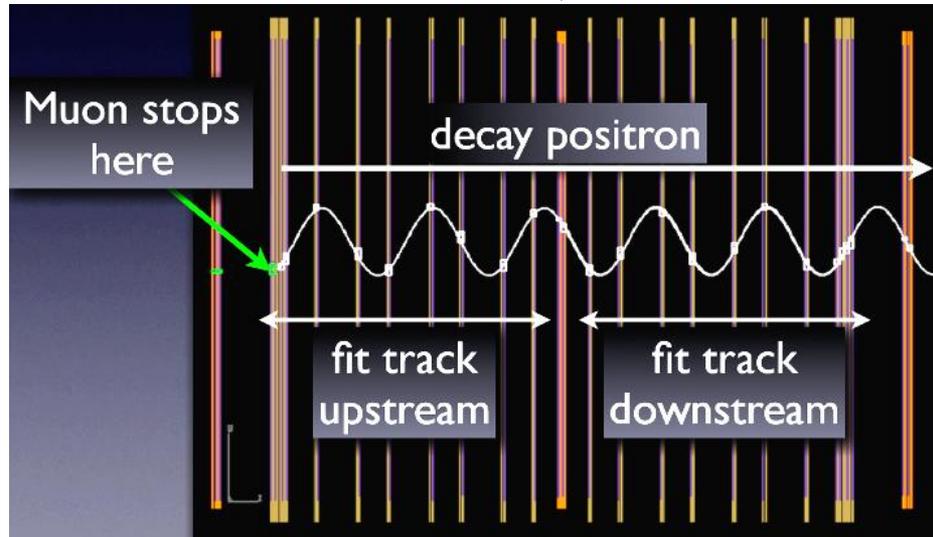
Momentum calibration



- The endpoint at 52.83 MeV/c provides a calibration point
- Improvements in the endpoint fitter reduced previous small biases
- Find a ~ 10 keV/c difference in the **absolute calibration** of the data vs Monte Carlo
- Propagate this through the entire spectrum twice
 - Shift by a constant
 - Rescale the momentum axis

Bremsstrahlung

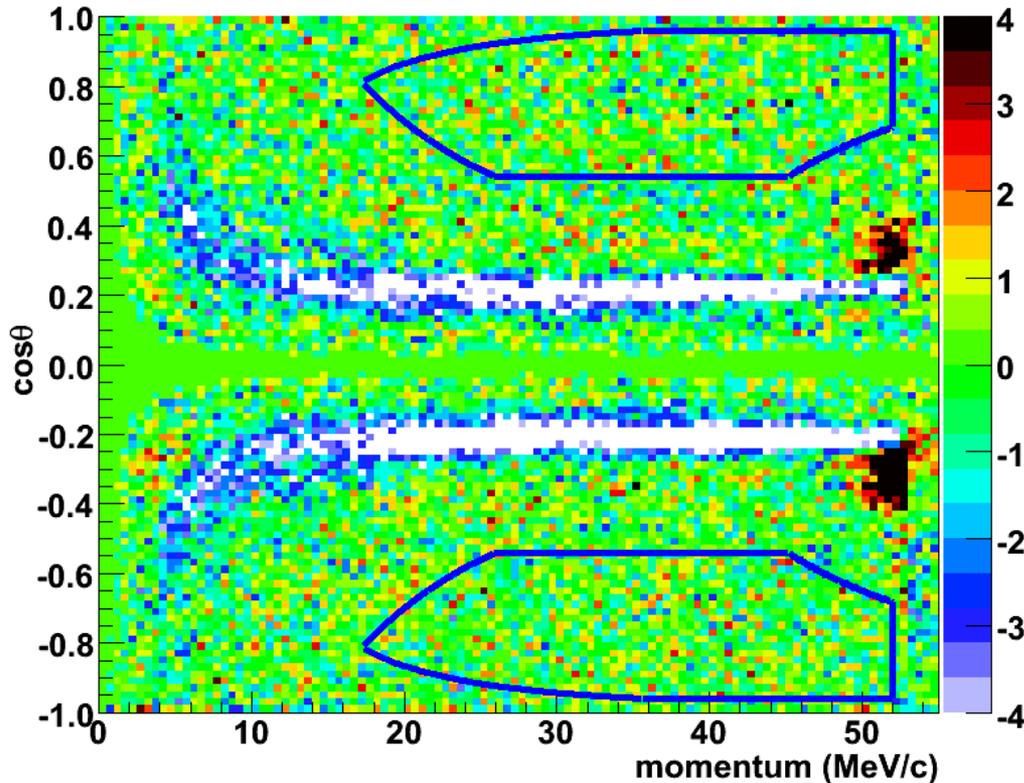
Normal muon
stopping target



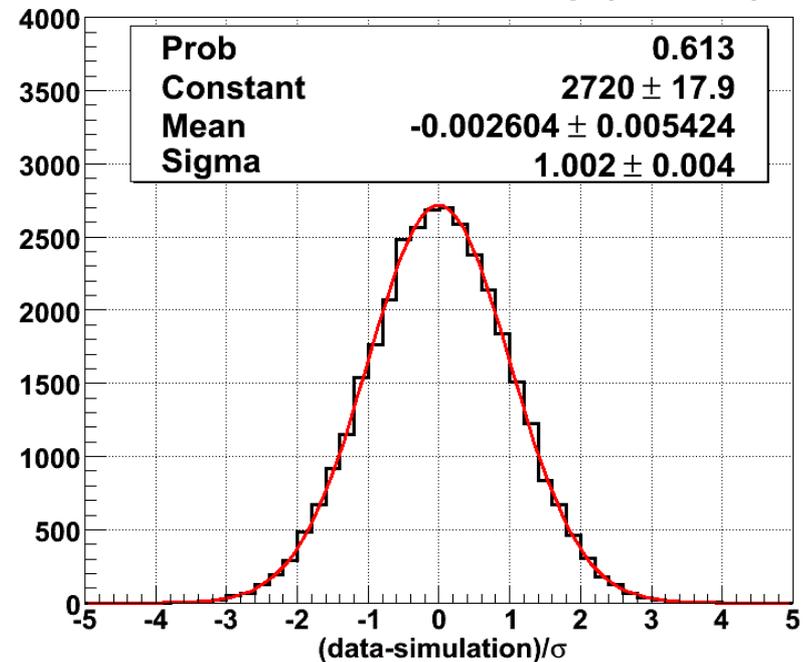
- Leading systematic for ρ and δ
- Two separate measurements
 - “Upstream stops”
 - “Broken” decay tracks
- Consistent results validate bremsstrahlung simulation in our GEANT3 Monte Carlo at the 2.5% level

Overall quality of data vs Monte Carlo match

Normalised residuals for nominal set (s87)



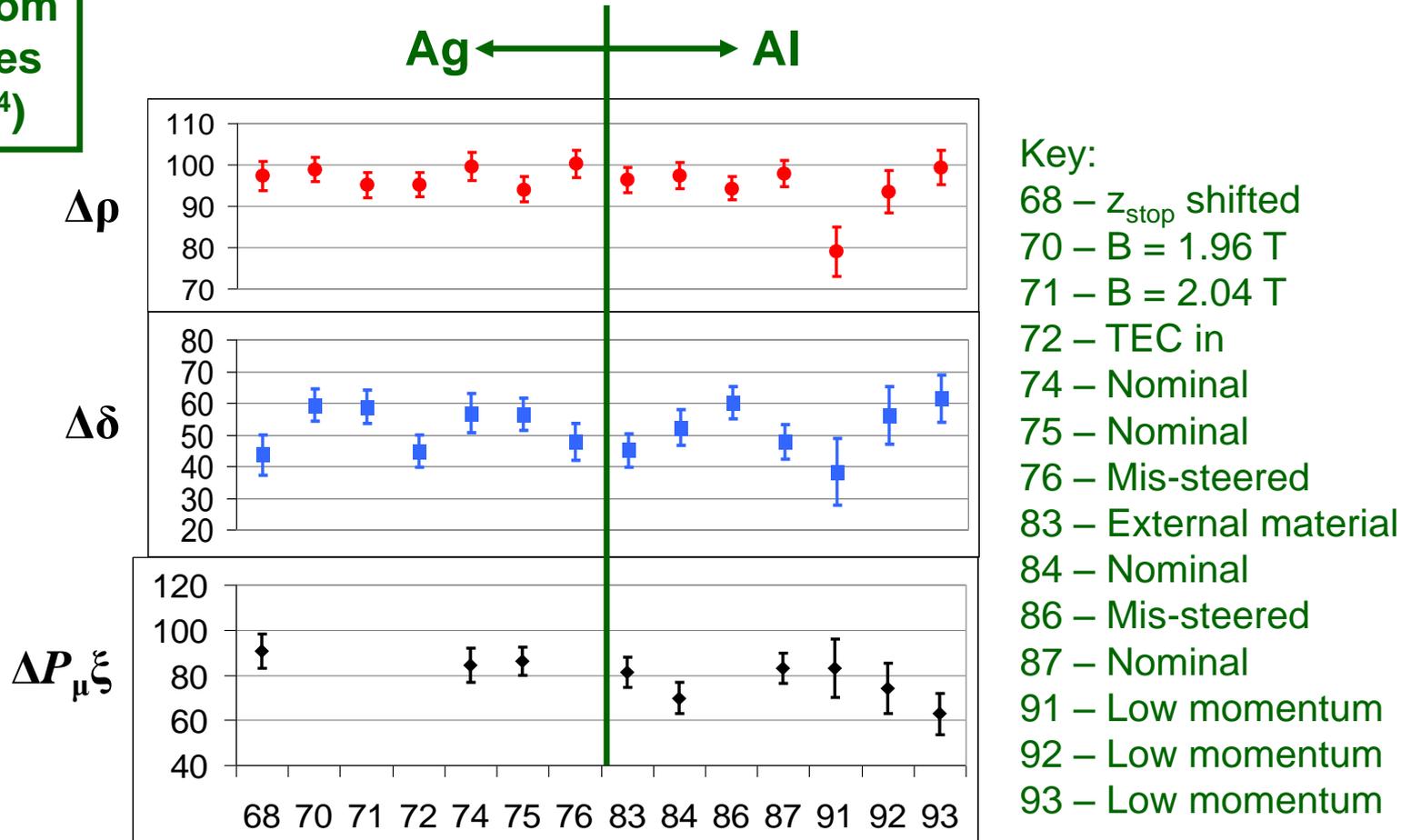
Residuals in fiducial only (all sets)



- Monte Carlo reproduces the data very well, even very far outside the fiducial region

Reproducibility of the results

Difference from hidden values (units = 10^{-4})



- 14 separate measurements under various experimental conditions
- All 14 data sets are used for ρ and δ , $\chi^2 = 14.0$ and 17.7
- 9 of the data sets are used for $P_{\mu\xi}$, $\chi^2 = 9.6$

Total uncertainty budget – ρ and δ

Comparison to 2004
leading systematics

Uncertainties	ρ ($\times 10^{-4}$)	δ ($\times 10^{-4}$)
Positron interactions	1.8	1.6
External uncertainties	1.3	0.6
Momentum calibration	1.2	1.2
Chamber response	1.0	1.8
Resolution	0.6	0.7
Spectrometer alignment	0.2	0.3
Beam stability	0.2	0.0
<i>Systematics added in quadrature</i>	2.8	2.9
Statistical uncertainty	0.9	1.6
<i>Total uncertainty</i>	3.0	3.3

Similar magnitude

Down factor of ~3

Down factor of ~3

Down factor of ~2

Total uncertainty budget – $P_{\mu\xi}$

Comparison to 2004
leading systematics

Uncertainties	$P_{\mu\xi} (\times 10^{-4})$
Depolarization in fringe field	+15.8, -4.0
Depolarization in stopping material	3.2
Background muons	1.0
Depolarization in production target	0.3
Chamber response	2.3
Resolution	1.5
Momentum calibration	1.5
External uncertainties	1.2
Positron interactions	0.7
Beam stability	0.3
Spectrometer alignment	0.2
<i>Systematics added in quadrature</i>	+16.5, -6.2
Statistical uncertainty	3.5
<i>Total uncertainty</i>	+16.9, -7.2

Down factor of ~3

Down factor of ~4

Down factor of ~3

Final TWIST results

- $\rho = 0.74991 \pm 0.00009$ (stat) ± 0.00028 (syst)
- $\delta = 0.75072 \pm 0.00016$ (stat) ± 0.00029 (syst)
- $P_{\mu\xi} = 1.00084 \pm 0.00035$ (stat) $\begin{matrix} + 0.00165 \\ - 0.00063 \end{matrix}$ (syst)
- Correlations:
 - $\text{Corr}(\rho, \delta) = +0.69$
 - $\text{Corr}(\rho, P_{\mu\xi}) = -0.06$ (for $P_{\mu\xi}$ high) and -0.14 (for $P_{\mu\xi}$ low)
 - $\text{Corr}(\delta, P_{\mu\xi}) = -0.18$ (for $P_{\mu\xi}$ high) and -0.43 (for $P_{\mu\xi}$ low)
- Can combine the above to find $P_{\mu\xi}\delta/\rho = 1.00192 \begin{matrix} + 0.00167 \\ - 0.00066 \end{matrix}$
 - This is asymmetry at endpoint \rightarrow must be ≤ 1
 - This 2.9σ surprise is currently under investigation

Implications for the muon decay matrix element

- The final **TWIST** results have been included in a new muon decay global analysis together with all previous muon decay parameter measurements
- Find significantly tighter 90% c.l. upper limits on the coupling of right-handed muons to right- or left-handed electrons:
 - $|g_{RR}^S| < 0.031$
 - $|g_{RR}^V| < 0.015$
 - $|g_{LR}^S| < 0.041$
 - $|g_{LR}^V| < 0.018$
 - $|g_{LR}^T| < 0.012$

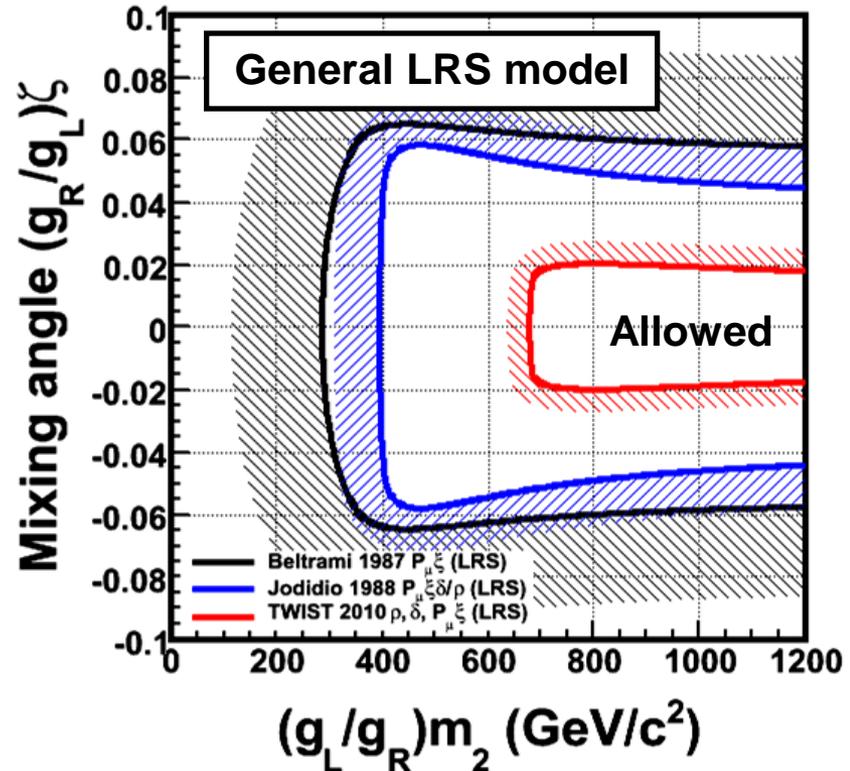
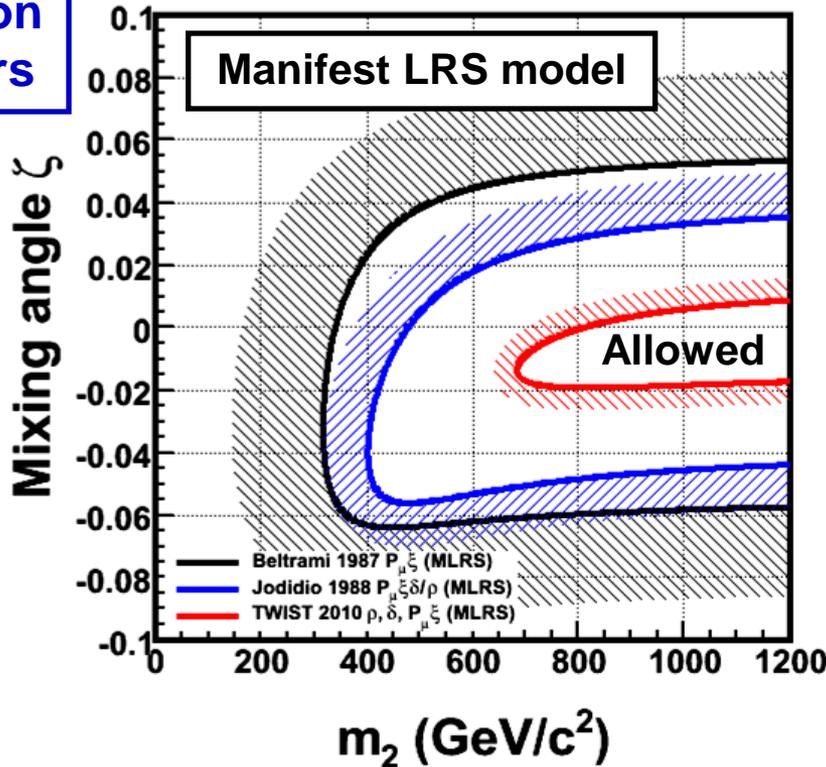
Factor of ~2 smaller than pre-**TWIST** values

Factor of ~3 smaller than pre-**TWIST** values
- New limit on right-handed muon couplings: $Q_R^\mu < 5.8 \times 10^{-4}$ (90% c.l.)
 - Factor of ~9 smaller than pre-**TWIST** value
- Uncertainty for η reduced by 1/3 compared to 2005 global analysis
 - $\eta = -0.0033 \pm 0.0046$
 - Important for the determination of G_F

Implications for left-right symmetric models

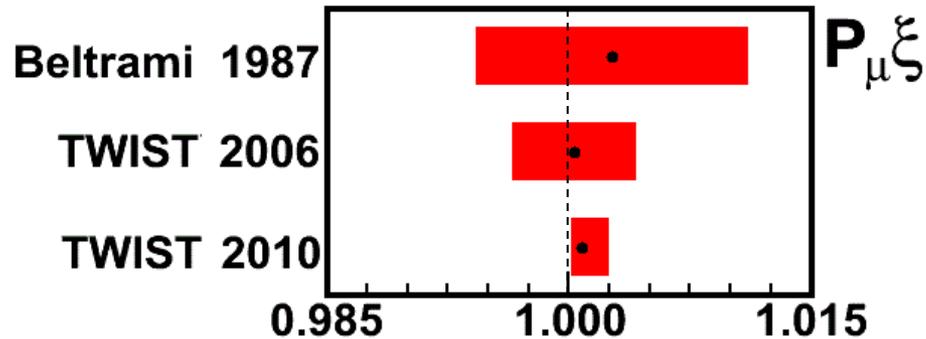
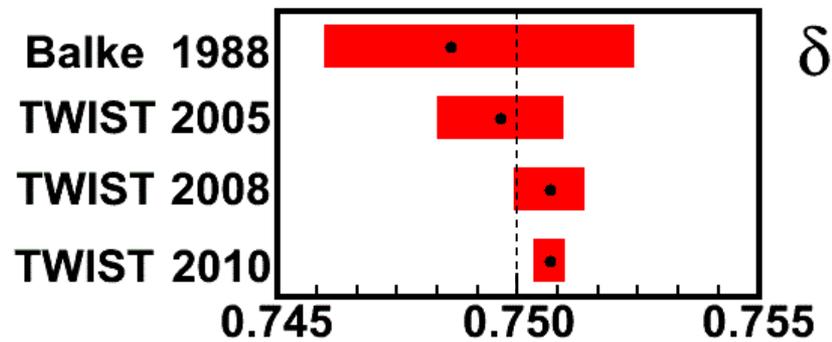
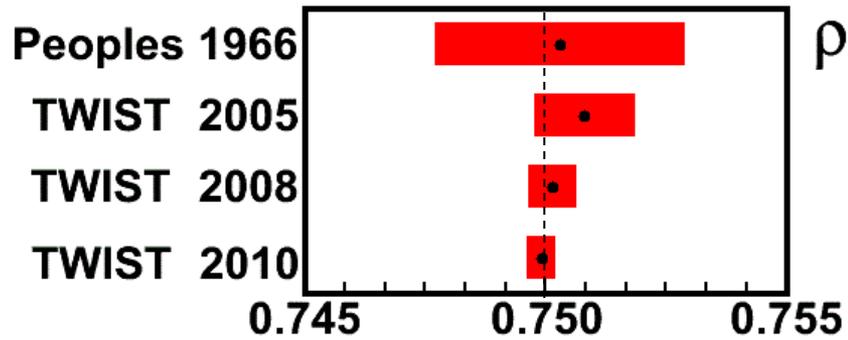
$$W_L = W_1 \cos \zeta + W_2 \sin \zeta \quad W_R = e^{i\omega} (-W_1 \sin \zeta + W_2 \cos \zeta)$$

90% c.l.
exclusion
contours



- Significantly improved limits on both W_R mass and L - R mixing angle ζ
- Many **other limits** also apply if V_{ud}^R is near 1; for example:
 - Direct production at the Tevatron finds $M(W_R) > 1$ TeV
 - $0^+ \rightarrow 0^+$ nuclear beta decay finds $|\zeta| < 0.0005$
- **TWIST** limits in the general case make **no assumption about V_{ud}^R**

Conclusions



The **TWIST** Collaboration (<http://twist.triumf.ca>)

TRIUMF

Ryan Bayes*†
Yuri Davydov
Wayne Faszler
Makoto Fujiwara
David Gill
Alexander Grossheim
Peter Gumplinger
Anthony Hillairet*†
Robert Henderson

Jingliang Hu

Glen Marshall
Dick Mischke
Konstantin Olchanski
Art Olin†
Robert Openshaw
Jean-Michel Poutissou
Renée Poutissou
Grant Sheffer
Bill Shin‡‡

Alberta

Andrei Gaponenko*
Robert MacDonald*

British Columbia

James Bueno*
Mike Hasinoff

Texas A&M

Carl Gagliardi
Bob Tribble

Regina

Ted Mathie
Roman Tacik

Kurchatov Institute

Vladimir Selivanov

Montréal

Pierre Depommier

Valparaiso

Don Koetke
Shirvel Stanislaus

* = graduate student

† = also UVIC

‡‡ = also Saskatchewan



Supported by NSERC, the National Research Council of Canada, the Russian Ministry of Science, and the US Department of Energy. Computing resources provided by WestGrid.

Coupling constants and Michel parameters

- The Michel parameters are bilinear combinations of the coupling constants:

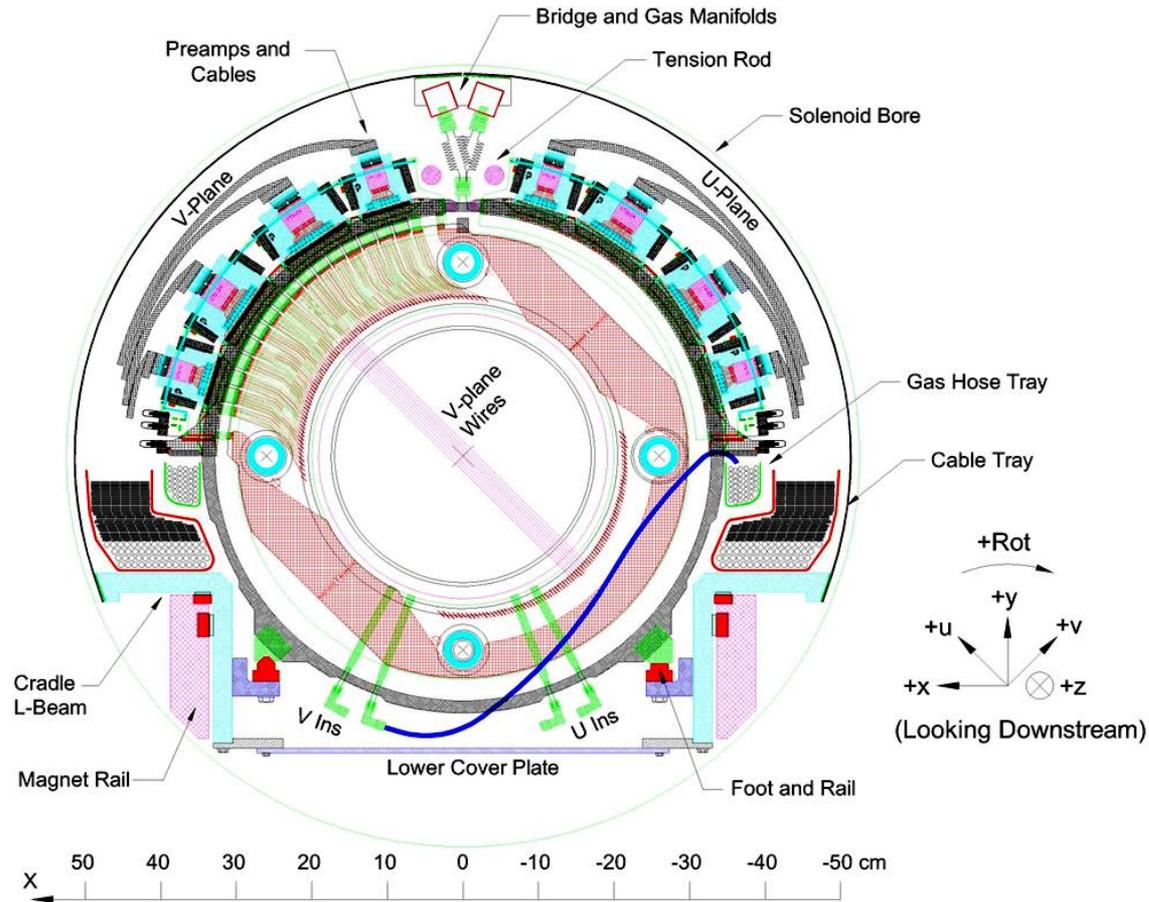
$$\rho = \frac{3}{4} - \frac{3}{4} [|g_{RL}^V|^2 + |g_{LR}^V|^2 + 2 |g_{RL}^T|^2 + 2 |g_{LR}^T|^2 + \text{Re}(g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*})]$$

$$\eta = \frac{1}{2} \text{Re}[g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} + g_{RL}^V (g_{LR}^{S*} + 6g_{LR}^{T*}) + g_{LR}^V (g_{RL}^{S*} + 6g_{RL}^{T*})]$$

$$\xi = 1 - \frac{1}{2} |g_{LR}^S|^2 - \frac{1}{2} |g_{RR}^S|^2 - 4 |g_{RL}^V|^2 + 2 |g_{LR}^V|^2 - 2 |g_{RR}^V|^2 + 2 |g_{LR}^T|^2 - 8 |g_{RL}^T|^2 + 4 \text{Re}(g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*})$$

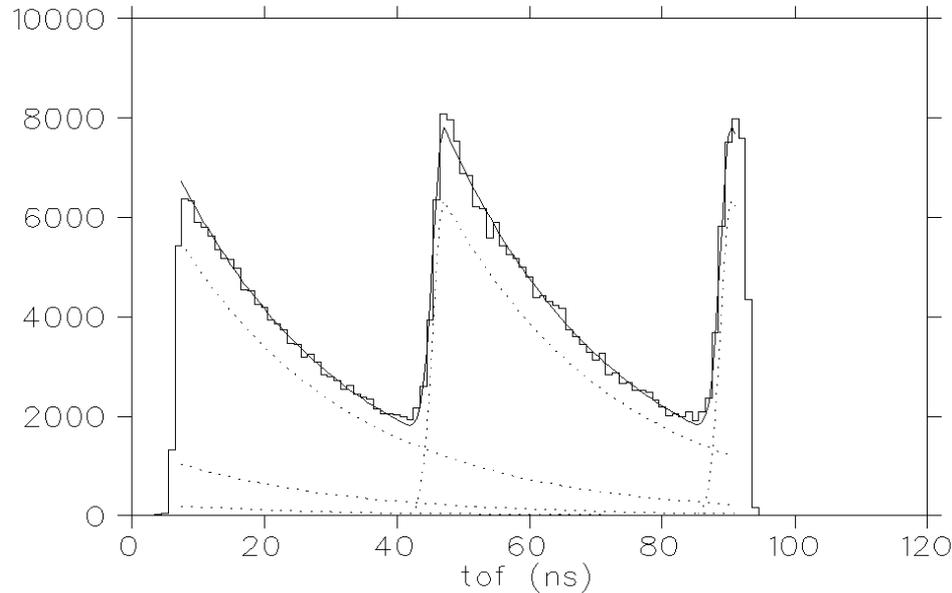
$$\xi\delta = \frac{3}{4} - \frac{3}{8} |g_{RR}^S|^2 - \frac{3}{8} |g_{LR}^S|^2 - \frac{3}{2} |g_{RR}^V|^2 - \frac{3}{4} |g_{RL}^V|^2 - \frac{3}{4} |g_{LR}^V|^2 - \frac{3}{2} |g_{RL}^T|^2 - 3 |g_{LR}^T|^2 + \frac{3}{4} \text{Re}(g_{LR}^S g_{LR}^{T*} - g_{RL}^S g_{RL}^{T*})$$

Precision detector construction



- Very low mass ($\sim 10^{-4} X_0$ per U-V pair), high precision chambers
 - Longitudinal alignment by engineering
 - Transverse alignment using particle tracks
 - >5000 wires, efficiency >99.8%
- } To a few parts in 10^5

“Good muons” vs “bad muons”



- Positive muons can arise from pion decay at rest or in flight
 - At rest (“surface”) muons have large (~ 1) negative polarization
 - In flight (“cloud”) muons have small ($\sim 20\%$) positive polarization
- In TWIST, we (usually) want the former and not the latter
- Achieved by cutting on the muon arrival time at the spectrometer

Previous muon decay parameter results

- From Fall, 2002 run:
 - $\rho = 0.75080 \pm 0.00032$ (stat) ± 0.00097 (syst) ± 0.00023 (η)
PRL 94, 101805
 - $\delta = 0.74964 \pm 0.00066$ (stat) ± 0.00112 (syst)
PRD 71, 071101
- New global analysis (PRD 72, 073002) using the ρ and δ results together with previous measurements, plus recent e^+ transverse polarization measurements from PSI (PRL 94, 021802):
 - Significant improvements in the limits for $g^{S,V,T}_{LR}$
 - $\eta = -0.0036 \pm 0.0069$
- From Fall, 2004 run:
 - $P_{\mu}\xi = 1.0003 \pm 0.0006$ (stat) ± 0.0038 (syst)
PRD 74, 072007
 - $\rho = 0.75014 \pm 0.00017$ (stat) ± 0.00044 (syst) ± 0.00011 (η)
 - $\delta = 0.75067 \pm 0.00030$ (stat) ± 0.00067 (syst)
PRD 78, 032010
- Factors of 2 ($P_{\mu}\xi$) to 5 (ρ, δ) increased precision vs. pre-TWIST

Systematic uncertainties for 2004 data

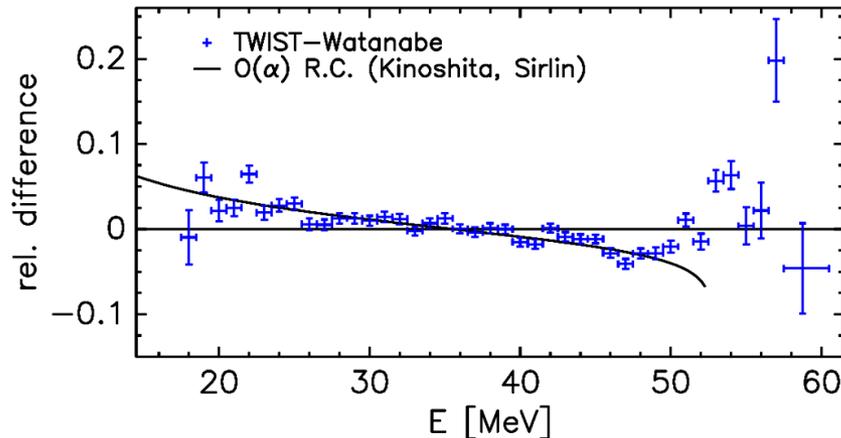
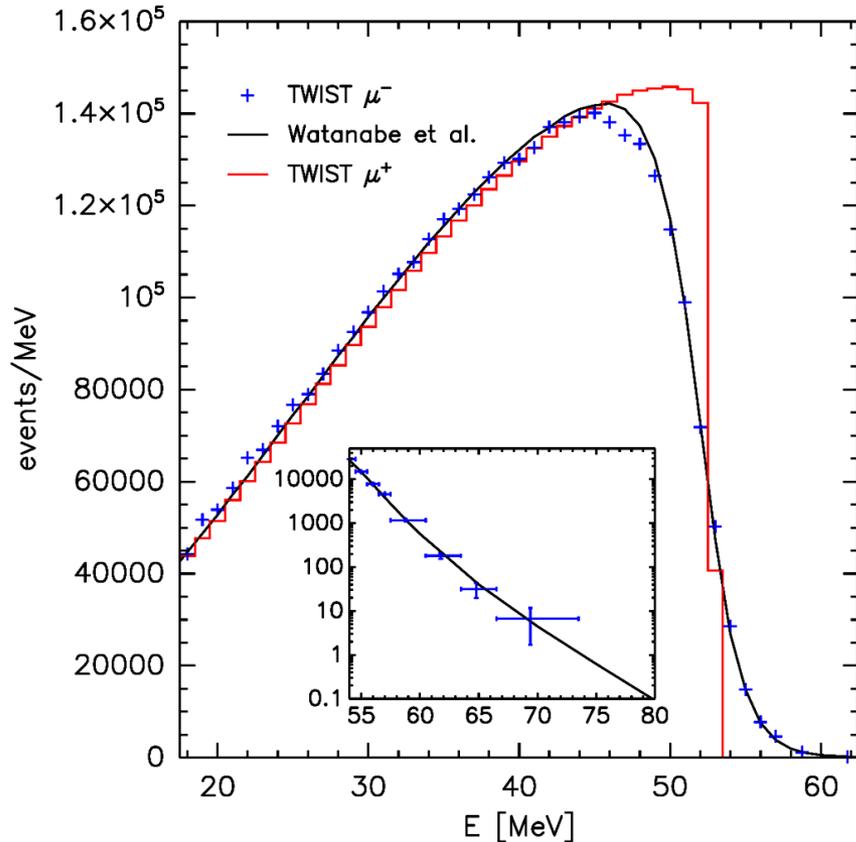
TABLE II. Summary of systematic uncertainties by category.

Category	$\Delta\rho$	$\Delta\delta$
Chamber response	0.000 29	0.000 52
Energy scale	0.000 29	0.000 41
Positron interactions	0.000 16	0.000 09
Resolution	0.000 02	0.000 03
Alignment and lengths	0.000 03	0.000 03
Beam intensity	0.000 01	0.000 02
Correlations with η	0.000 11	0.000 01
Theory	0.000 03	0.000 01
Total	0.000 46	0.000 67

TABLE III. Contributions to the systematic uncertainty for $P_{\mu\xi}^{\pi}$.

Effect	Uncertainty
Depolarization in fringe field (ave)	0.0034
Depolarization in stopping material (ave)	0.0012
Chamber response (ave)	0.0010
Spectrometer alignment	0.0003
Positron interactions (ave)	0.0003
Depolarization in production target	0.0002
Momentum calibration	0.0002
Upstream-downstream efficiency	0.0002
Background muon contamination (ave)	0.0002
Beam intensity (ave)	0.0002
Michel parameter η	0.0001
Theoretical radiative corrections	0.0001

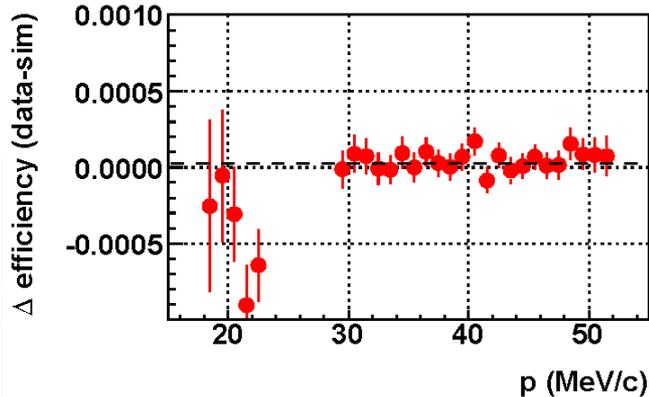
Negative muon decay-in-orbit



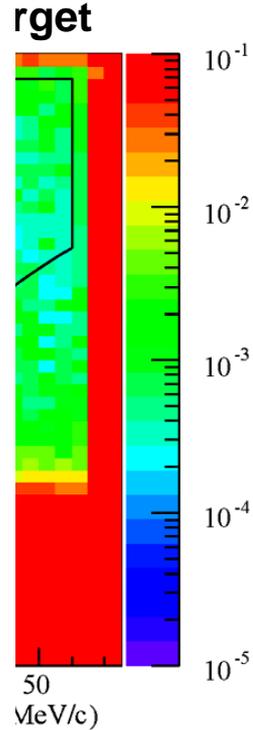
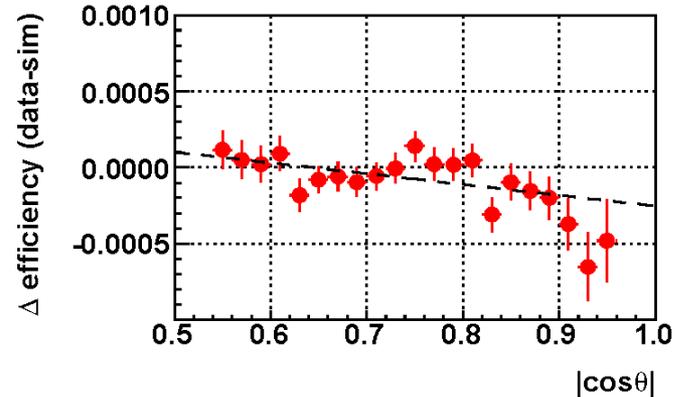
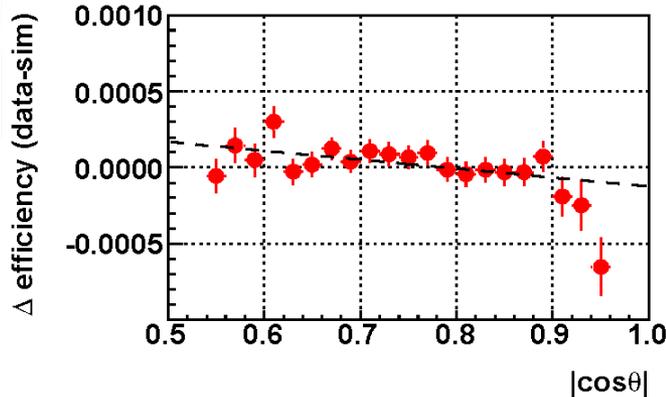
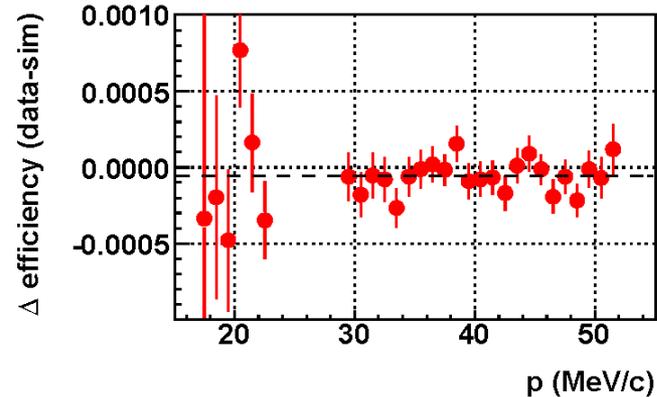
- Future $\mu \rightarrow e$ conversion experiments plan to study negative muons bound to Al
- Most precise measurement ever of the muon decay-in-orbit spectrum
- Theoretical predictions include higher-order contributions from the muon+nucleus potential
- Need to include the $O(\alpha)$ radiative corrections that arise from the interaction between the muon and the outgoing electron

Tracking (in)efficiency

Upstream, silver upstream stops (set73)

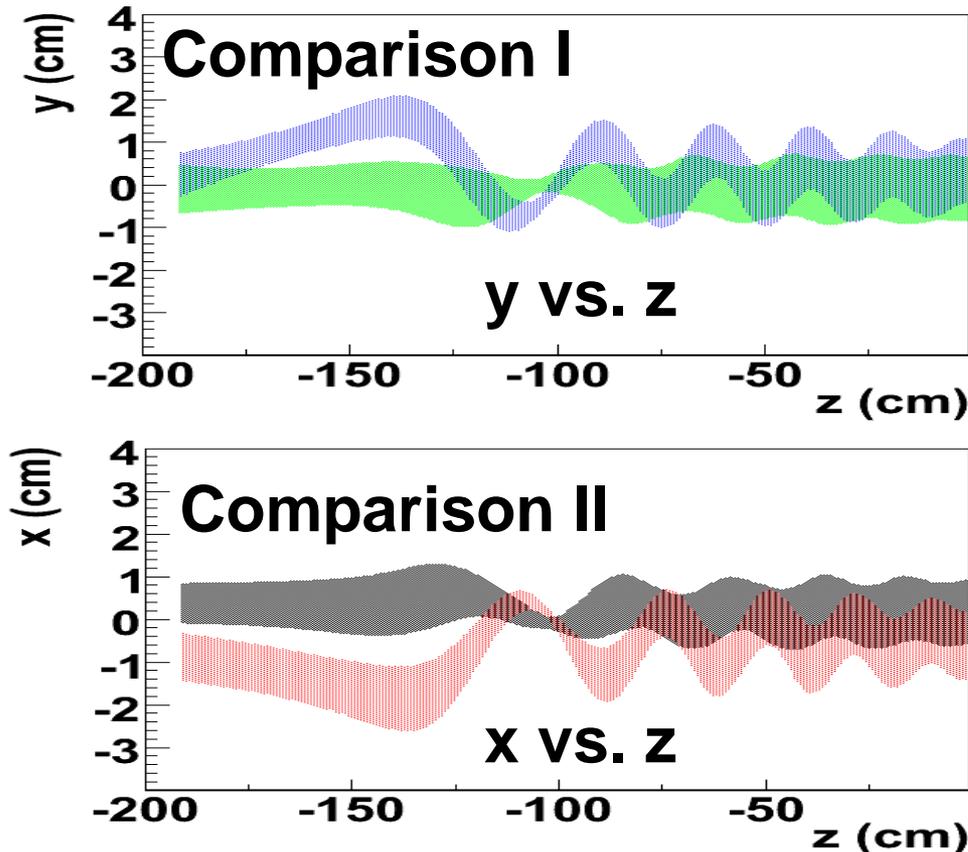


Downstream, silver upstream stops (set73)



- Measure the tracking efficiency with “upstream stops” data
 - Reconstruct a track with a particular $(p, \cos(\theta))$ in one half
 - Do we reconstruct a track in the other half?
- Inefficiency in data is $<0.5\%$ throughout the fiducial region
- Inefficiency in data and Monte Carlo match to $<0.05\%$

Fringe field and mis-steered beam

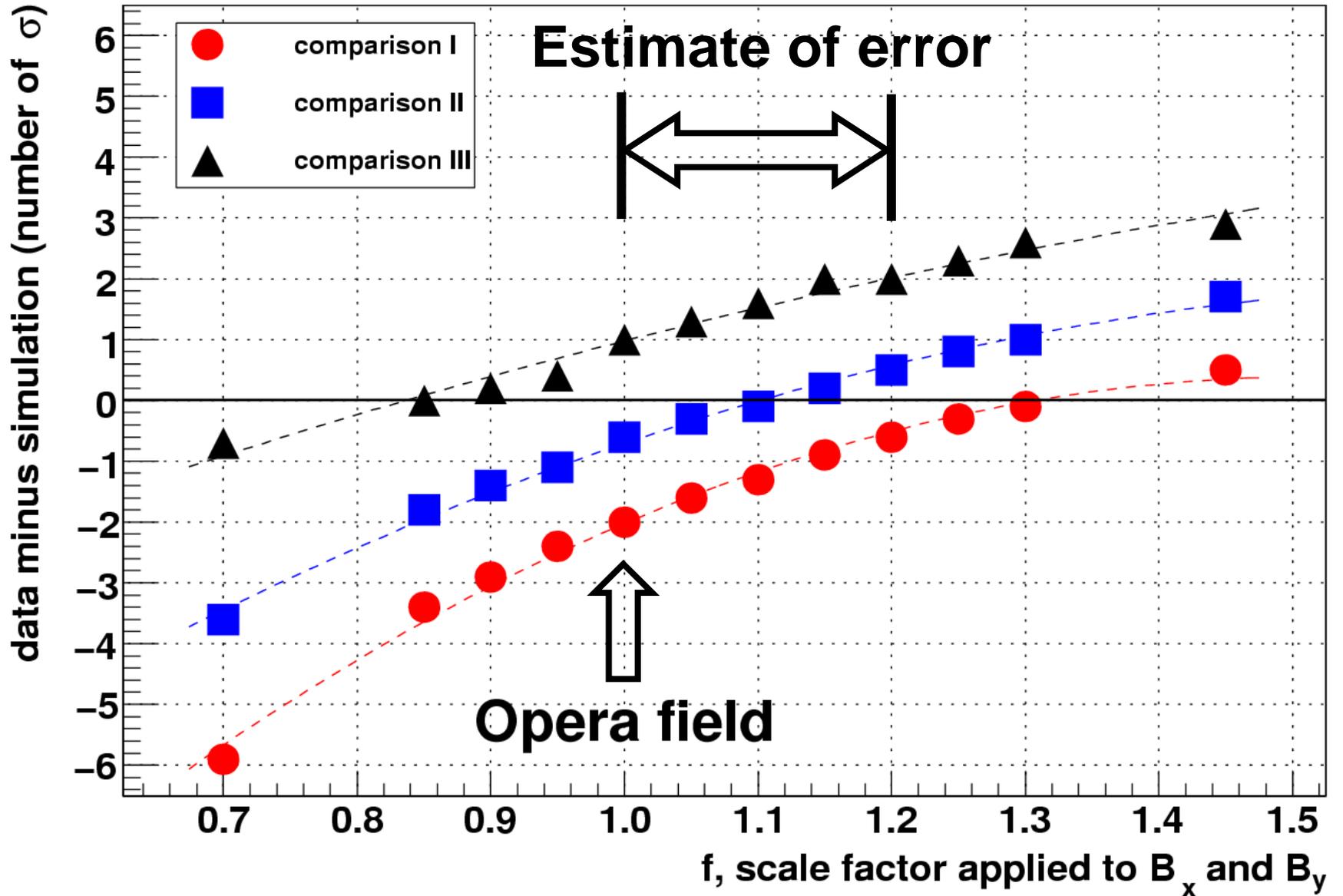


Move beam away from optimum position and/or angle to observe change in polarization:

- Comparison I: mis-steer y direction by 28 mrad
Find: $\Delta P_{\mu} = -105 \pm 9 \times 10^{-4}$
- Comparison II: mis-steer x position by 10 mm and direction by 10 mrad
Find: $\Delta P_{\mu} = -62 \pm 8 \times 10^{-4}$
- Comparison III: leave TEC in to introduce scattering
Find: $\Delta P_{\mu} = -18 \pm 9 \times 10^{-4}$

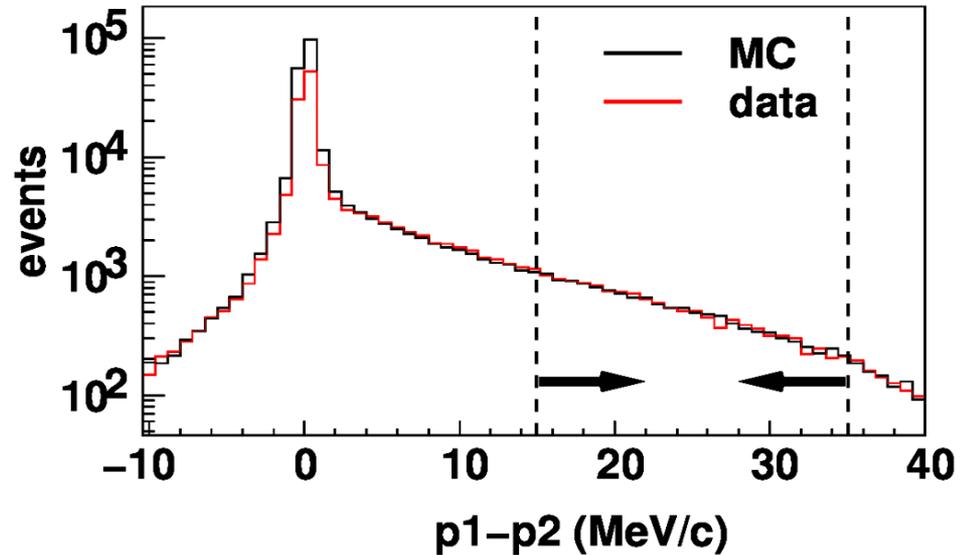
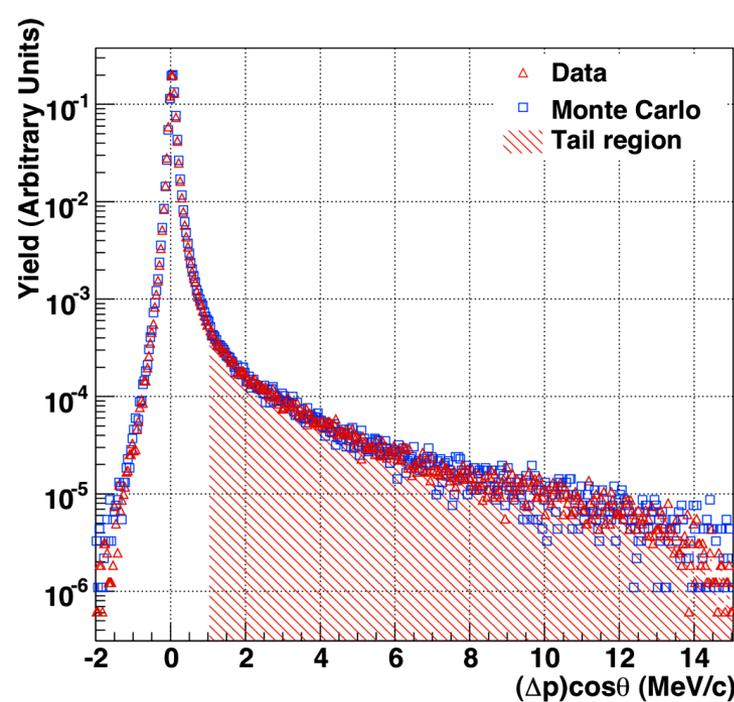
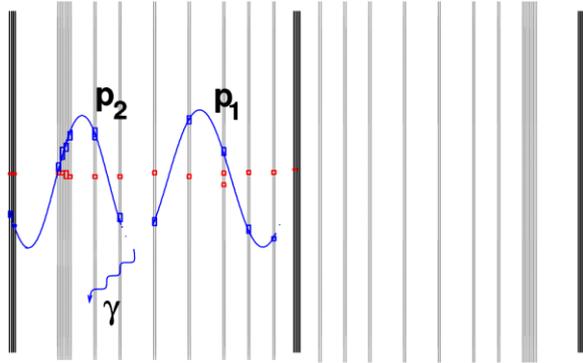
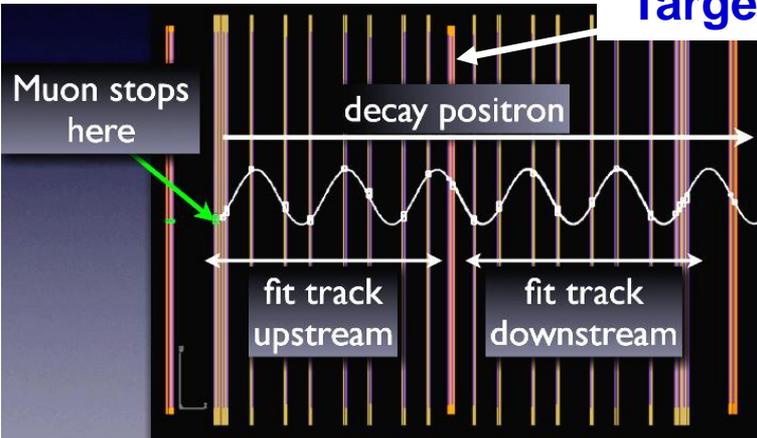
Compare differences with simulation to check fringe field systematic

Sensitivity to fringe field transverse components



Bremsstrahlung

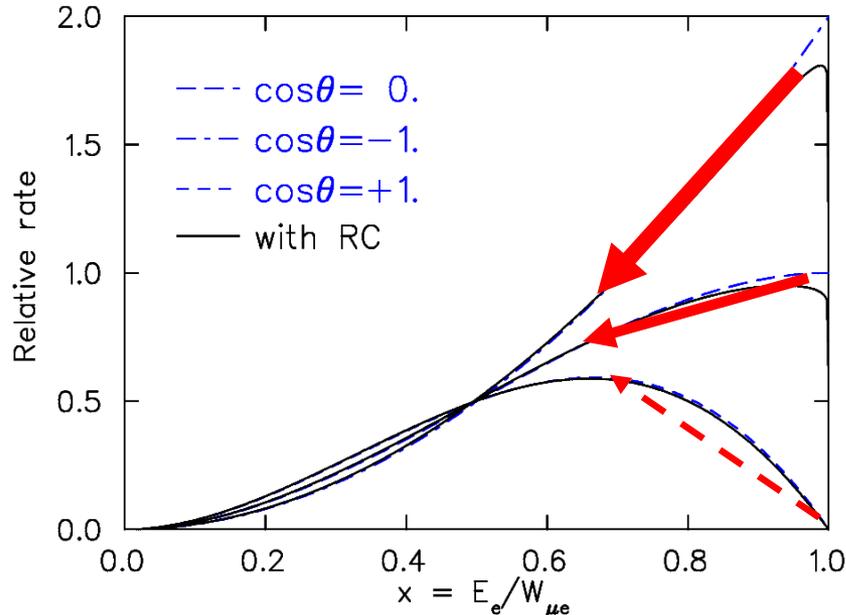
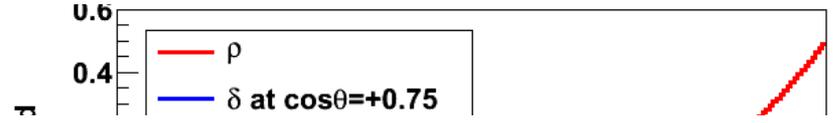
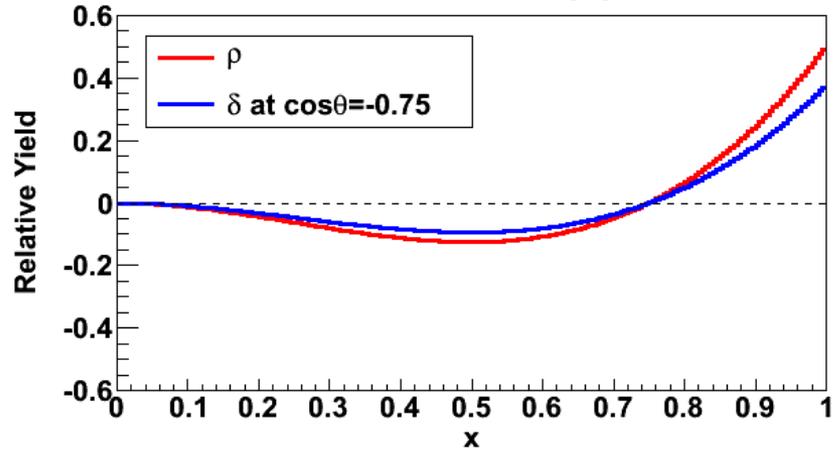
Target



- Leading systematic for ρ and δ
- Two separate measurements
 - “Upstream stops”
 - “Broken tracks”
- Consistent results validate bremsstrahlung simulation in our GEANT3 Monte Carlo at the 2.5% level

Why are ρ and δ systematics correlated?

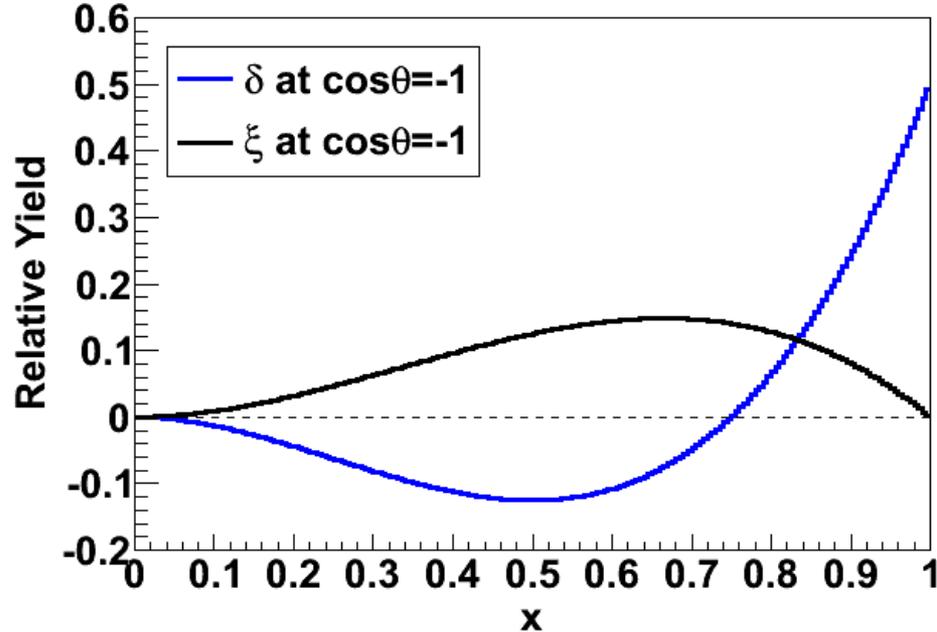
Derivatives at $\cos(\theta) = \pm 0.75$



- ρ and δ involve the momentum-dependence of the yield and asymmetry
- They have:
 - Same upstream shapes
 - Opposite downstream shapes
- Effects that
 - Distort the momentum, and
 - Couple to the yield
- Distort ρ and δ similarly
- Example: **bremsstrahlung**

Why are δ and $P_\mu \xi$ anti-correlated?

Fit derivatives for $P_\mu \xi \delta$ and $P_\mu \xi$



In **TWIST**, the fit parameters are

$P_\mu \xi$ and $P_\mu \xi \delta$

$$\delta = \frac{P_\mu \xi \delta}{P_\mu \xi}$$

- Anti-correlation between statistical uncertainties for δ and $P_\mu \xi$
- Three types of systematics influence the asymmetry measurements
 - Distort P_μ ; only impact $P_\mu \xi$
 - Distort contribution of $P_\mu \xi \delta$ derivative; only impact δ
 - Distort contribution of $P_\mu \xi$ derivative; impact **BOTH** $P_\mu \xi$ and δ

Muon decay parameters in the global analysis

(Phys Rev D 72, 073002)

$$Q_{RR} = \frac{1}{4} |g_{RR}^S|^2 + |g_{RR}^V|^2,$$

$$Q_{LR} = \frac{1}{4} |g_{LR}^S|^2 + |g_{LR}^V|^2 + 3|g_{LR}^T|^2,$$

$$Q_{RL} = \frac{1}{4} |g_{RL}^S|^2 + |g_{RL}^V|^2 + 3|g_{RL}^T|^2,$$

$$Q_{LL} = \frac{1}{4} |g_{LL}^S|^2 + |g_{LL}^V|^2,$$

$$B_{LR} = \frac{1}{16} |g_{LR}^S + 6g_{LR}^T|^2 + |g_{LR}^V|^2,$$

$$B_{RL} = \frac{1}{16} |g_{RL}^S + 6g_{RL}^T|^2 + |g_{RL}^V|^2,$$

$$I_\alpha = \frac{1}{4} [g_{LR}^V (g_{RL}^S + 6g_{RL}^T)^* + (g_{RL}^V)^* (g_{LR}^S + 6g_{LR}^T)] \\ = (\alpha + i\alpha')/2A,$$

$$I_\beta = \frac{1}{2} [g_{LL}^V (g_{RR}^S)^* + (g_{RR}^V)^* g_{LL}^S] = -2(\beta + i\beta')/A.$$

$$0 \leq Q_{\epsilon\mu} \leq 1, \quad \text{where } \epsilon, \mu = R, L,$$

$$0 \leq B_{\epsilon\mu} \leq Q_{\epsilon\mu}, \quad \text{where } \epsilon\mu = RL, LR,$$

$$|I_\alpha|^2 \leq B_{LR} B_{RL}, \quad |I_\beta|^2 \leq Q_{LL} Q_{RR},$$

$$Q_{RR} + Q_{LR} + Q_{RL} + Q_{LL} = 1.$$

$$\rho = \frac{3}{4} + \frac{1}{4} (Q_{LR} + Q_{RL}) - (B_{LR} + B_{RL}),$$

$$\xi = 1 - 2Q_{RR} - \frac{10}{3} Q_{LR} + \frac{4}{3} Q_{RL} + \frac{16}{3} (B_{LR} - B_{RL}),$$

$$\xi\delta = \frac{3}{4} - \frac{3}{2} Q_{RR} - \frac{7}{4} Q_{LR} + \frac{1}{4} Q_{RL} + (B_{LR} - B_{RL}),$$

$$\xi' = 1 - 2Q_{RR} - 2Q_{RL},$$

$$\xi'' = 1 - \frac{10}{3} (Q_{LR} + Q_{RL}) + \frac{16}{3} (B_{LR} + B_{RL}),$$

$$\bar{\eta} = \frac{1}{3} (Q_{LR} + Q_{RL}) + \frac{2}{3} (B_{LR} + B_{RL}),$$

$$\eta = (\alpha - 2\beta)/A, \quad \eta'' = (3\alpha + 2\beta)/A.$$

- The global analysis combines the final **TWIST** results with all non-**TWIST** muon decay parameter measurements
- The fit parameters are Q_{RR} , Q_{LR} , Q_{RL} , B_{LR} , B_{RL} , α/A , β/A , α'/A , β'/A