# TWIST

## The TRIUMF Weak Interaction Symmetry Test

## Precision Muon Decay at TRIUMF

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University of Alberta

TWIST: Universities of Alberta, British Columbia, Northern British Columbia, Montreal, Saskatchewan; TRIUMF, Texas A&M, Valporaiso, KIAE - Russia

# TWIST - Personnel

**TRIUMF**
- ❖ Willy Andersson
- ● Yuri Davydov
- ● Jaap Doornbos
- ❖ Wayne Faszer
- ● Dave Gill
- ● Peter Gumplinger
- ● Richard Helmer
- ● Robert Henderson
- ● John Macdonald
- ● Glen Marshall
- ● Art Olin
- ❖ David Ottewell
- ❖ Robert Openshaw
- ● Jean-Michel Poutissou
- ● Renee Poutissou
- ❖ Grant Sheffer
- ● Hans-Christian Walter
- ● Dennis Wright

**Alberta**
- ✻ Andrei Gaponenko
- ● Peter Green
- ● Peter Kitching
- ✻ Rob MacDonald
- ● Maher Quraan
- ● Nathan Rodning
- ❖ John Schaapman
- ✻ Farhana Sobratee
- ❖ Jan Soukup
- ● Glen Stinson

**British Columbia**
- ✻ Blair Jamieson
- ❖ Doug Maas
- ● Mike Hasinoff

**Northern British Columbia**
- ● Elie Korkmaz
- ● Tracy Porcelli

**Montreal**
- ● Pierre Depommier

**Regina**
- ● Ted Mathie
- ✻ George Price
- ● Roman Tacik

**Saskatchewan**
- ● Bill Shin

**Texas A&M**
- ● Carl Gagliardi
- ● John Hardy
- ✻ Jim Musser
- ● Robert Tribble
- ● Maxim Vasiliev

**Valparaiso**
- ● Don Koetke
- ● Robert Manweiler
- ❖ Paul Nord
- ● Shirvel Stanislaus

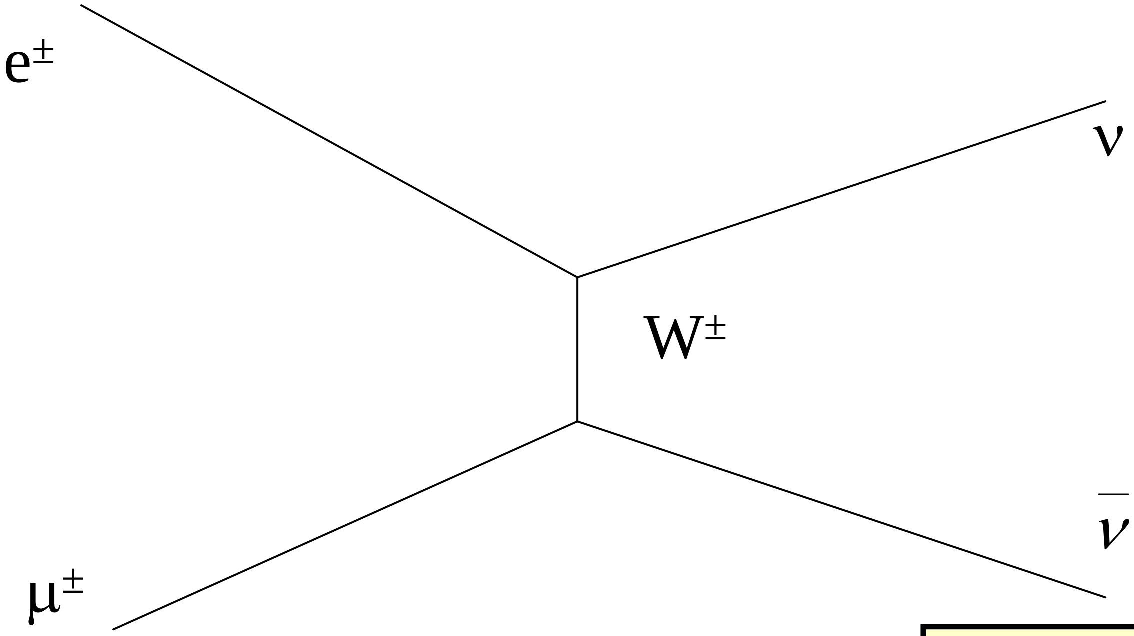
**KIAE (Russia)**
- ● Arkadi Khruchinsky
- ● Vladimir Selivanov
- ● Vladimir Torokhov

✻ Students
❖ Professional Staff

# Outline

- **Background on muon decay**
- **The E614 Experiment**
- **Sensitivity to new physics**

# The Standard Model for μ decay




(V-A) Interaction is built in

- parity violation is perfect
- exchange particle is known

**Only one coupling is non-zero in the Standard Model**

| | | |
|---|---|---|
| $\lvert g^{S}_{RR} \rvert \equiv 0$ | $\lvert g^{V}_{RR} \rvert \equiv 0$ | $\lvert g^{T}_{RR} \rvert = \textit{zero}$ |
| $\lvert g^{S}_{LR} \rvert \equiv 0$ | $\lvert g^{V}_{LR} \rvert \equiv 0$ | $\lvert g^{T}_{LR} \rvert \equiv 0$ |
| $\lvert g^{S}_{RL} \rvert \equiv 0$ | $\lvert g^{V}_{RL} \rvert \equiv 0$ | $\lvert g^{T}_{RL} \rvert \equiv 0$ |
| $\lvert g^{S}_{LL} \rvert \equiv 0$ | $\lvert g^{V}_{LL} \rvert \equiv 1$ | $\lvert g^{T}_{LL} \rvert = \textit{zero}$ |

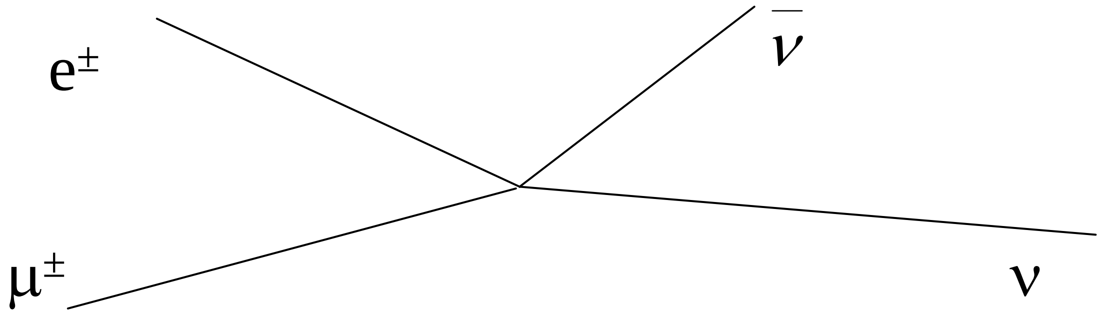
- The operator (V-A) satisfies the requirement that the Weak interaction violates parity.
- (V-A) violates parity perfectly
- The (V-A) operator projects out the left-handed (negative chirality) component of the wave function

$$\bar{\psi}\gamma^{\mu}(1-\gamma^{5})\psi = \bar{\psi}\gamma^{\mu}(1-\gamma^{5})\begin{bmatrix}\psi_{+}\\ \psi_{-}\end{bmatrix}$$

$$= \bar{\psi}\gamma^{\mu}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\psi_{+}\\ \psi_{-}\end{bmatrix} = \bar{\psi}\gamma^{\mu}\psi_{-}$$

- the (V-A) theory therefore states that leptons with positive chirality do not undergo weak interactions.

# A more general interaction - which does not presuppose the W




$$rate \sim \left| \sum_{\substack{\gamma=S,V,T \\ i,j=R,L}} g_{ij}^{\gamma} \langle \bar{\psi}_{ei} | \Gamma^{\gamma} | \psi_{\nu_e} \rangle \langle \bar{\psi}_{\nu_\mu} | \Gamma_{\gamma} | \psi_{\mu j} \rangle \right|^2$$

| | |
|---|---|
| Scalar | $\bar{\psi}\psi$ |
| Vector | $\bar{\psi}\gamma^{\mu}\psi$ |
| Tensor | $\bar{\psi}\sigma^{\mu\nu}\psi$ |
| Axial Vector | $\bar{\psi}\gamma^{5}\gamma^{\mu}\psi$ |
| Pseudoscal ar | $\bar{\psi}\gamma^{5}\psi$ |

Allows for possible

- scalar
- vector
- tensor

interactions of right-handed and left-handed leptons

# The preceding - in terms of the Michel parameters

$$rate \sim x^2\left[3-3x+\frac{2}{3}\rho(4x-3)+P_\mu\xi\cos(\theta)\left(1-x+\frac{2}{3}\delta(4x-3)\right)\right]$$

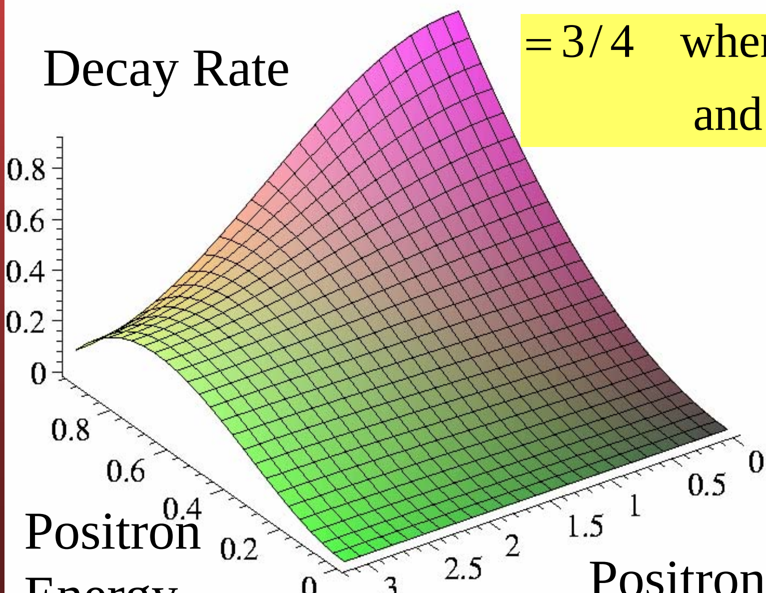
For example-

$$\rho \equiv \frac{3}{4}\left[|g^V_{LL}|^2+|g^V_{RR}|^2+|g^T_{LR}|^2+|g^T_{RL}|^2\right]$$
$$+\frac{3}{16}\left[|g^S_{LL}|^2+|g^S_{RR}|^2+|g^S_{LR}|^2+|g^S_{RL}|^2\right]$$
$$-\frac{3}{4}\left[\mathrm{Re}\left(g^S_{LR}g^{T*}_{LR}\right)+\mathrm{Re}\left(g^S_{RL}g^{T*}_{RL}\right)\right]$$

$= 3/4$ when $|g^V_{LL}|^2=1$ and other couplings are zero

Similar expressions exist defining ξ and δ.

A fourth parameter, η, contributes to order $(m_e/m_\mu)$




Above expression is modified by radiative corrections, required to second order

# The Expression becomes considerably simpler in the Standard Model

3/4

$$rate \sim x^2 \left[ 3 - 3x + \frac{2}{3}\rho(4x-3) + P_\mu \xi \cos(\theta) \left( 1 - x + \frac{2}{3}\delta(4x-3) \right) \right]$$

For example-

**Standard Model Values**

1 0 0 0

$$\rho \equiv \frac{3}{4}\left[ |g^V_{LL}|^2 + |g^V_{RR}|^2 + |g^T_{LR}|^2 + |g^T_{RL}|^2 \right]$$

0 0 0 0

$$+ \frac{3}{16}\left[ |g^S_{LL}|^2 + |g^S_{RR}|^2 + |g^S_{LR}|^2 + |g^S_{RL}|^2 \right]$$

0 0 0 0

$$- \frac{3}{4}\left[ \mathrm{Re}\left( g^S_{LR} g^{T*}_{LR} \right) + \mathrm{Re}\left( g^S_{RL} g^{T*}_{RL} \right) \right]$$

$= 3/4$ *when* $|g^V_{LL}|^2 = 1$

*and other couplings are zero*

Similar expressions exist defining ξ, δ, and η.

# This simple model may be too simple

*exchange particle:*

| *spin 0* | *spin 1* | *spin 2* |
|---|---|---|
| $\lvert g^S_{RR} \rvert < 0.066$ | $\lvert g^V_{RR} \rvert < 0.033$ | $\lvert g^T_{RR} \rvert \equiv 0$ |
| $\lvert g^S_{LR} \rvert < 0.125$ | $\lvert g^V_{LR} \rvert < 0.060$ | $\lvert g^T_{LR} \rvert < 0.036$ |
| $\lvert g^S_{RL} \rvert < 0.424$ | $\lvert g^V_{RL} \rvert < 0.110$ | $\lvert g^T_{RL} \rvert < 0.122$ |
| $\lvert g^S_{LL} \rvert < 0.55$ | $\lvert g^V_{LL} \rvert > 0.96$ | $\lvert g^T_{LL} \rvert \equiv 0$ |

**All but one of these terms has been set to zero in the Standard model for simplicity**

**The Weak Interaction may not be purely (V-A)**

# *TWIST* – Goals

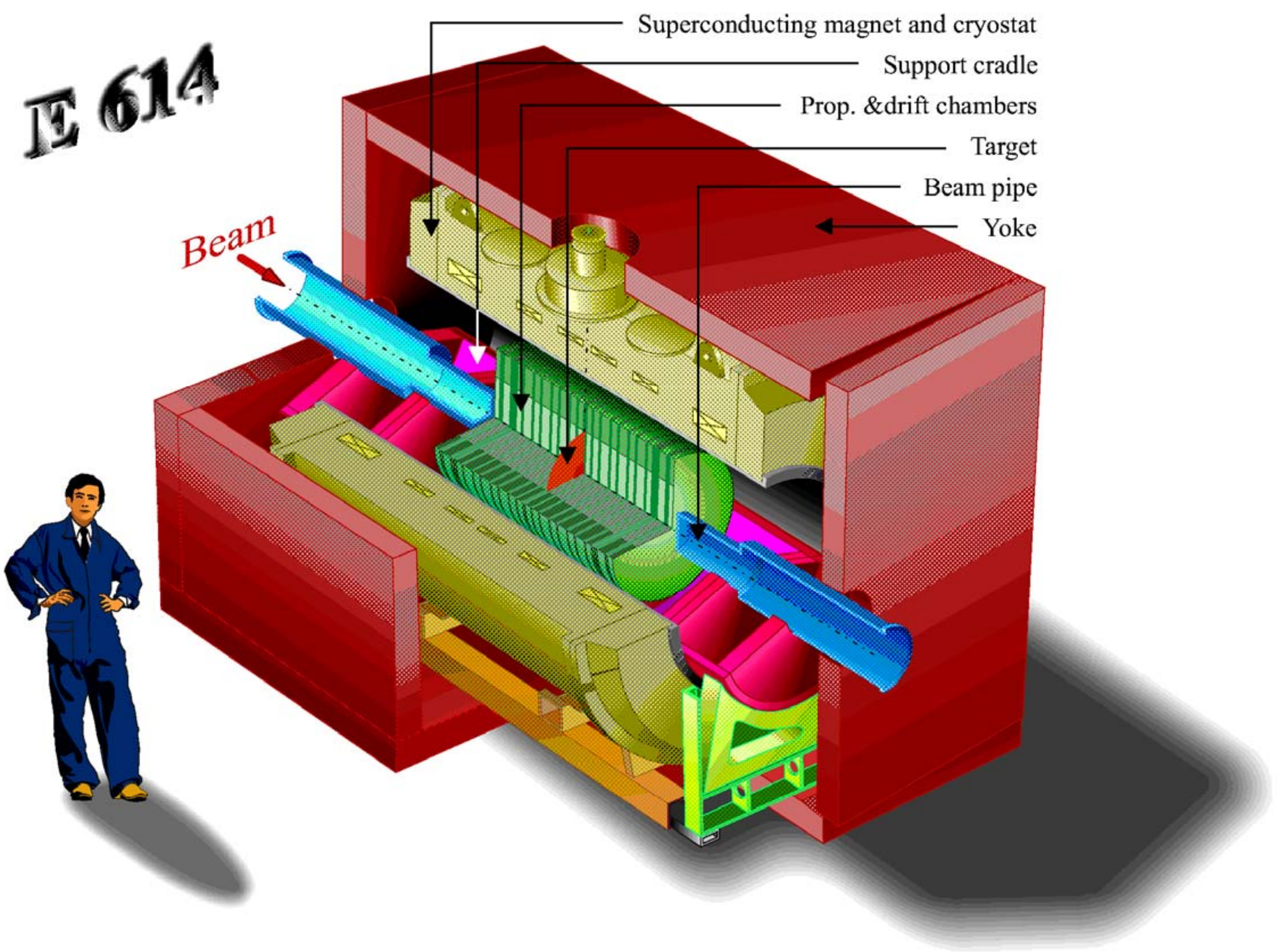
We propose to study $10^9$ $\mu^+$ decays

Goal:

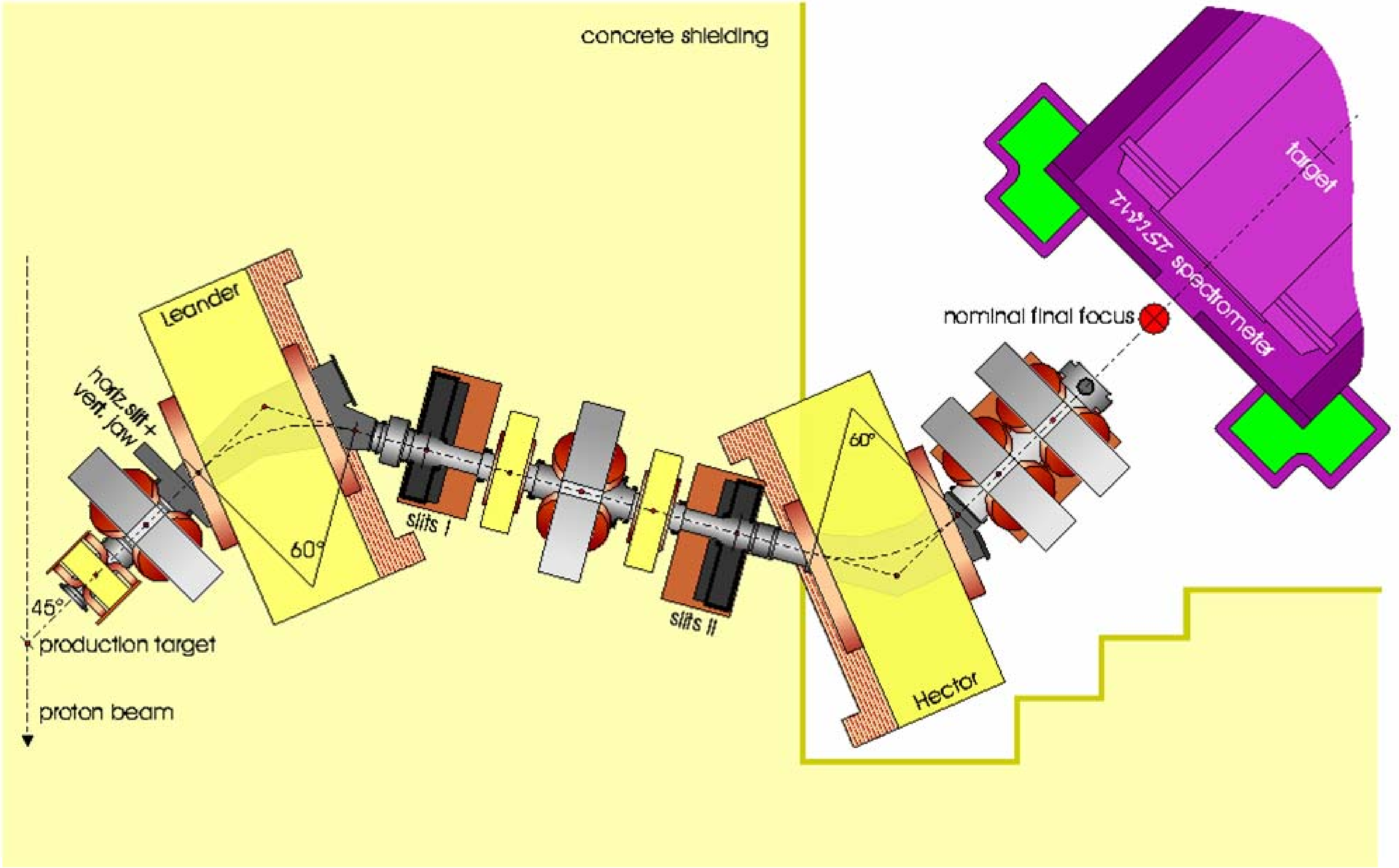
- to determine the Michel parameters to a few parts in $10^4$

- to test for weak couplings inconsistent with the Standard Model

# *TWIST* – Spectrometer

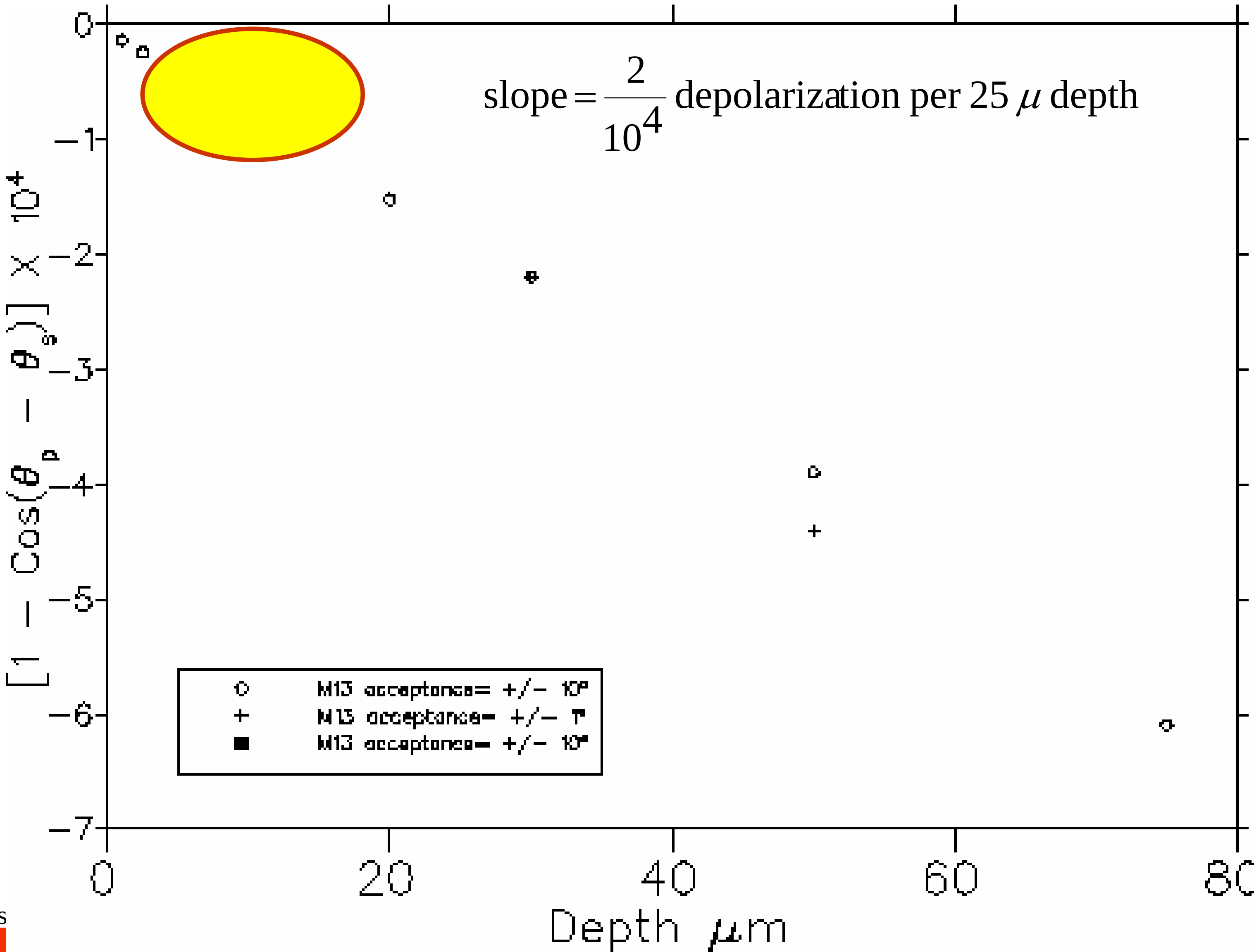


# TWIST – Beamline



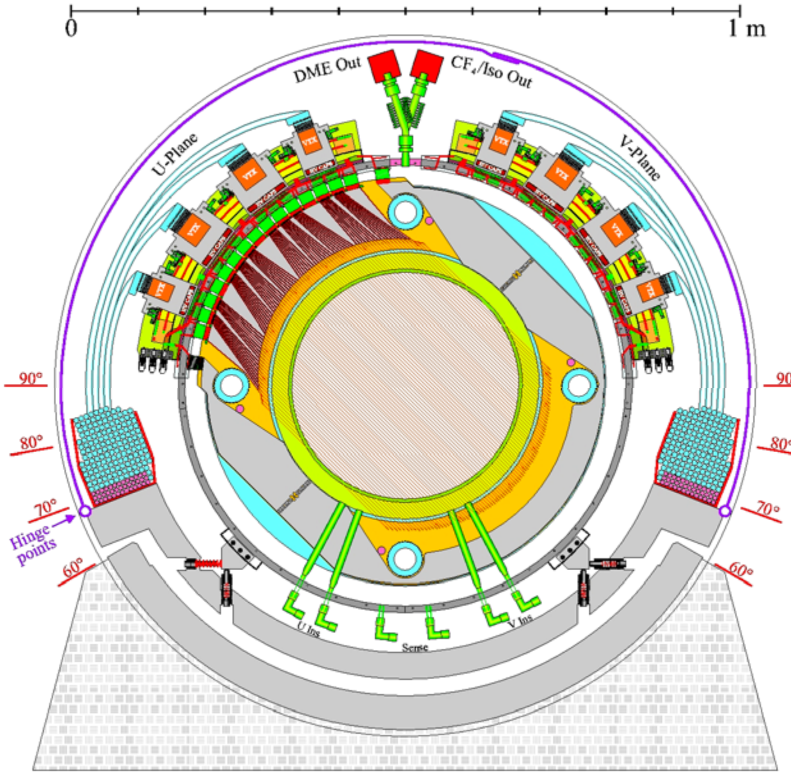
# TWIST – 1AT1 depolarization

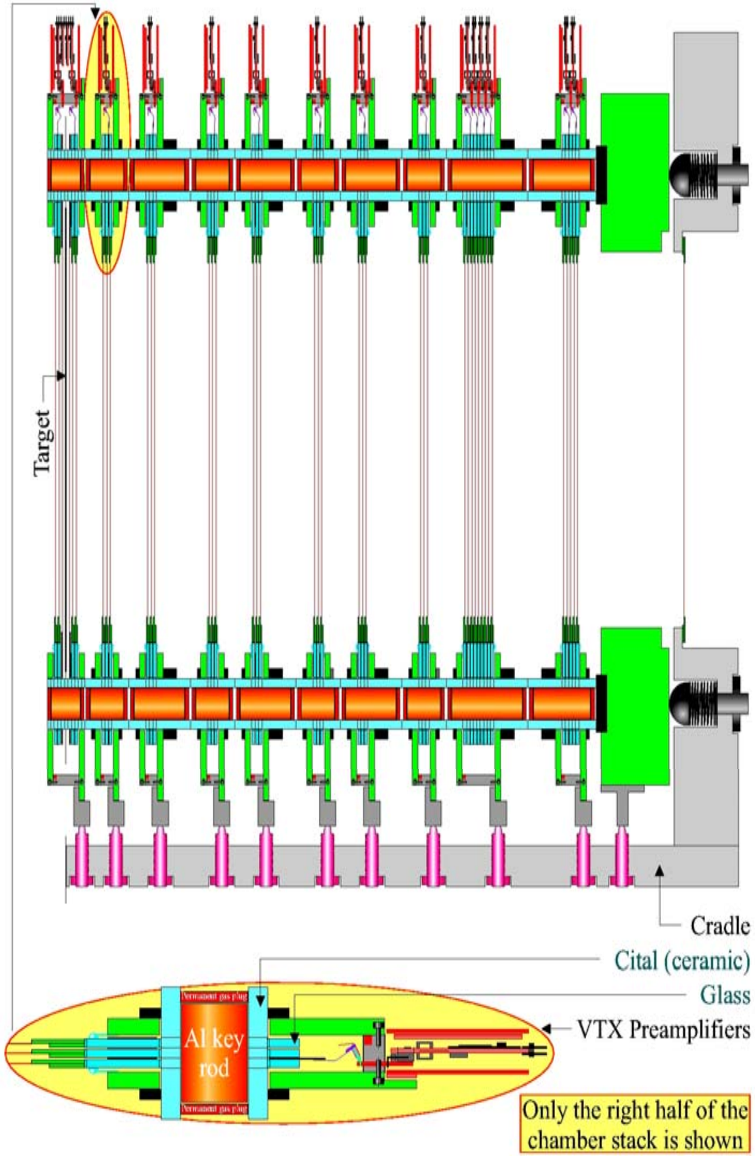
1AT1 Scatter => ~ 0.0001 depolarization



# *TWIST* – Chambers & half detector

Planar drift chambers sample positron track

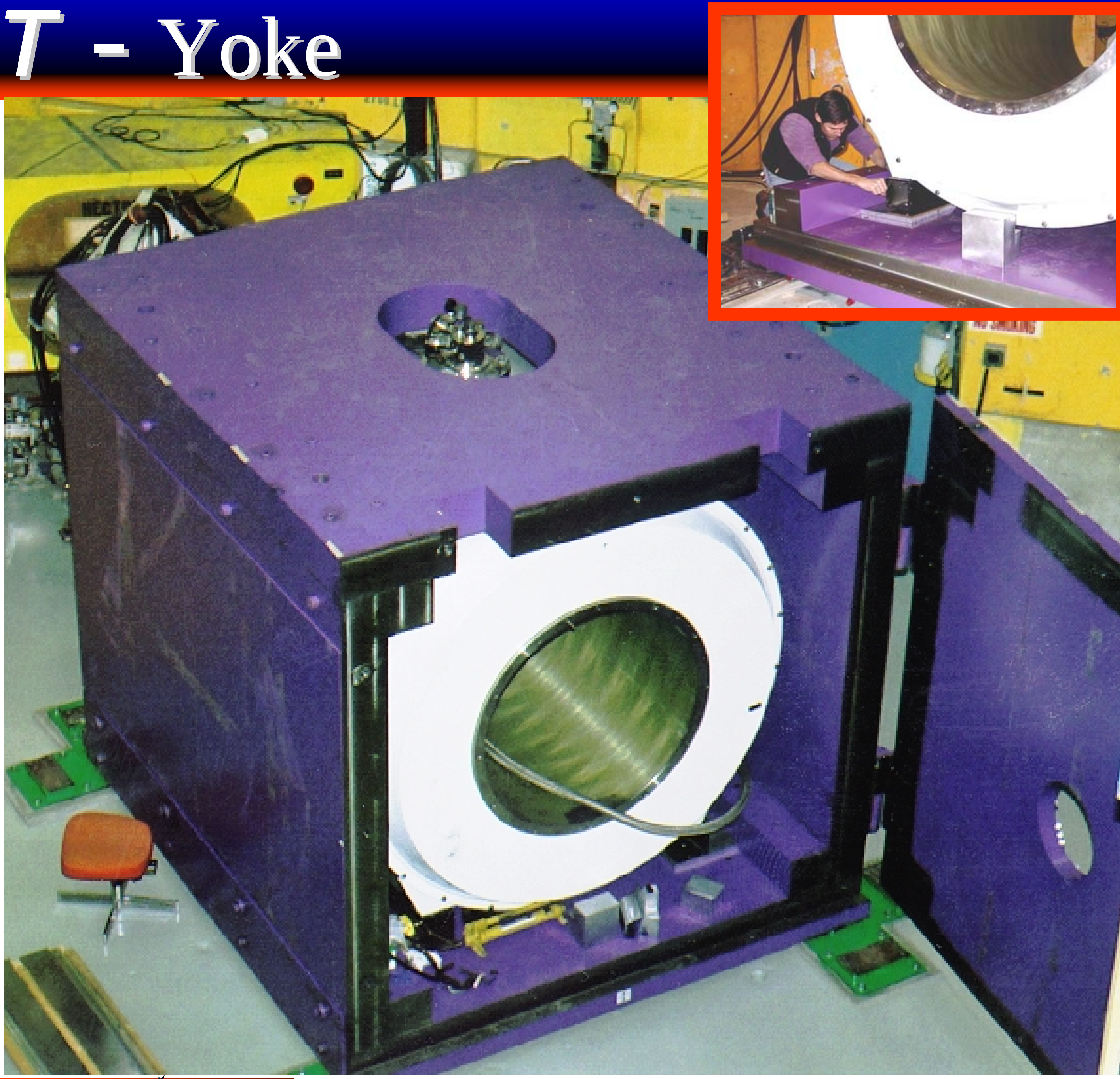


# *TWIST* - Yoke

The TWIST yoke pieces were delivered and assembled before Christmas

Alignment was completed in the first week of January



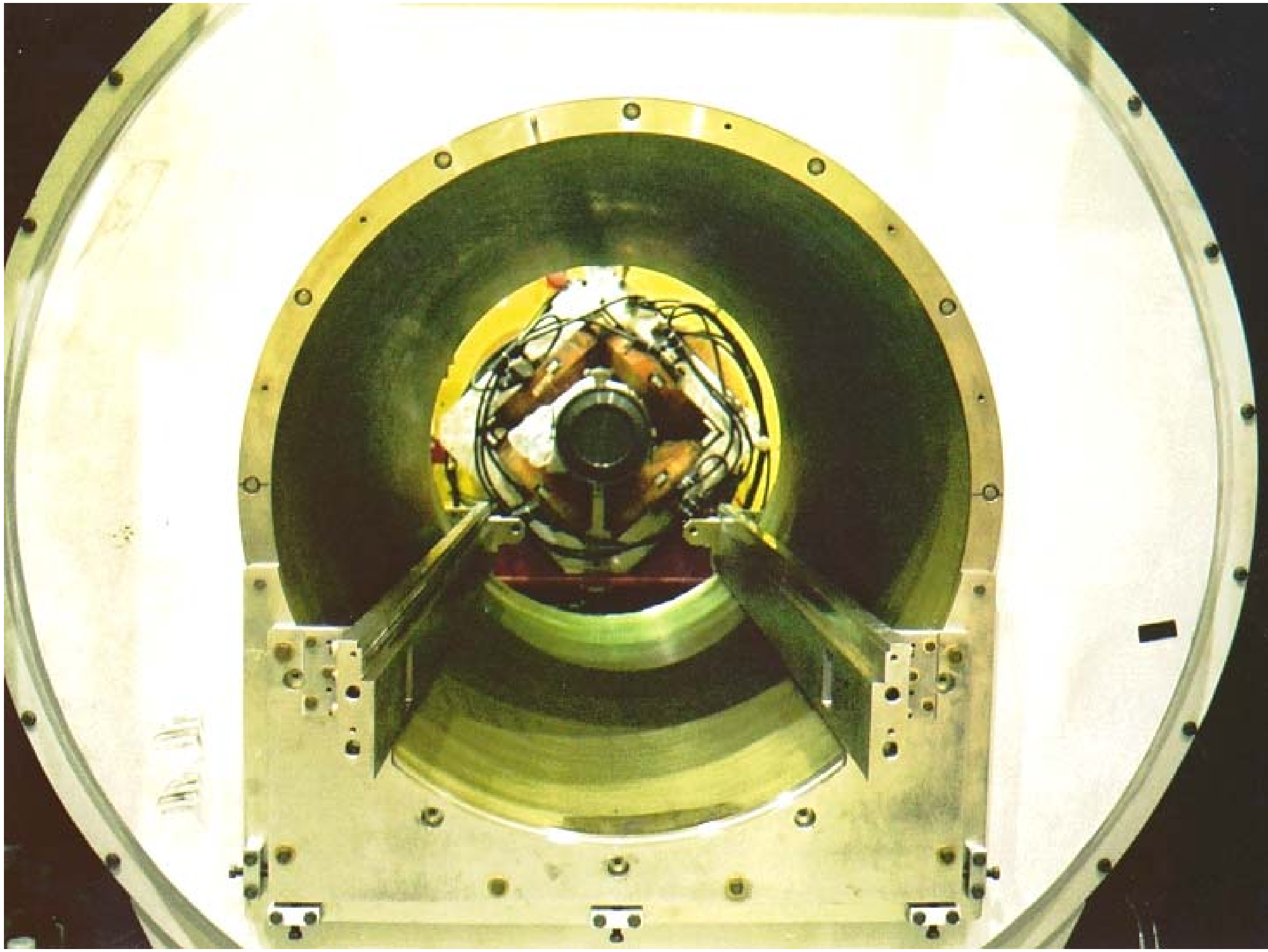
# *TWIST* – Solenoid and WC track

Track is in place and aligned to accept detector cradle and stack

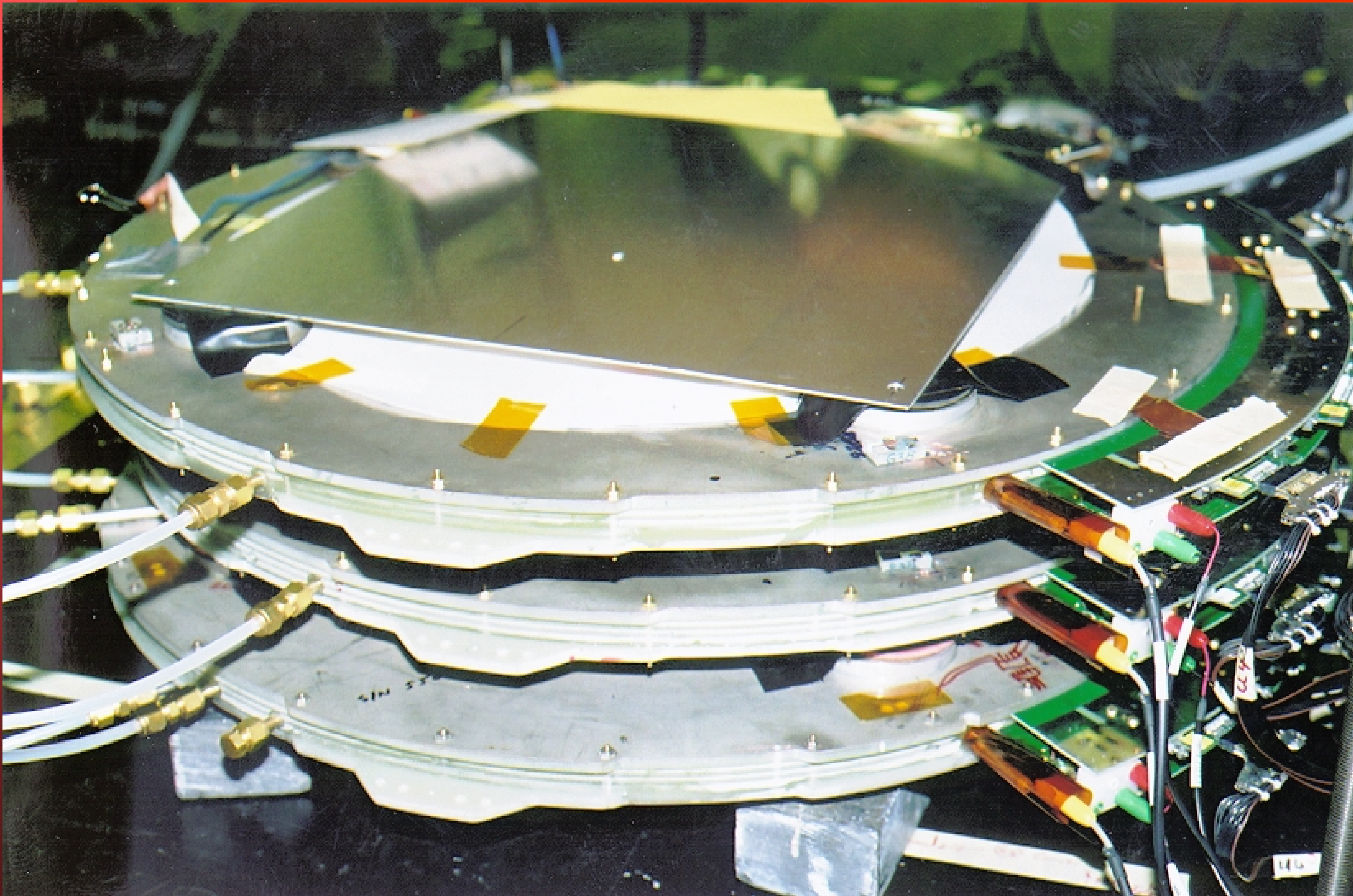
Magnet is cooling

Commissioning begins this week

Mapping complete by end of March

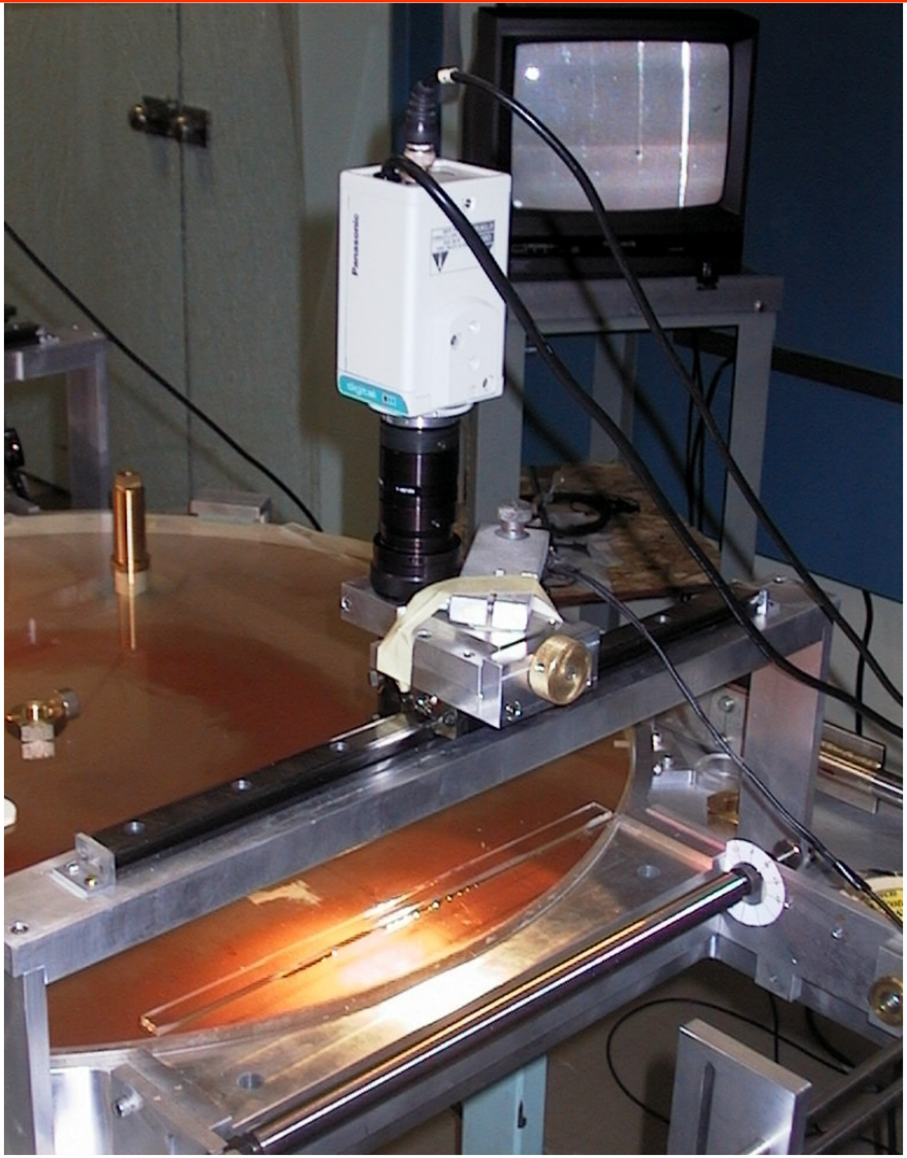


# *TWIST* – Chambers

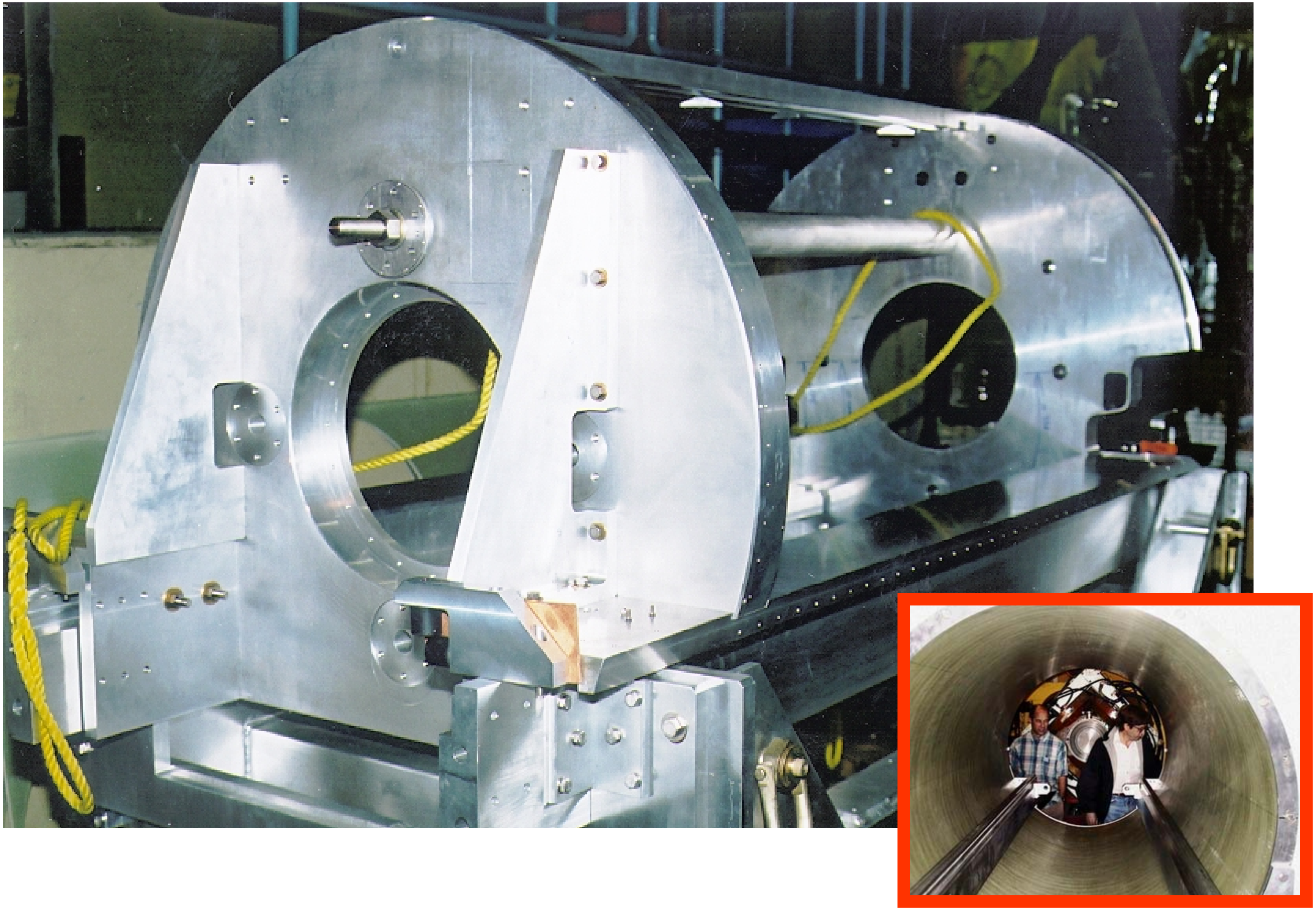


# TWIST Glass Planes

Planes are assembled on glass plates with optical precision relative to longitudinal coordinate



# *TWIST* – Chamber Support Cradle



# E614 Precision

*Accepted Experimental Values*

$$\rho = 0.7518 \pm 0.0026$$

$$P_\mu \xi = 1.0027 \pm 0.0085$$

$$\delta = 0.7486 \pm 0.0038$$

$$\eta = -0.007 \pm 0.013$$

*E614 Proposal*

$$\sigma_\rho = \pm 0.00005 \pm 0.00009$$

$$\sigma_{P_\mu \xi} = \pm 0.00010 \pm 0.00010$$

$$\sigma_\delta = \pm 0.00008 \pm 0.00010$$

$$\sigma_\eta \approx \pm 0.003$$

25-60 fold improvement in precision on the Michel parameters

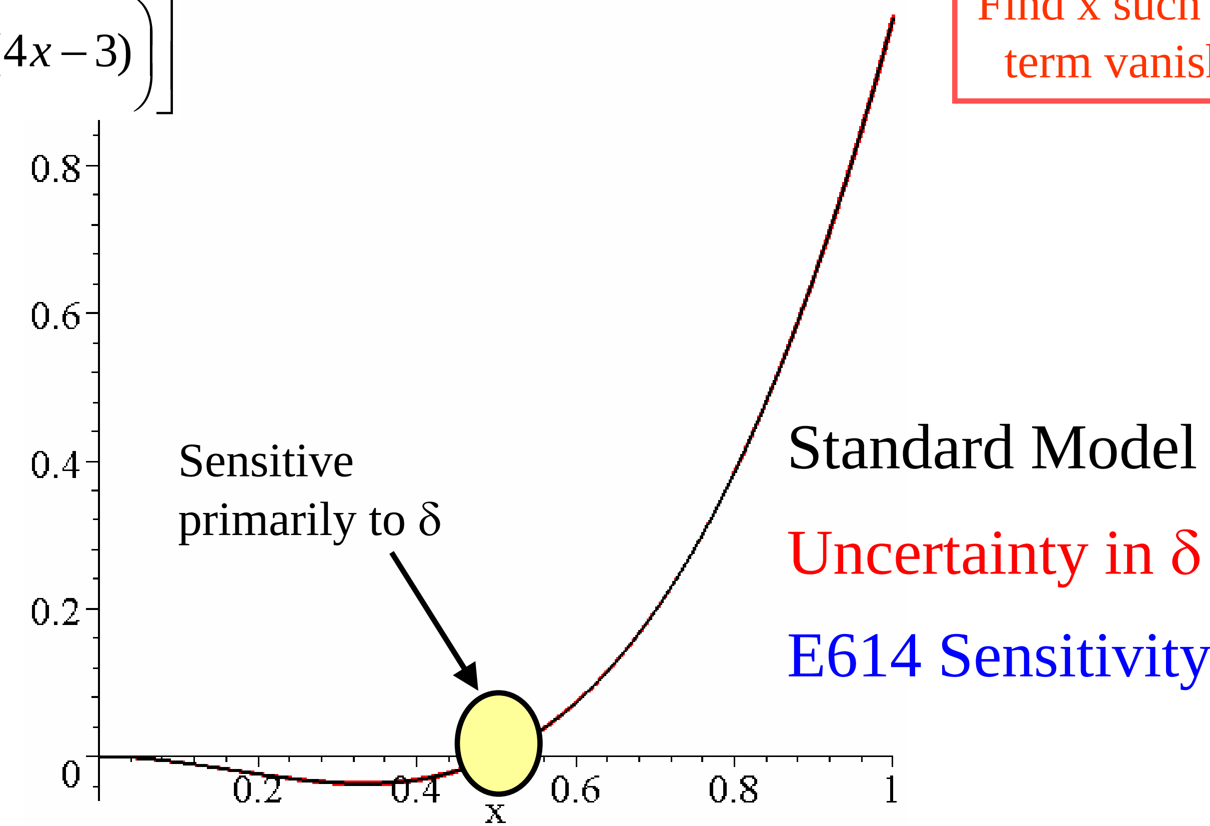
3-10 fold improvement in couplings

# The (forward - backward) distribution goes flat at a value of x dependant (only) upon δ

$$[Forward - Backward] \sim x^2 \left[ 2P_\mu \xi \cos(\theta) \left( 1 - x + \frac{2}{3}\delta(4x-3) \right) \right]$$

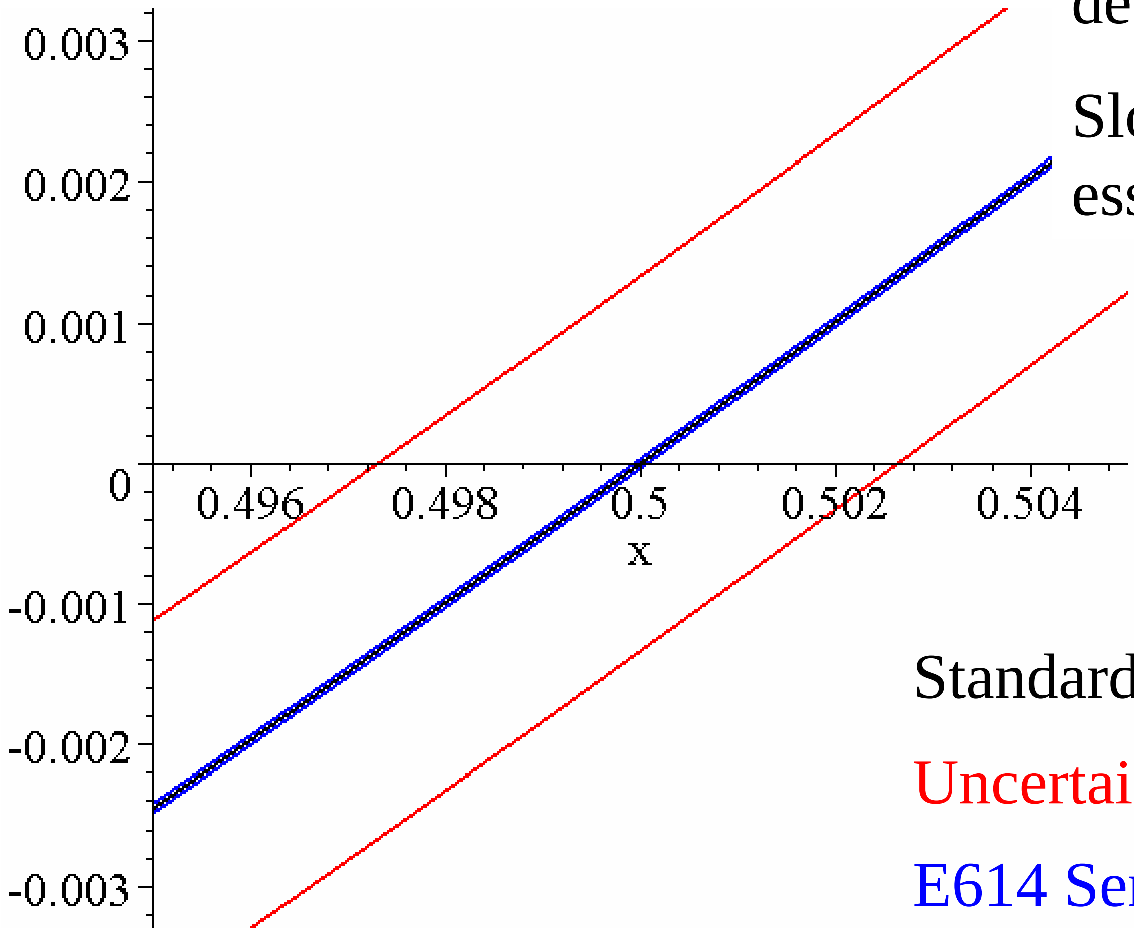
Find x such that term vanishes

$$x^2 \left[ 2P_\mu \xi \left( 1 - x + \frac{2}{3}\delta(4x-3) \right) \right]$$



# Same as the previous slide - on expanded scale

$$x^2\left[2P_\mu\xi\left(1-x+\frac{2}{3}\delta(4x-3)\right)\right]$$




Zero crossing determines $\delta$

Slope is essentially $P_\mu\xi$

Standard Model

Uncertainty in $\delta$

E614 Sensitivity

# Minimal extensions to the Standard Model

Allowing only vector couplings result in simplified Michel parameters

$$\rho \equiv \frac{3}{4}\left[ |g^V_{LL}|^2 + |g^V_{RR}|^2 + |\cancel{g^T_{LR}}|^2 + |\cancel{g^T_{RL}}|^2 \right] + \frac{3}{16}\left[ |\cancel{g^S_{LL}}|^2 + |\cancel{g^S_{RR}}|^2 + |\cancel{g^S_{LR}}|^2 + |\cancel{g^S_{RL}}|^2 \right] - \frac{3}{4}\left[ \mathrm{Re}\left(\cancel{g^S_{LR} g^{T*}_{LR}}\right) + \mathrm{Re}\left(\cancel{g^S_{RL} g^{T*}_{RL}}\right) \right]$$

In the context of the model,
Four parameters and four unknowns

$$\xi \equiv |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 + 5|\cancel{g^T_{LR}}|^2 - 5|\cancel{g^T_{RL}}|^2 + \frac{1}{4}|\cancel{g^S_{LL}}|^2 - \frac{1}{4}|\cancel{g^S_{LR}}|^2 + \frac{1}{4}|\cancel{g^S_{RL}}|^2 - \frac{1}{4}|\cancel{g^S_{RR}}|^2 + 4\mathrm{Re}(\cancel{g^S_{LR} g^{T*}_{LR}}) - 4\mathrm{Re}(\cancel{g^S_{RL} g^{T*}_{RL}})$$

$$\xi\delta \equiv \frac{3}{4}\left[ |g^V_{LL}|^2 - |g^V_{RR}|^2 - |\cancel{g^T_{LR}}|^2 + |\cancel{g^T_{RL}}|^2 \right] + \frac{3}{16}\left[ |\cancel{g^S_{LL}}|^2 - |\cancel{g^S_{RR}}|^2 - |\cancel{g^S_{LR}}|^2 + |\cancel{g^S_{RL}}|^2 \right] - \frac{3}{4}\left[ \mathrm{Re}(\cancel{g^S_{LR} g^{T*}_{LR}}) - \mathrm{Re}(\cancel{g^S_{RL} g^{T*}_{RL}}) \right]$$

$$\eta \equiv \frac{1}{2}\mathrm{Re}\left[ g^V_{LL} \cancel{g^{S*}_{RR}} + g^V_{RR} \cancel{g^{S*}_{LL}} \right] + \frac{1}{2}\mathrm{Re}\left[ g^V_{RL}\left(\cancel{g^{S*}_{LR}} + 6\cancel{g^{T*}_{LR}}\right) + g^V_{LR}\left(\cancel{g^{S*}_{RL}} + 6\cancel{g^{T*}_{RL}}\right) \right]$$

# Anticipated sensitivity to new couplings

| | Current Limits | E614(A) | E614(B) | E614(C) | E614(D) |
|---|---|---|---|---|---|
| $\lvert g^S_{RR}\rvert$ | <0.066 | — | — | 0.020 | 0.045 |
| $\lvert g^V_{RR}\rvert$ | <0.033 | 0.012 | 0.014 | 0.013 | 0.022 |
| $\lvert g^S_{LR}\rvert$ | <0.125 | — | — | 0.027 | 0.046 |
| $\lvert g^V_{LR}\rvert$ | <0.060 | 0.012 | 0.013 | 0.012 | 0.018 |
| $\lvert g^T_{LR}\rvert$ | <0.036 | — | 0.009 | — | 0.013 |
| $\lvert g^S_{RL}\rvert$ | <0.424 | — | — | — | — |
| $\lvert g^V_{RL}\rvert$ | <0.110 | 0.012 | 0.012 | 0.011 | — |
| $\lvert g^T_{RL}\rvert$ | <0.122 | — | 0.008 | — | — |
| $\lvert g^S_{LL}\rvert$ | <0.55 | — | — | — | — |
| $\lvert g^V_{LL}\rvert$ | >0.96 | >0.99977 | >0.99953 | — | — |

Upper limits (90% CL) for weak coupling constants with current limits taken from the Particle Data Group. Improved limits expected from TWIST based on measurements of $\rho$, $\xi$, $\delta$ and $\eta$ assume:

- (A) V, A couplings only,
- (B) V, A and T couplings,
- (C) V, A and S couplings or
- (D) most general V, A, S, and T derivative-free couplings.

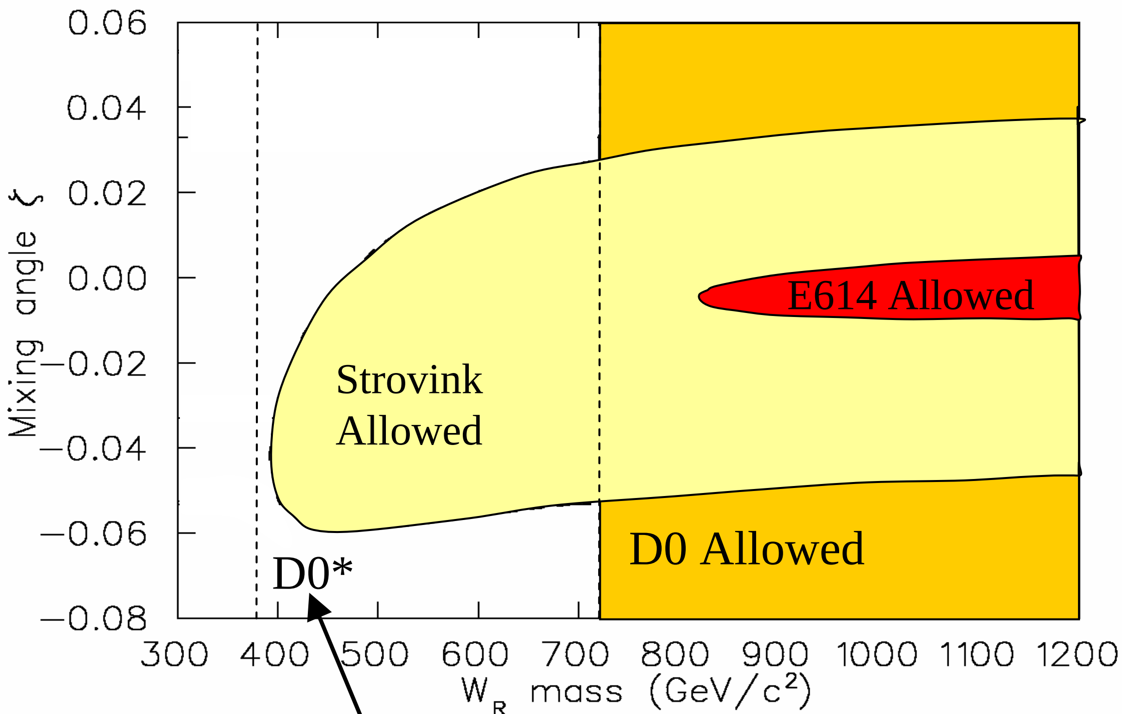
# One way of looking at the discovery potential

Assume manifest L-R Symmetry

ie $g_R = g_L$

$CKM_R = CKM_L$

and no cp violation




$$V_{ud}^{R} = 0$$

$$V_{us}^{R} = 1$$

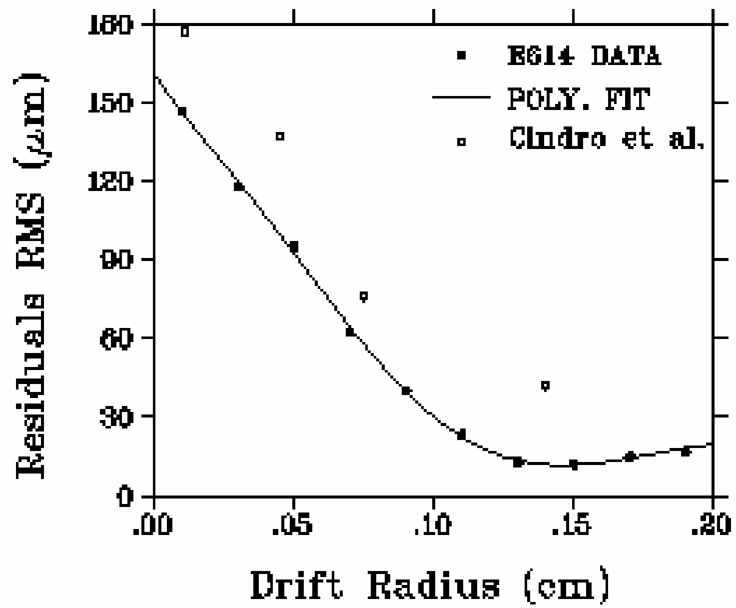
Beta decay, $p\bar{p}$ direct production, and muon decay are complimentary

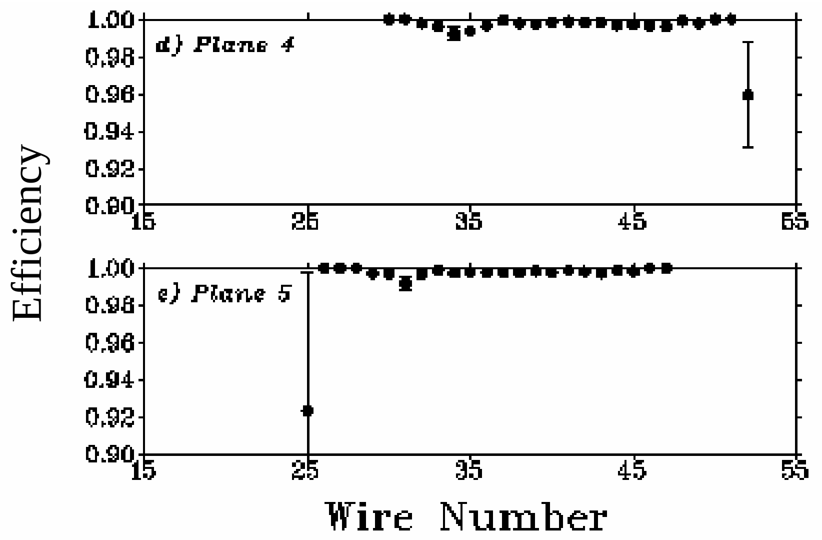
# E614 Timeline

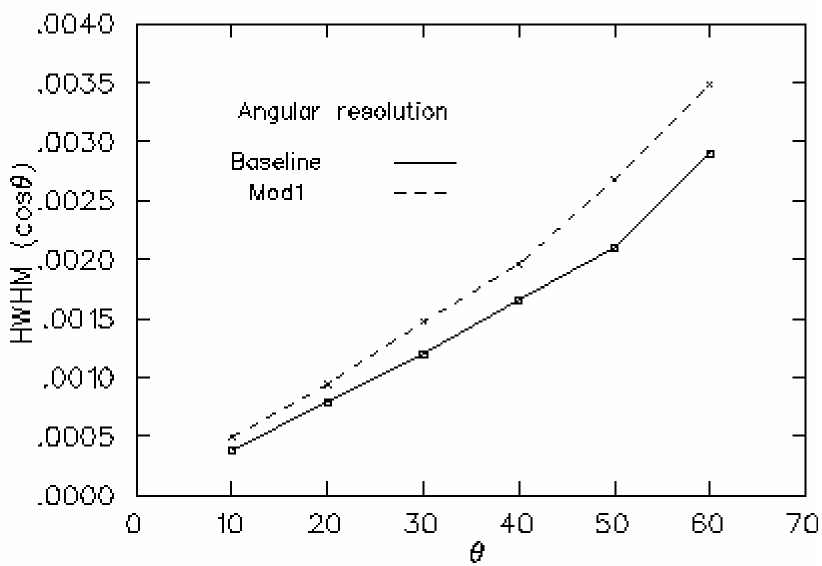
- ✓ **High Priority at TRIUMF – 1993**
- ✓ **First Capital Funding – April 1997**
- ✓ **WC Review - January 1999**
- ✓ **Mechanical Review - June 1999**
- ✓ **Beam Tests - final prototype - August 1999**
- ✓ **Full WC Production underway - March 2000**
- ➢ **WC Module Completion May 2000 – April 2001**
- ✓ **WC Bench tests beginning June 2000**
- ✓ **Yoke assembly December 2000**
- **Yoke, Solenoid, and cryogenics Commissioning: February - April 2001**
- **First beam: Summer of 2001**

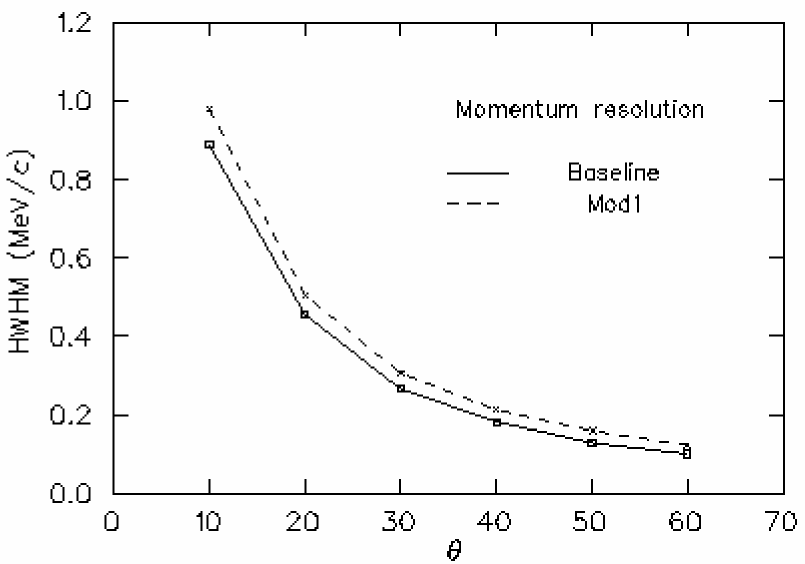
- **Preliminary Physics: December 2002**

# Spectrometer Resolution



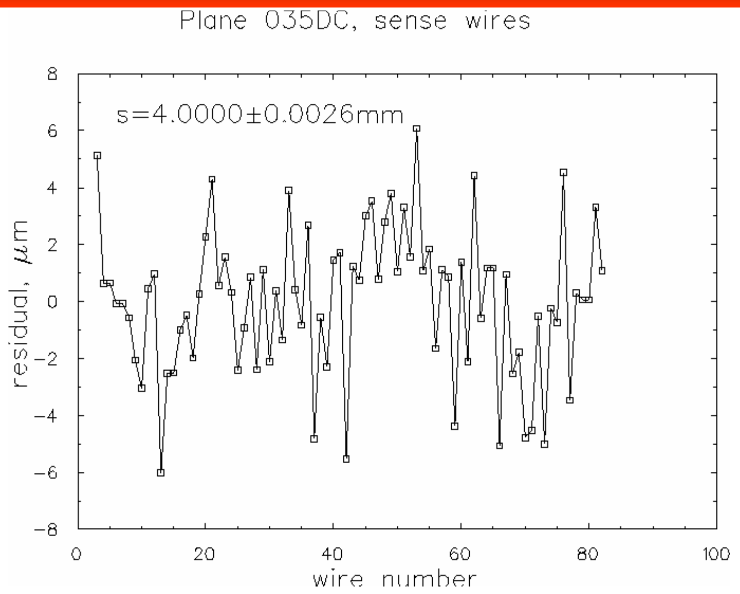



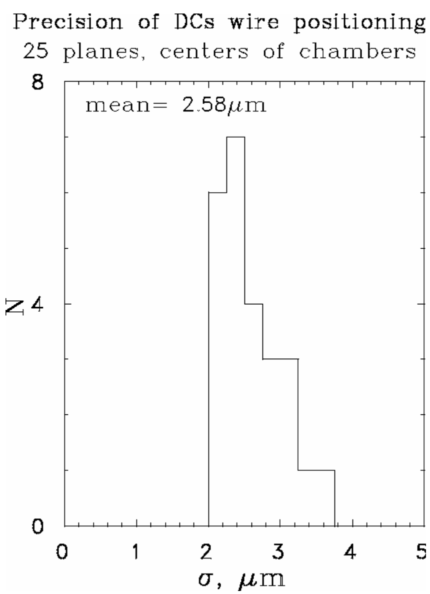
# *TWIST* – Chambers

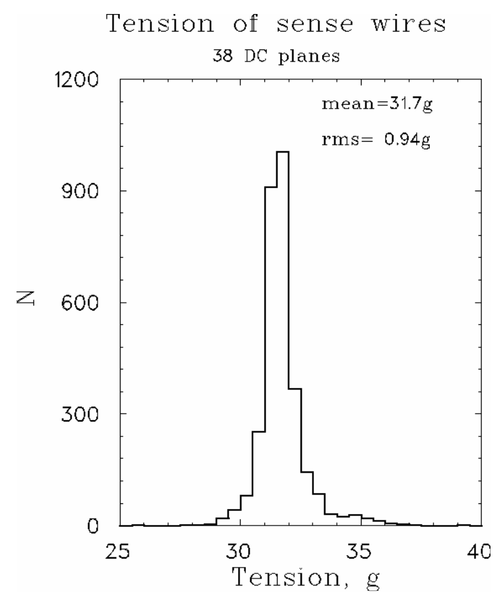
Quality Control on stringing of Wire Planes

The figures show:

1. Wire-to-wire variation in z position for a typical plane; σ = 2.6 μ
2. Average error in wire position over 25 drift planes; σ = 2.58 μ
3. Average wire tension over 38 drift planes; rms = 0.94g



# TWIST - Electronics

TWIST Requires

• 240 preamplifiers

• 268 postamplifiers

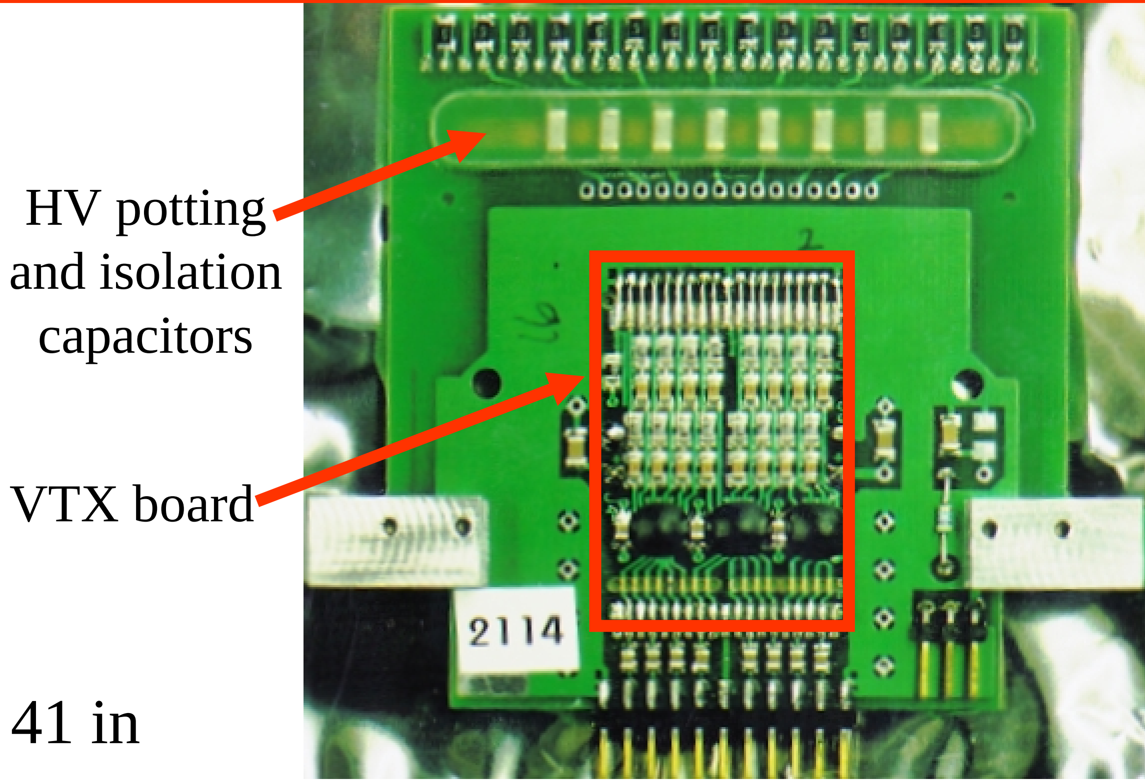
• 42 TDC's

Status

• 86 preamplifiers tested, 41 in mid-production

• 120 postamplifiers tested, 180 more in production

• 47 TDC's in hand




TWIST preamplifier

16 and 24 channel versions based on Fermilab CDF VTX boards

Cross talk is minimal (0.8% amplitude), and is easily rejected in software by cutting on pulse width

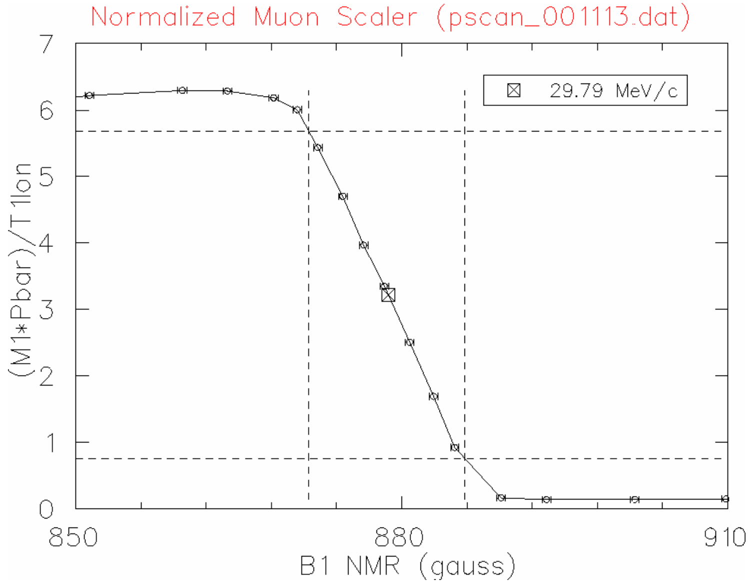
# TWIST – Positron Background

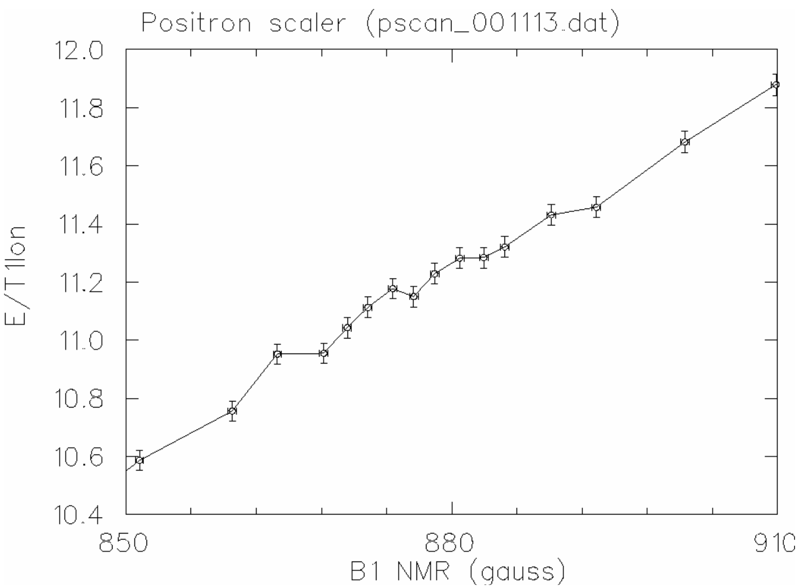
Beamline studies from October/November 2000

- Backgrounds
  - Rates: $e^+/\mu^+ \sim 4$

(as expected)

A pyrolytic graphite target will give us a 33% improvement in the rate relative to the positrons



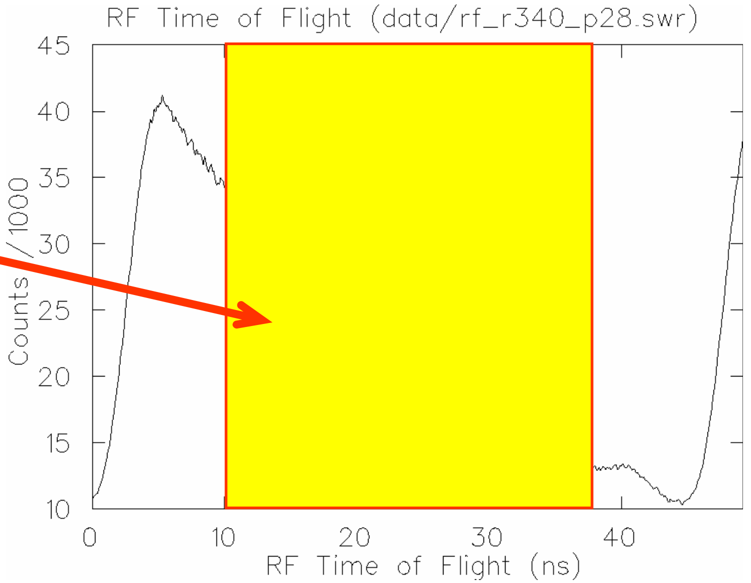



# *TWIST* – RF Cuts

Flight time through beamline

Surface Muons gated on cyclotron RF

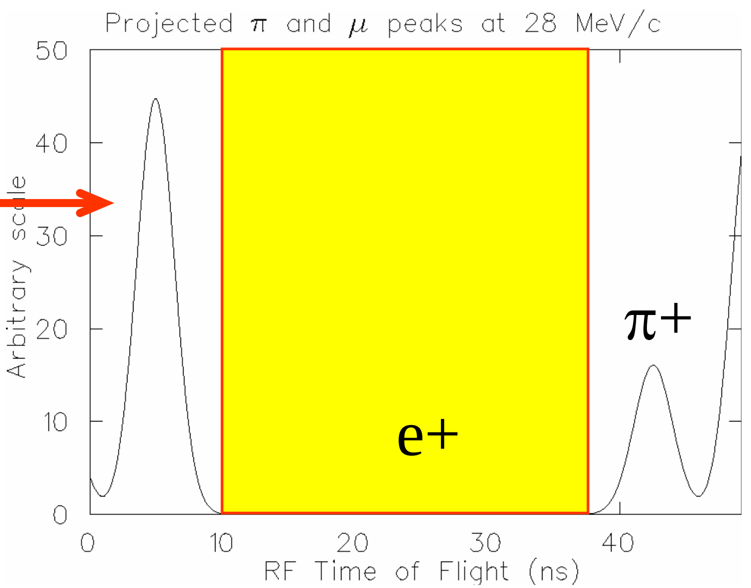
Time characteristic of π decay




Backgrounds (extrapolated from higher momentum)

Cloud Muons

Rate: 9% that of surface muons



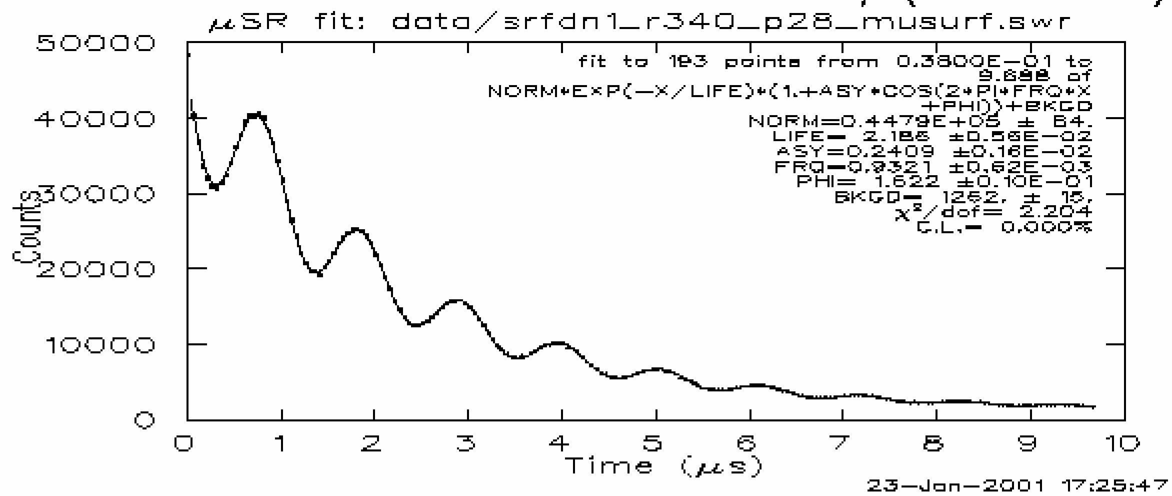
# *TWIST* – Cloud muon polarization

Surface muons

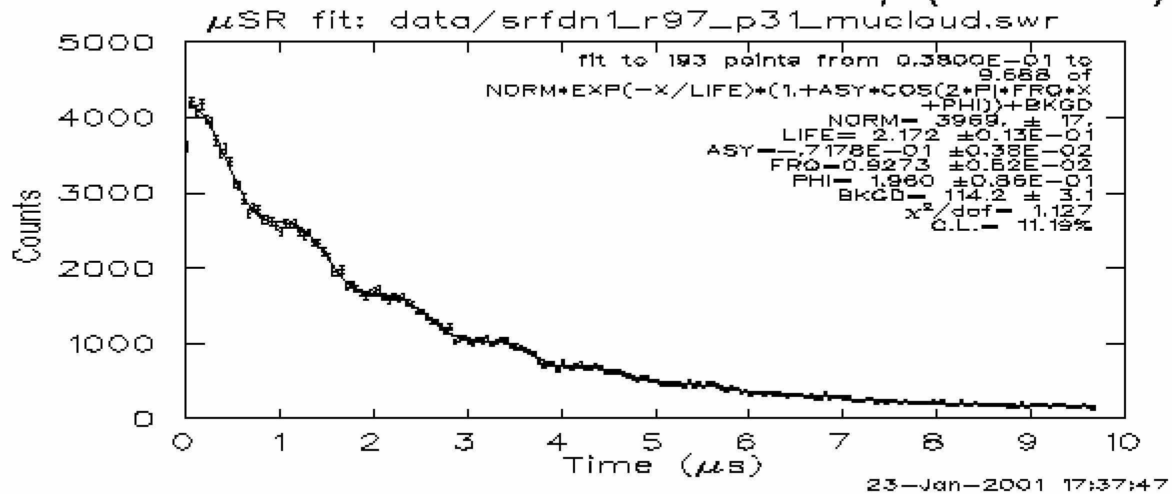
Polarization of the cloud muons is approximately 0.30 (opposite to the surface muon polarization of –1.0)

Cloud muon flux is 9% that of the surface muons

**28 MeV/c (Surface Muons)**




**31 MeV/c (Cloud Muons)**

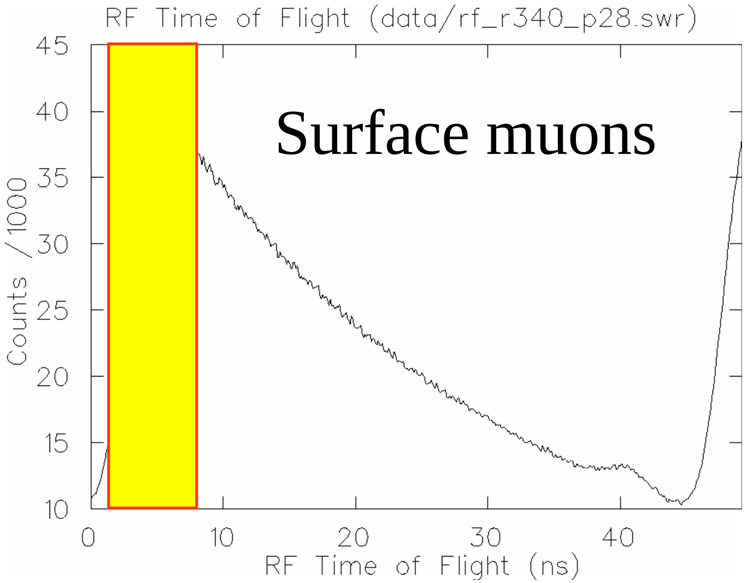


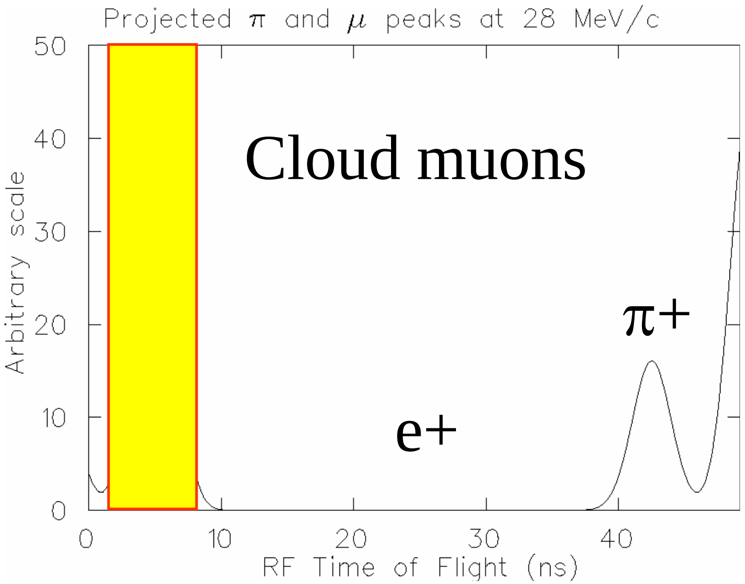

(Rob MacDonald – MSc data)

# *TWIST* – Unpolarized data sample

Flight time through beamline

By selecting a data sample with an appropriate RF gate, we can select an unpolarized sample of muons

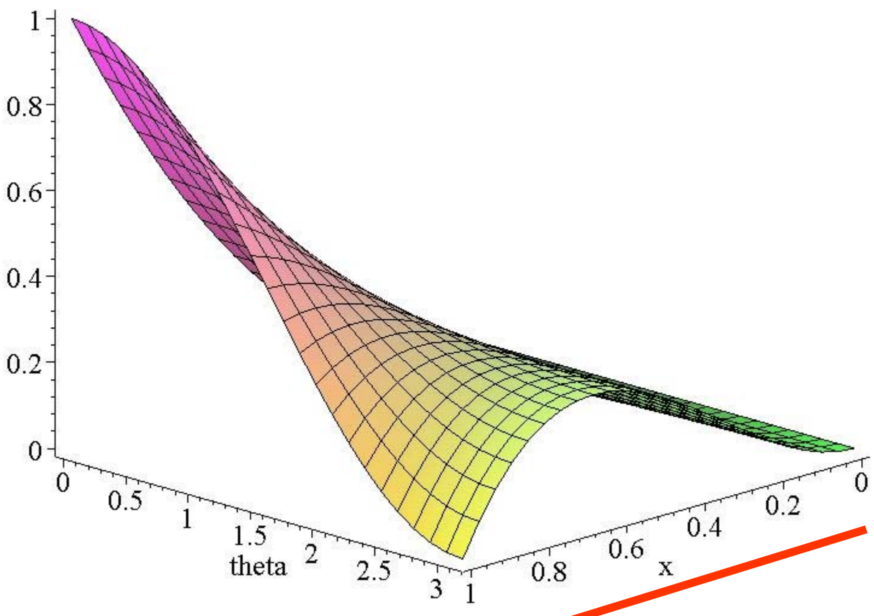


# TWIST – Energy Calibration

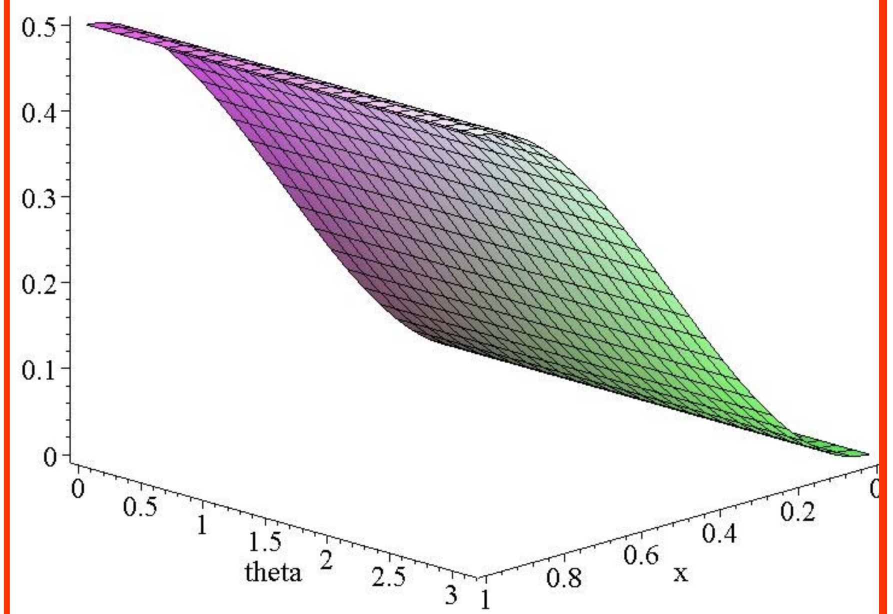
The edge of the distribution is used to calibrate the energy scale at large x

The polarized distribution has no edge at forward angles




52.8 MeV

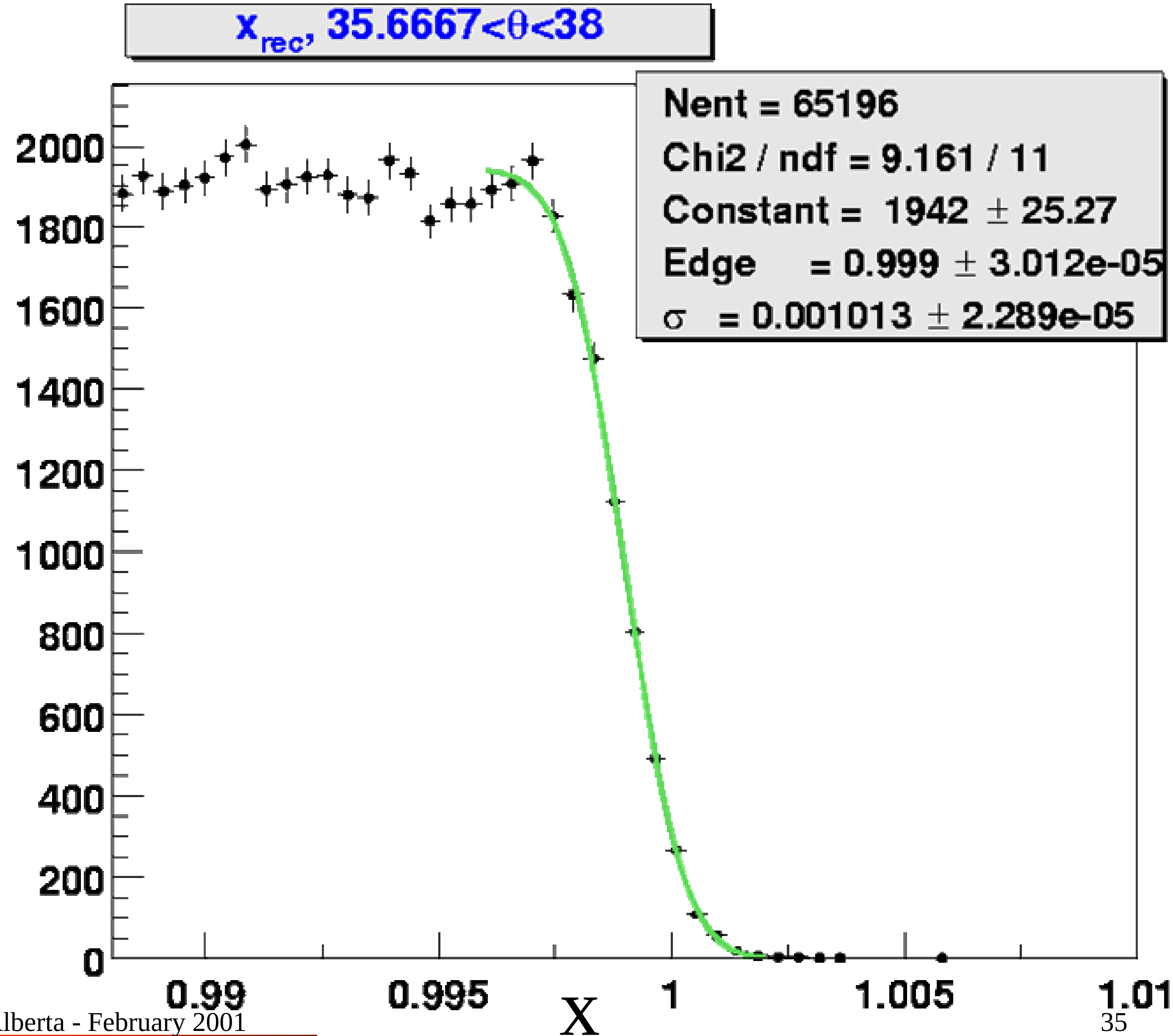
The unpolarized distribution will be used to calibrate the energy scale at all angles



# TWIST – Energy Calibration

Endpoint energy calibrations can be done to a precision of approximately 2 keV (where ~10 keV is needed).

Unpolarized beam will be used to provide energy calibrations independent of angle.

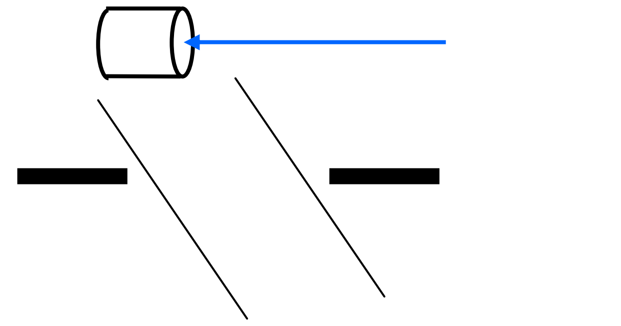


# *TWIST* – 1AT1 modifications

The surface muon beam is produced in part on the surface at which the protons enter, and in part along the length of the target cylinder.

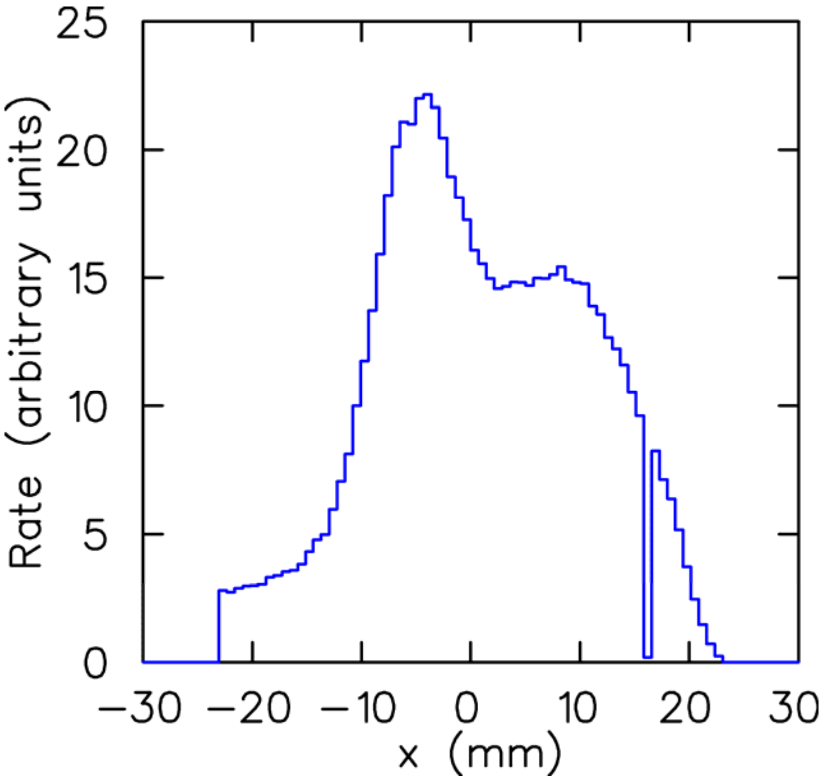
A shorter target would reduce the size of the beam spot

A hidden proton entry point would reduce sensitivity to wander in the proton beam

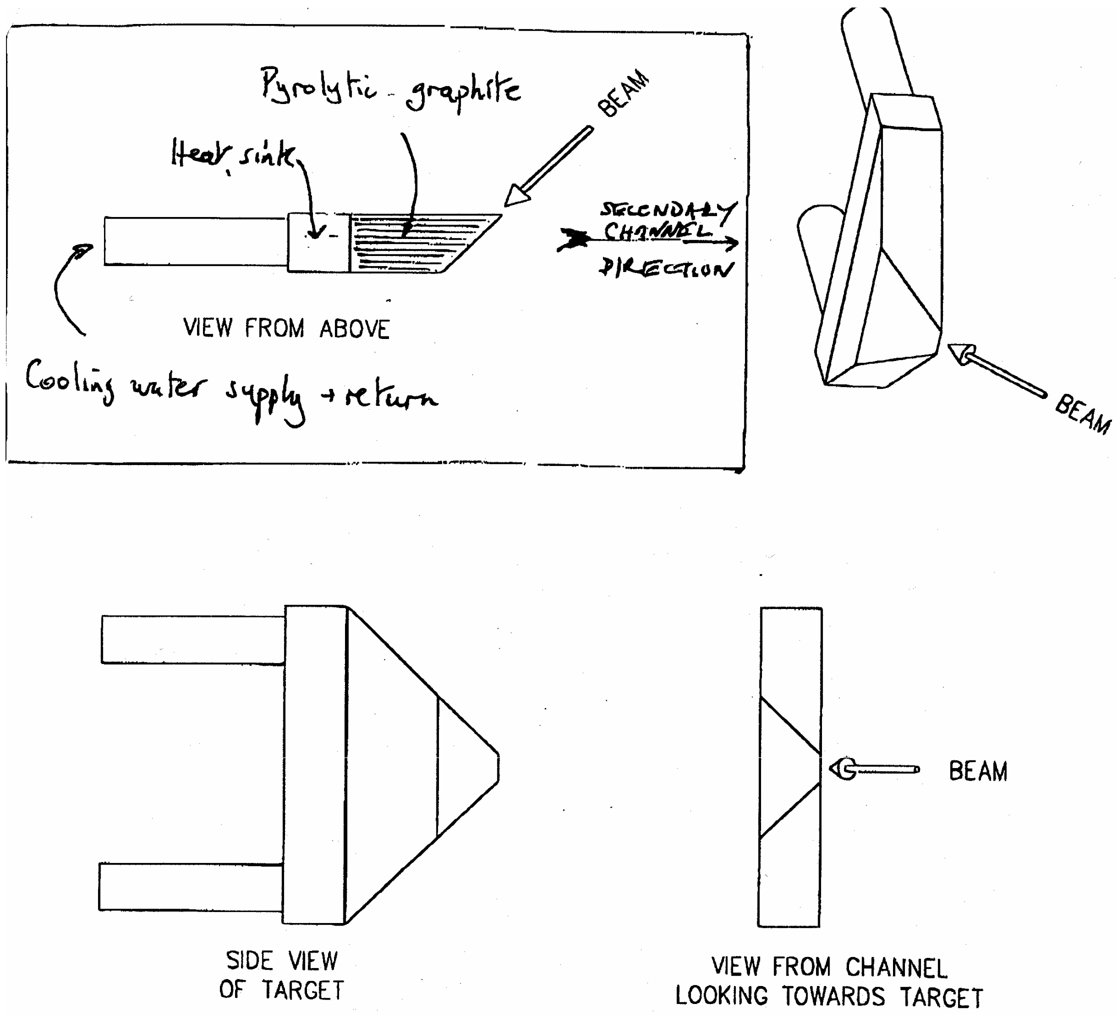



1AT1 target as imaged by M13

Surface muons



# *TWIST* – Modified target



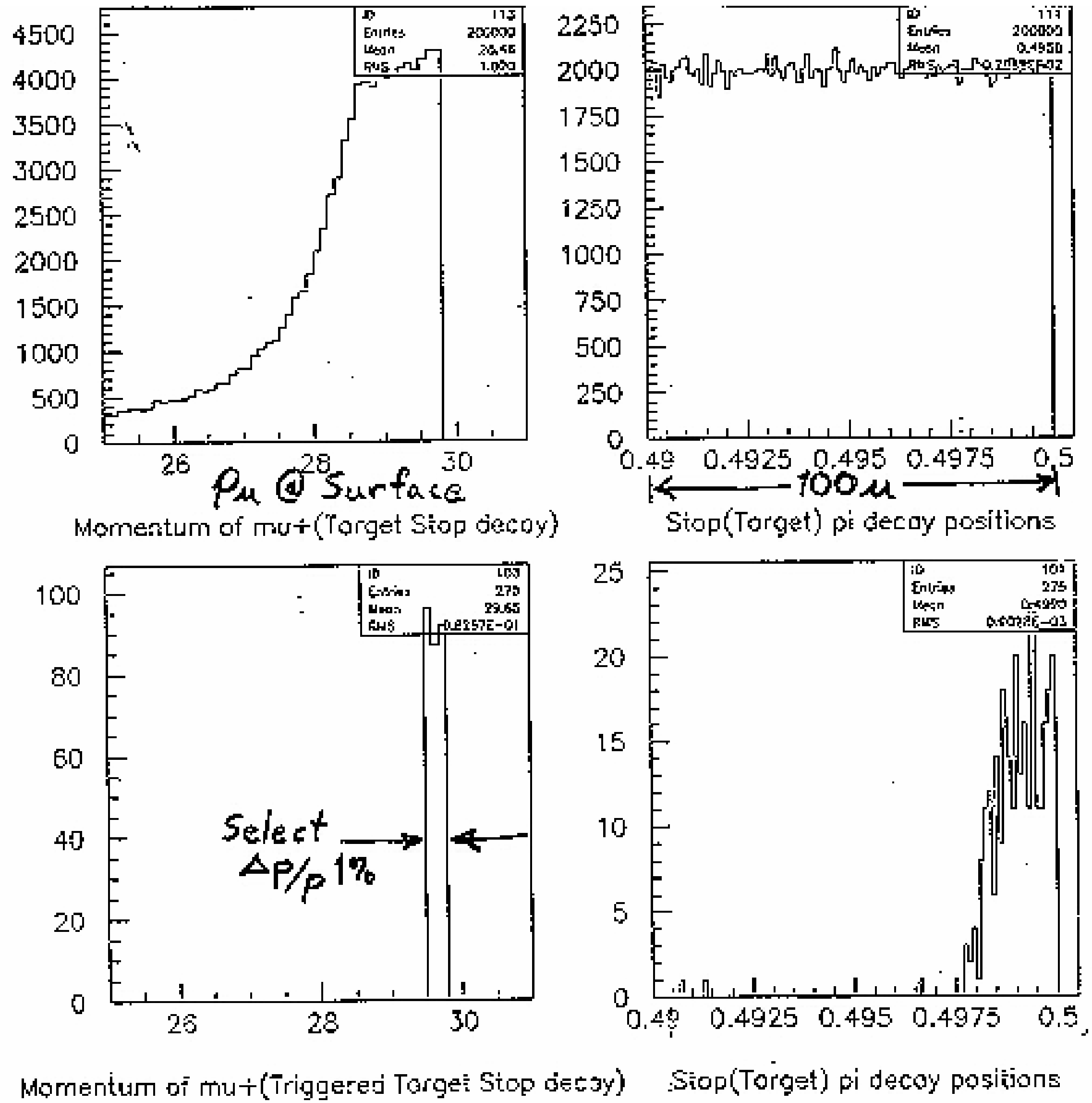
# *TWIST* – Goals

| | | |
|---|---|---|
| **Summer 2001** | **-** | **Commissioning data. Preliminary alignments and calibrations** |
| **End of 2001** | **-** | **Michel distributions on tape suitable for preliminary determination of $\rho$ and $\delta$** |
| **2002** | **-** | **Installation of the TEC** |
| | **-** | **Modified production target** |
| | **-** | **Beamline improvements, including realignments** |
| | **-** | **Improved Michel distributions based upon experience with alignments and calibrations** |
| | **-** | **Field alignment studies** |
| **2003** | **-** | **Studies of depolarization in the stopping target** |
| | **-** | **Preliminary $P_\mu\xi$ data** |
| | **-** | **Precision measurements of $\rho$, $\delta$ and $\eta$** |

# *TWIST* – 1AT1 surface selection

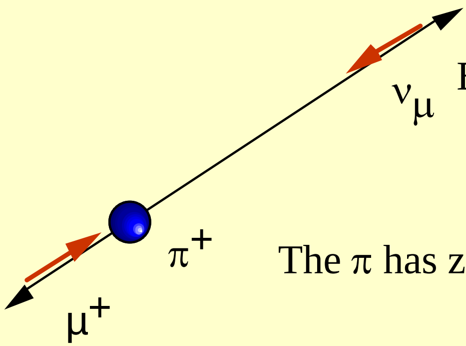
Δp/p of 1% selects muons from within about 20 microns of the surface.

These muons have limited multiple scattering, and little depolarization



# Secondary beams at TRIUMF

μ polarization due to 2-body decay

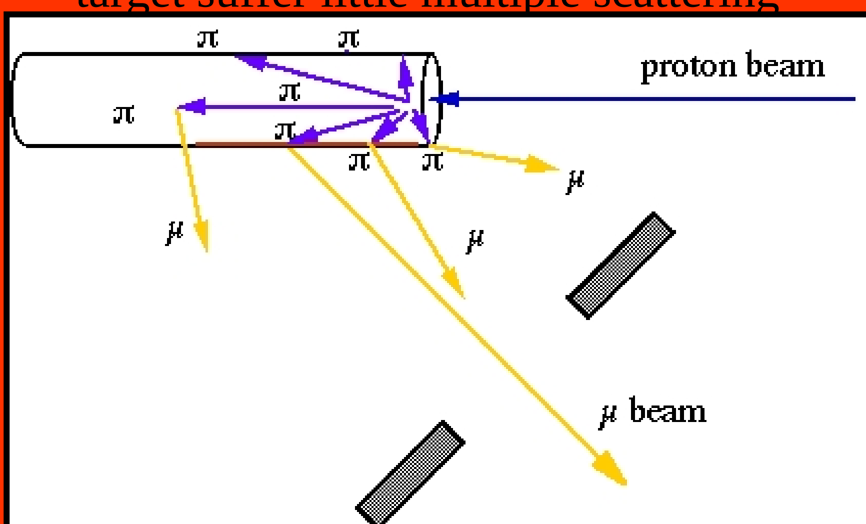



Back to back to conserve linear momentum

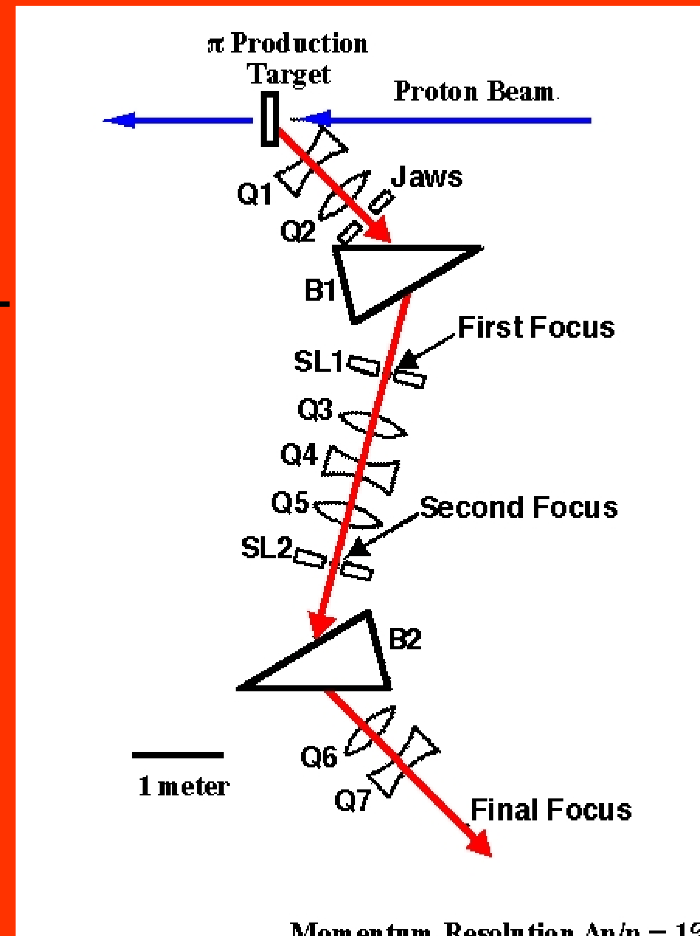
The π has zero angular momentum

=> no angular momentum in the final system

μ selected from the surface of the production target suffer little multiple scattering




Channel resolution ~ 1% allows selection of μ produced within 25 microns of target surface



# TWIST – average energy loss

Planar chambers give us a simple dependence of energy loss on 1/cos(theta).

Each successive curve is the result of a track fit using only four successive chambers.

The difference between successive curves demonstrates the small incremental energy loss per plane of ~10 keV at 0 degrees

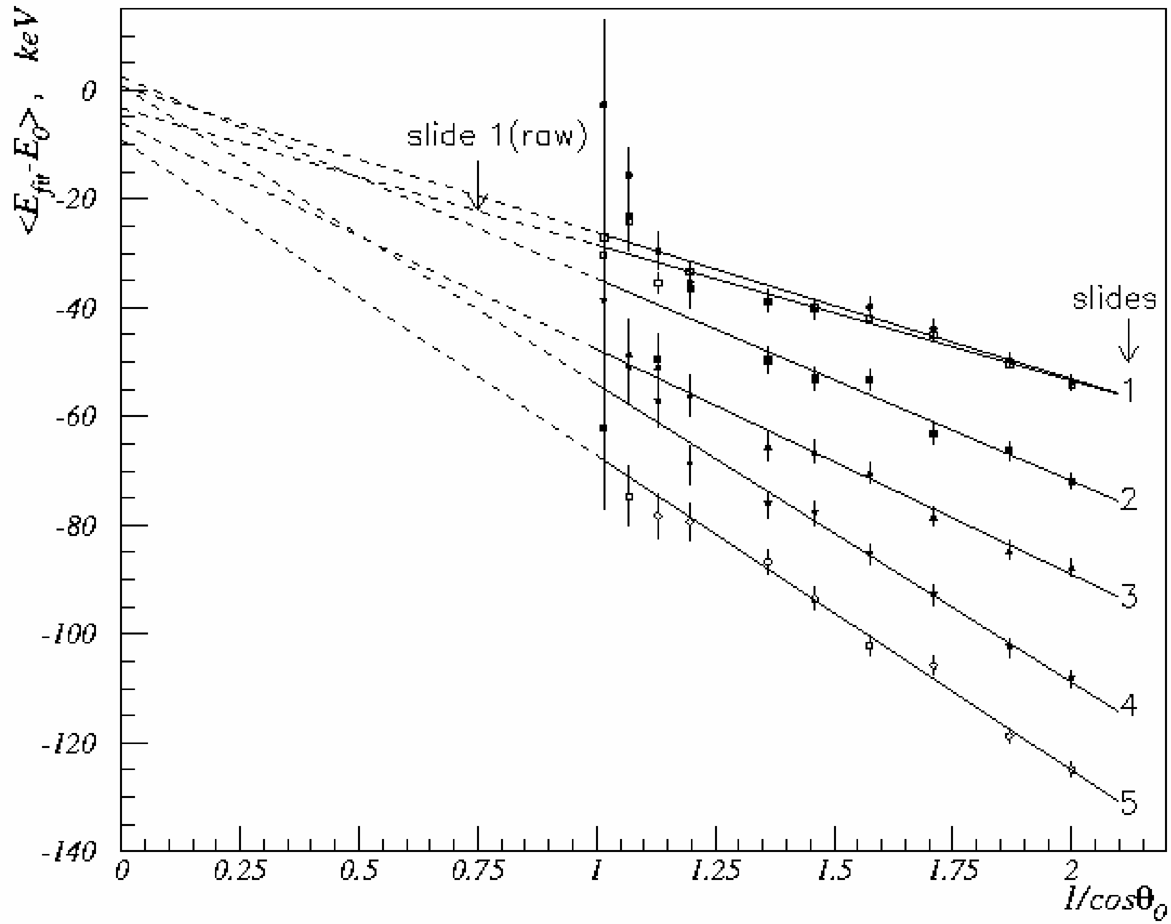
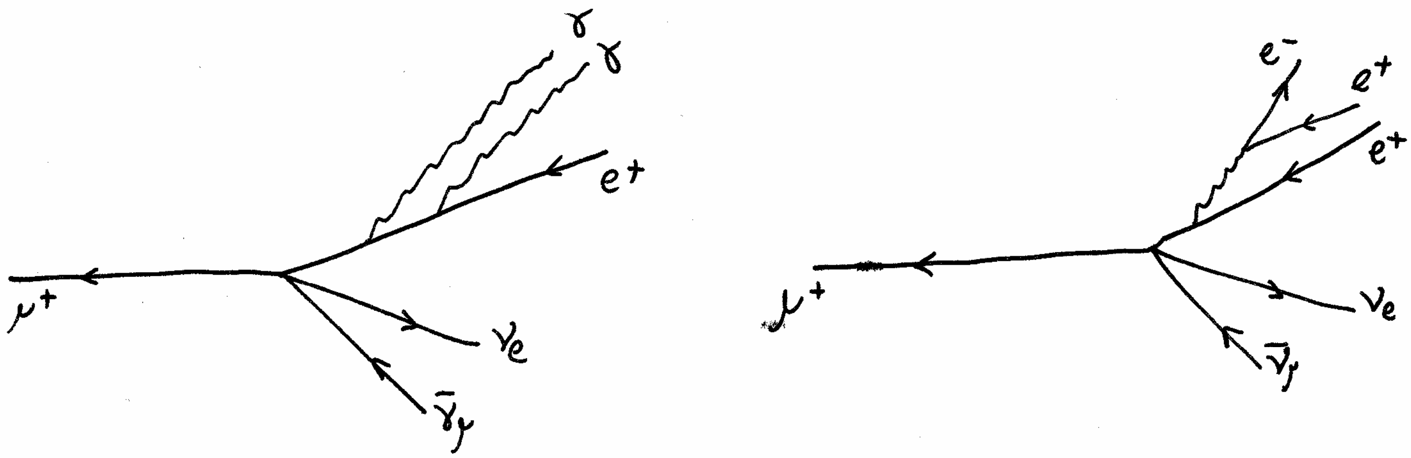



Fig. 9. Mean reconstructed positron energy $\overline{E_{fit}}$ as a function of $1/\cos\theta$. $E_0$=50 MeV, $\sigma_{PDC}$=50 $\mu$m. Straight lines are the fits with function $\overline{E_{fit}} = E_0 - \alpha/\cos\theta$.

# *TWIST* – Radiative corrections

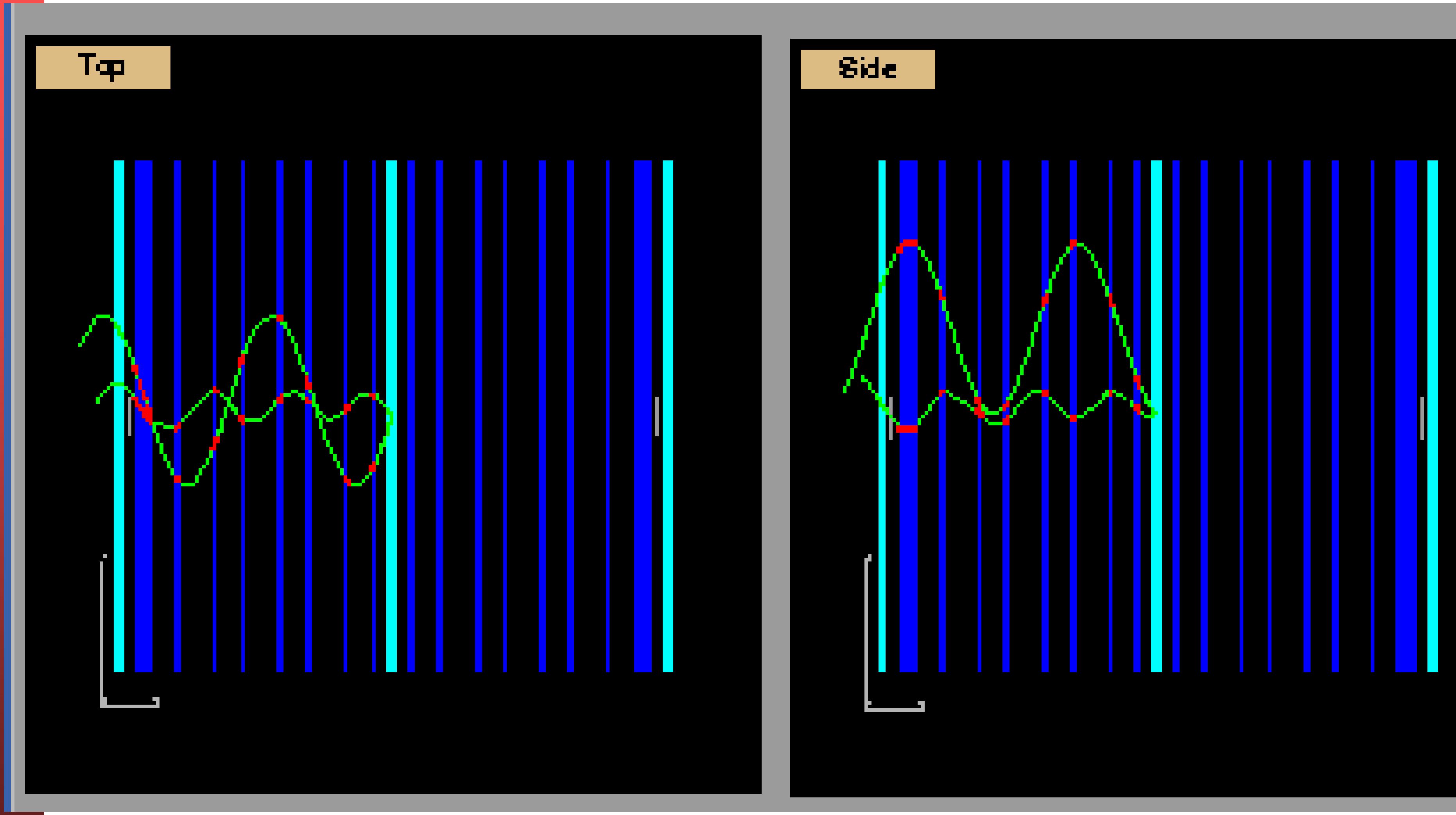



Diagrams responsible for the double-logarithmic $O\left(\frac{\alpha^2}{\pi^2}\ln^2\frac{m_\mu}{m_e}\right)$ corrections to the $e^+$ spectrum.

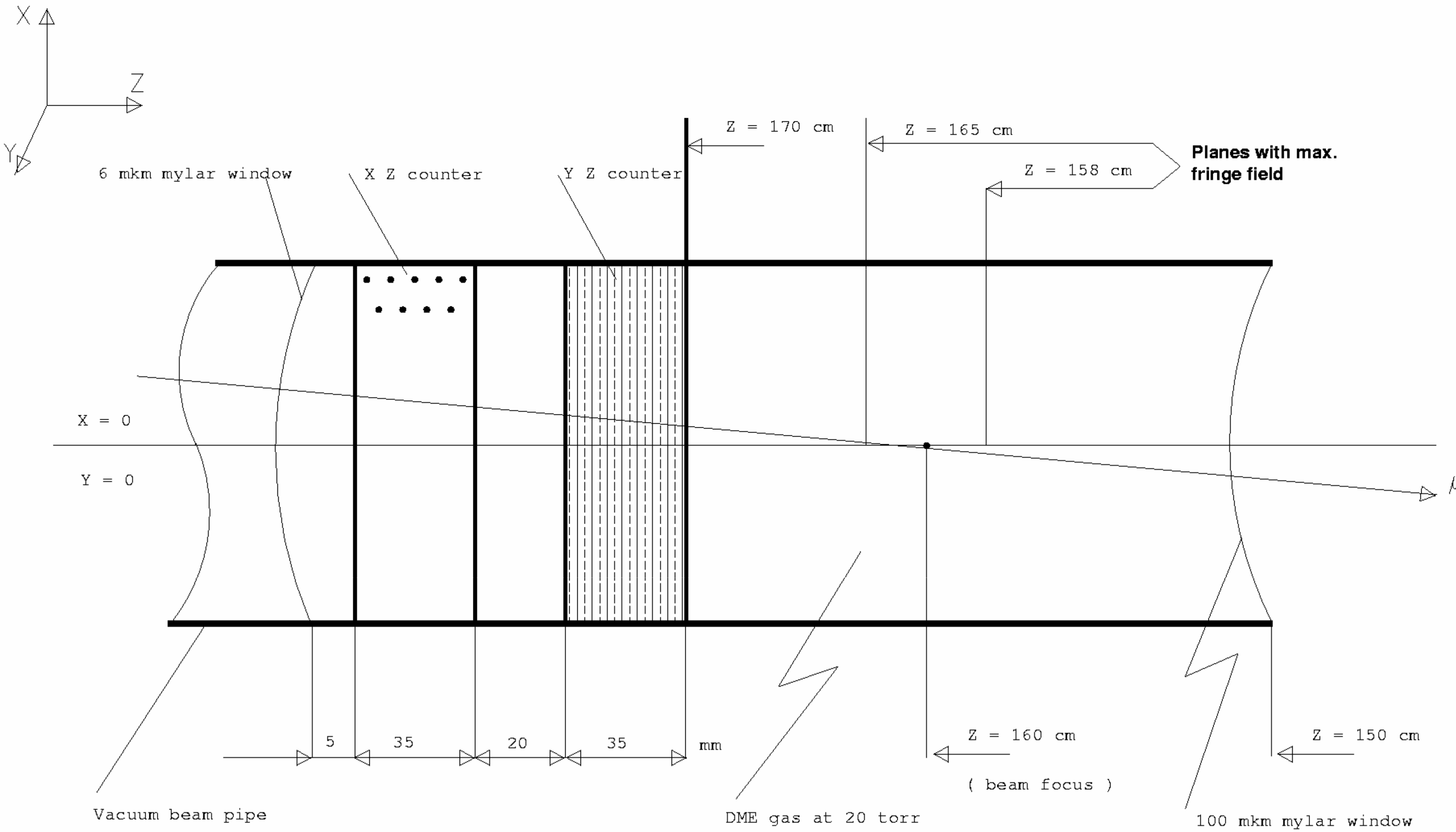
These diagrams have recently been calculated by Czarnecki and Arbusov (Alberta)

# Happy physicist with magnet in hand

# TWIST – Event Display



# TWIST – TEC Design Concept



# TWIST – TEC Design Concept

The TEC has been part of our planning since June 1998

Installation planned for Spring 2002

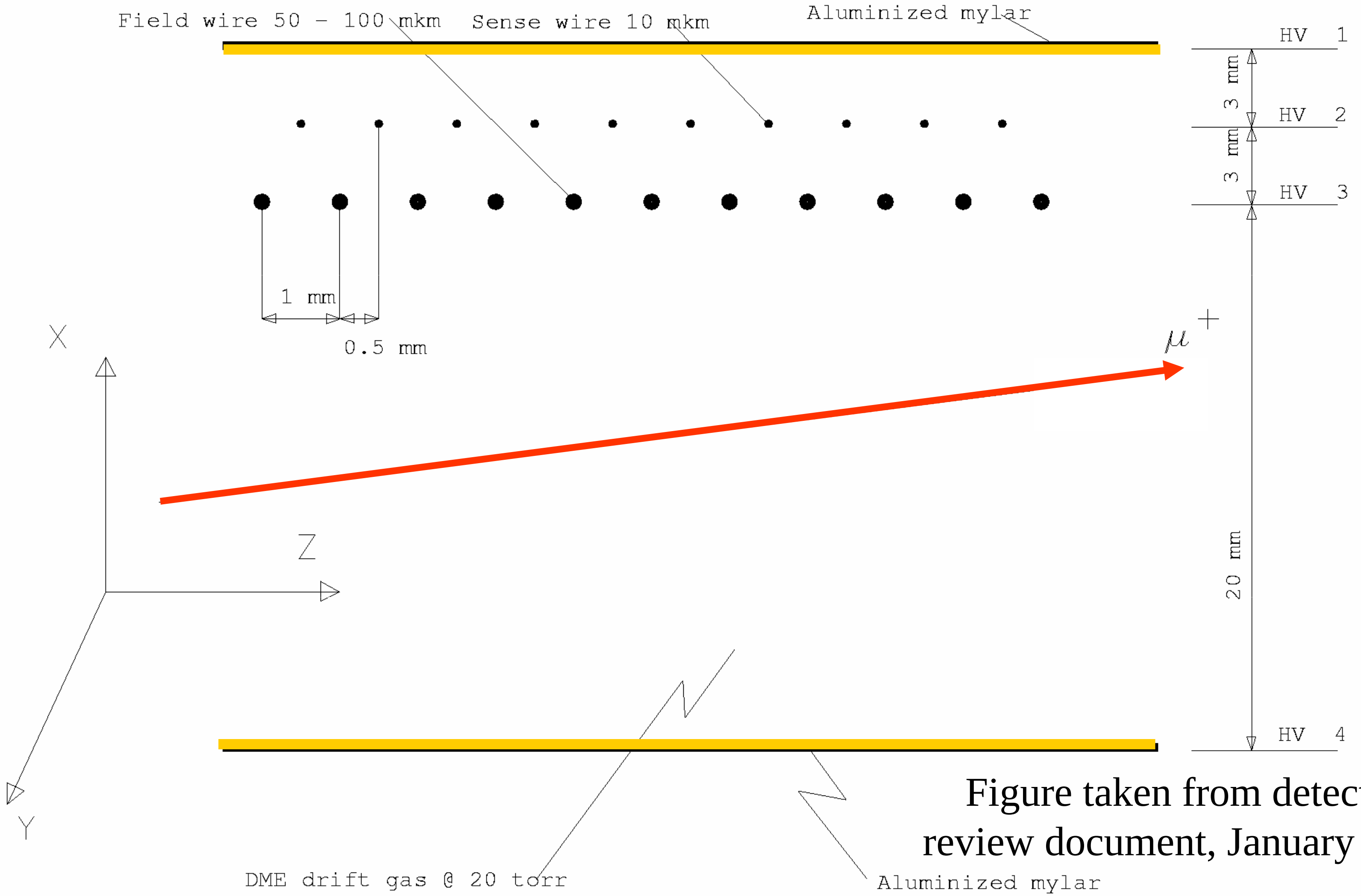
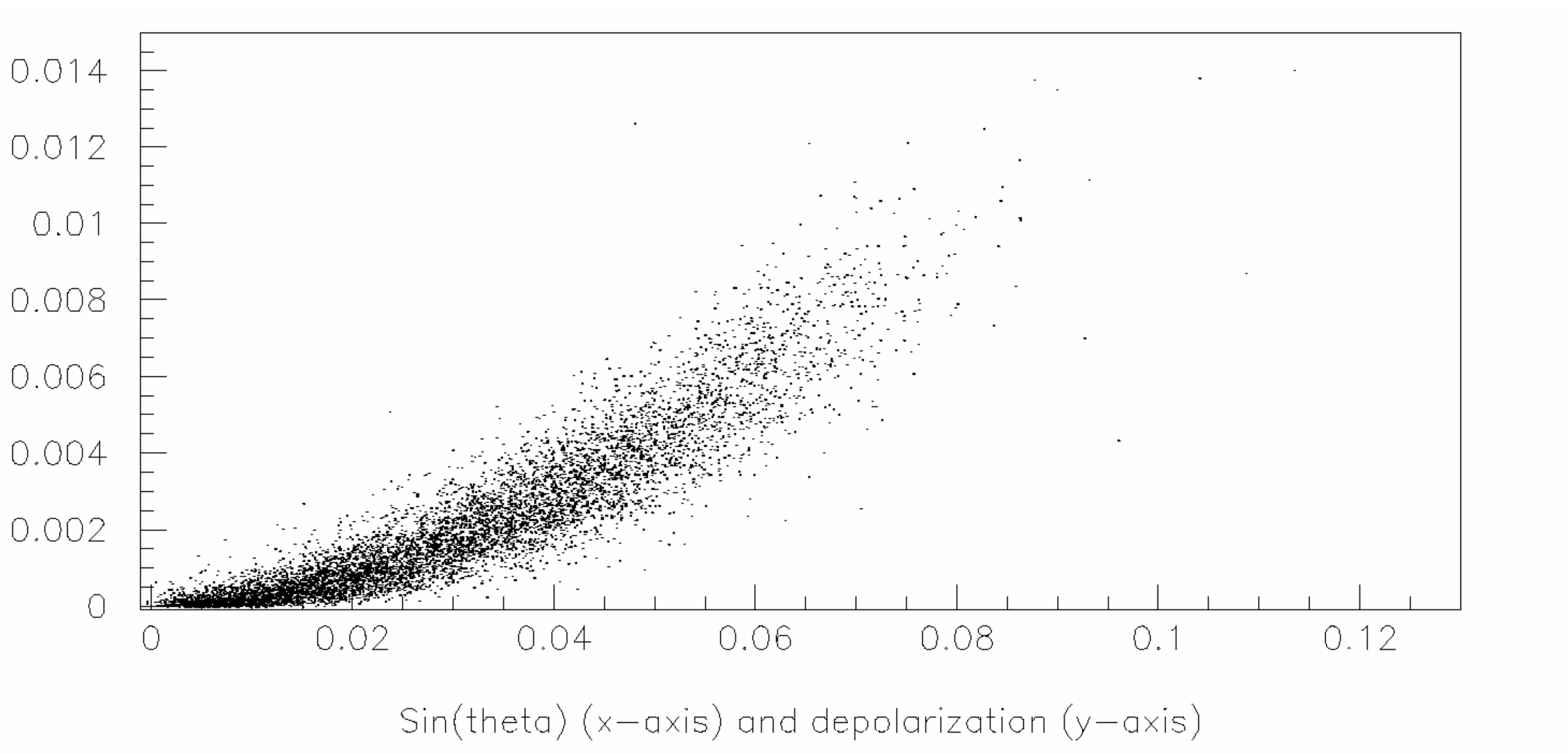



Figure taken from detector review document, January 1999

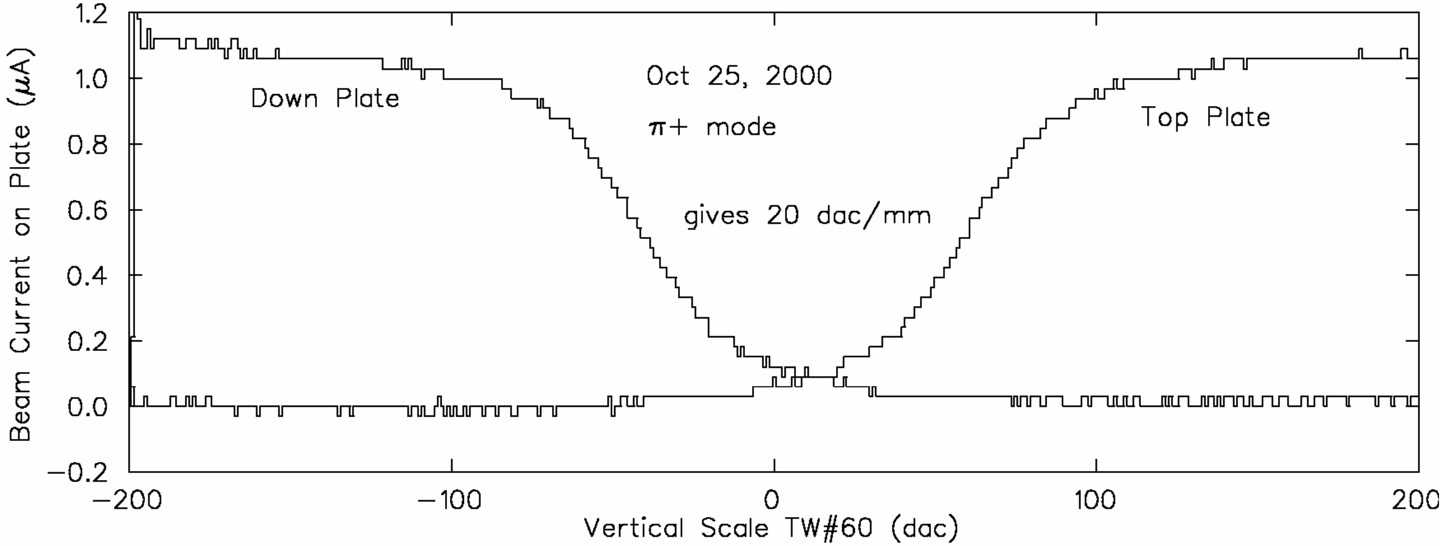
# *TWIST* – TEC Projected Performance

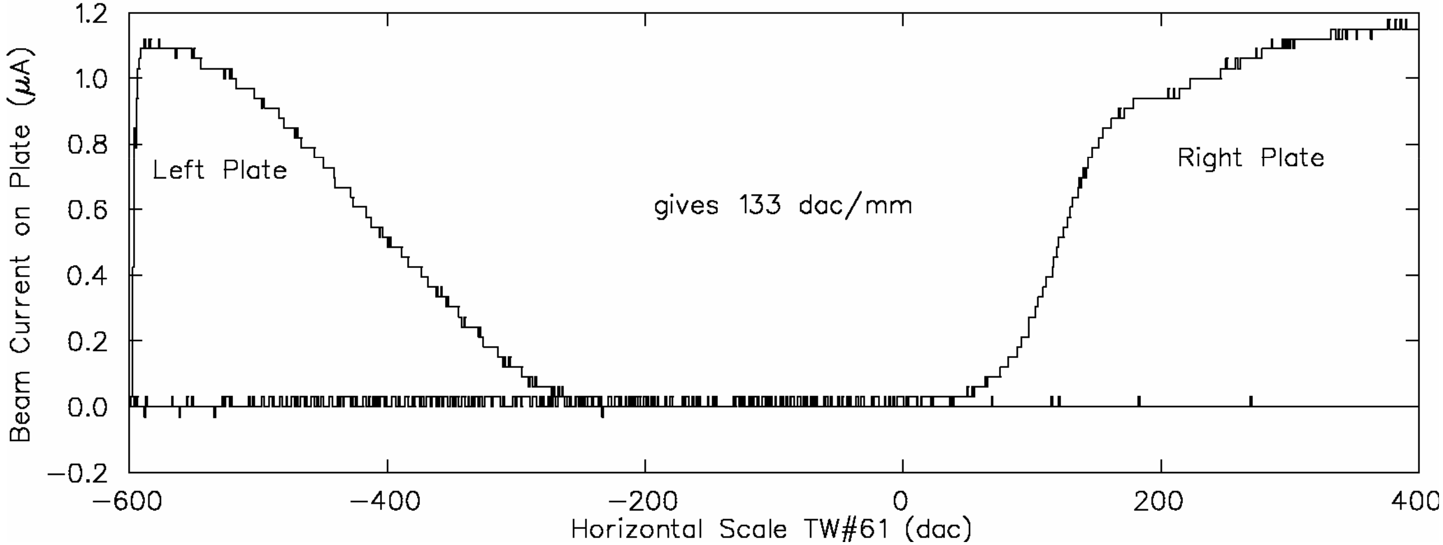
Effective Depolarization vs. TEC Tracking Angle




Correlation relies upon a highly convergent tune, focused at the peak in the radial fringe field

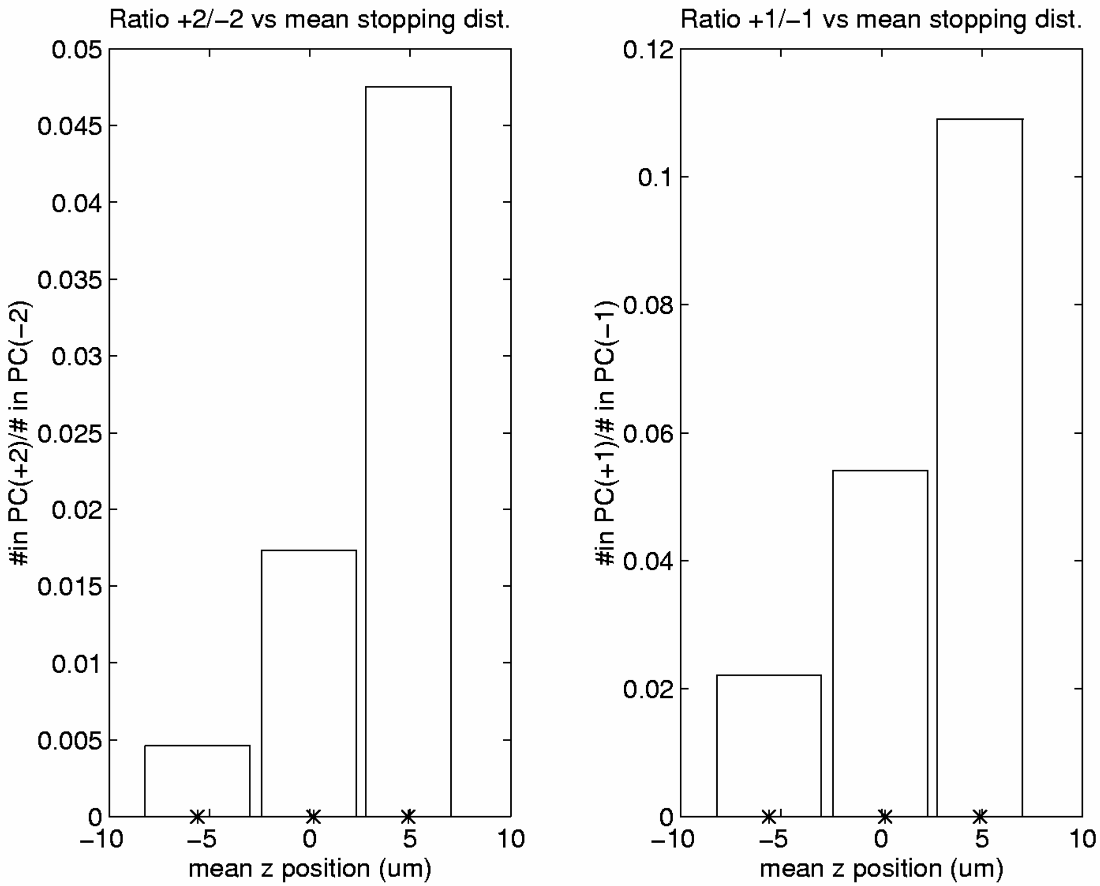
# TWIST – Proton Beam Monitor



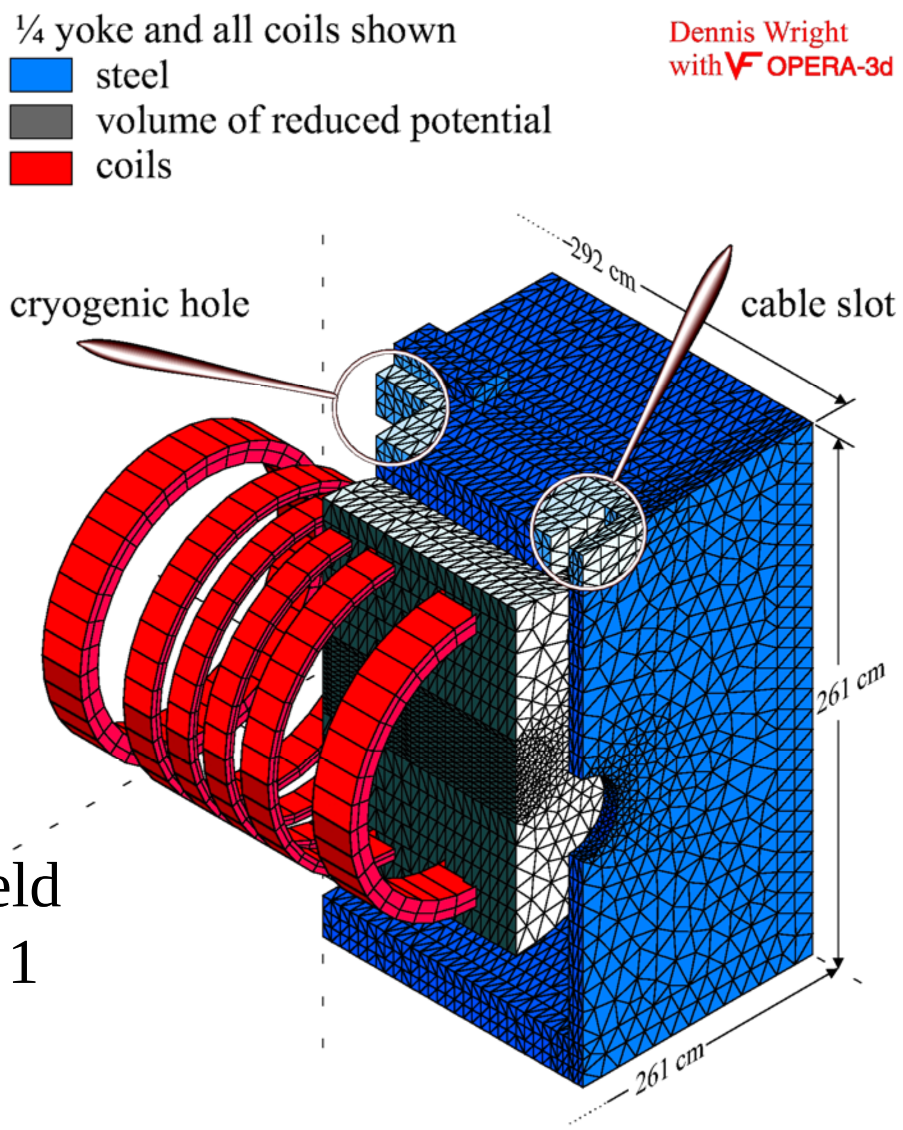



# *TWIST* – stopping distributions

Signal ratios in the target PC's can be used to monitor the stopping distribution



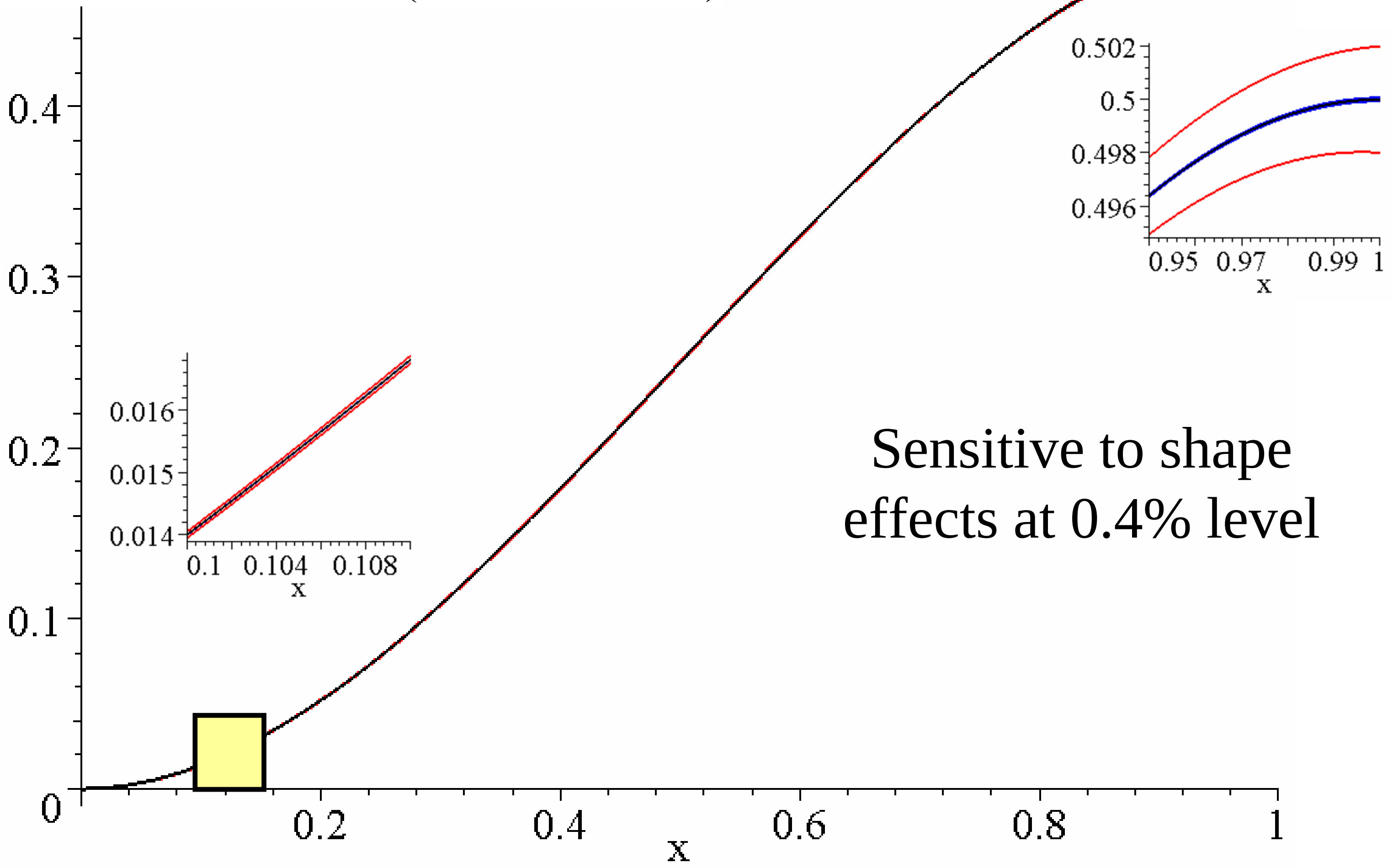
# TWIST – Field Calculations




Anticipated field uniformity to 1 part in $10^4$

# Unpolarized distribution in x

$$[Forward + Backward] \propto 2x^2\left(3(1-x) + \frac{2}{3}\rho(4x-3)\right)$$




Sensitive to shape effects at 0.4% level

# Michel Parameters - defined

$$\rho \equiv \frac{3}{4}\left[|g_{LL}^{V}|^2 + |g_{RR}^{V}|^2 + |g_{LR}^{T}|^2 + |g_{RL}^{T}|^2\right]$$
$$+ \frac{3}{16}\left[|g_{LL}^{S}|^2 + |g_{RR}^{S}|^2 + |g_{LR}^{S}|^2 + |g_{RL}^{S}|^2\right]$$
$$- \frac{3}{4}\left[\mathrm{Re}\left(g_{LR}^{S} g_{LR}^{T*}\right) + \mathrm{Re}\left(g_{RL}^{S} g_{RL}^{T*}\right)\right]$$

$$\xi \equiv |g_{LL}^{V}|^2 + 3|g_{LR}^{V}|^2 - 3|g_{RL}^{V}|^2 - |g_{RR}^{V}|^2 + 5|g_{LR}^{T}|^2$$
$$- 5|g_{RL}^{T}|^2 + \frac{1}{4}|g_{LL}^{S}|^2 - \frac{1}{4}|g_{LR}^{S}|^2 + \frac{1}{4}|g_{RL}^{S}|^2 - \frac{1}{4}|g_{RR}^{S}|^2$$
$$+ 4\,\mathrm{Re}(g_{LR}^{S} g_{LR}^{T*}) - 4\,\mathrm{Re}(g_{RL}^{S} g_{RL}^{T*})$$

$$\xi\delta \equiv \frac{3}{4}\left[|g_{LL}^{V}|^2 - |g_{RR}^{V}|^2 - |g_{LR}^{T}|^2 + |g_{RL}^{T}|^2\right]$$
$$+ \frac{3}{16}\left[|g_{LL}^{S}|^2 - |g_{RR}^{S}|^2 - |g_{LR}^{S}|^2 + |g_{RL}^{S}|^2\right]$$
$$- \frac{3}{4}\left[\mathrm{Re}(g_{LR}^{S} g_{LR}^{T*}) - \mathrm{Re}(g_{RL}^{S} g_{RL}^{T*})\right]$$